AN AUTOMATED SOLUTION FOR LINEAR ALGEBRAIC SYSTEMS

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CHAPTER ONE

1.0 BACKGROUND TO THE STUDY

1.1 INTRODUCTION

The study of systems of linear equations and their solutions is one of the major topics and plays an important role in linear Algebra. In recent years, linear Algebra has become an essential part of the mathematical background required of mathematicians, engineers, physicists and other scientists. This requirement reflects the importance and wide applications of the subject matter.

However, in practical application of linear equations, the need for automated procedures is very necessary and important. Some methods for performing computations are directly applicable to the computer solution of large – scale problems that arise in real – world application, while some methods are not directly applicable. On the basis of this, we shall limit ourselves to those methods that are applicable and consider the methods of solutions to systems of linear equation using Gauss-Jordan elimination methods and Gaussian elimination methods with back-substitution.

1.1.1 LINEAR EQUATION

We can represent a line in the xy-plane algebraically by an equation of the form:

$$a_1x + a_2y = b.$$

An equation of this kind is called a linear equation in the variables x and y. In general terms, we define a linear equation in the n variables $x_1, x_2, \ldots\ldots x_n$ to be one that can be expressed in the form.

$$a_1x_1 + a_2x_2 + \ldots\ldots + a_nx_n = b \text{ ----- } (1)$$

where the scalars $a_1, a_2 \ldots\ldots a_n$ (called the coefficient of the x_i's) and b are constants. The variables (Indeterminates) in a linear equation are also called unknowns.

A solution of a linear equation (1) is a sequence of n numbers $S_1, S_2, \ldots, S_n$ such that the equation is satisfied when we substitute $x_1 = s_1, x_2 = s_2, \ldots\ldots, x_n = s_n$. The set of all solutions of the equation is called the general solution of the equation.

Solutions of the equation (1) can be easily described and obtained. There are three cases:

CASE 1 one of the coefficients in (1) is not zero, say $a_1 \neq 0$

If we rewrite equation (1) as follows

$$a_1x_1 = b - a_2x_2 - \ldots\ldots\ldots\ldots - a_nx_n$$

$$x_1 = a_1^{-1}b - a_1^{-1}a_2x_2 - \ldots - a_1^{-1}a_n x_n$$

Then by arbitrarily assigning values to the unknowns $x_2, \ldots, x_n$ we obtain a value for x_i. These values form a solution of the equation.

However, the linear equation in one unknown

$$ax = b \text{ with } a \neq 0$$

Has unique solution $x = a^{-1}b$.

CASE (ii) All the coefficients in (1) are zero, but the constant is not zero. Thus, the equation is of the form

$$ox_1 + ox_2 + \ldots + ox_n = b \qquad \text{with } b \neq 0,$$

then the equation has no solution.

CASE (iii) All the coefficients in (1) are zero and the constant is also zero, that is, the equation is of the form

$$ox_1 + ox_2 + \ldots + ox_n = o$$

Then every n – tupple of scalars in R is a solution of the equation.

1.1.2 LINEAR SYSTEMS:

A finite set of linear equations in the variables $x_1, x_2, \ldots x_n$ is called a system of linear equations or simply a linear system.

1.1.3 SYSTEM OF LINEAR EQUATIONS

An arbitrary system of m linear equation in n unknowns will be written as

$$a_{11}x_1 + a_{12}x_2 + \dots\dots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots\dots + a_{2n}x_n = b_2$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \qquad \dots(2)$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$a_{m1}x_1 + a_{m2}x_2 + \dots\dots + a_{mn}x_m = b_m$$

where $x_1, x_2, \dots, x_n$ are the unknowns and the subscripted a's and b's(i.e a_{ij} and b_i) denote constants,and belongs to the real field R.

A sequence of numbers $s_1, s_2, \dots, s_n$ is called a solution of the system if $x_1 = s_1, \quad x_2 = s_2 \quad \dots\dots\dots, x_n = s_n$ is a solution (or particular solution) of every equation in the system. The set of all such solutions is termed the solution set or the general solution.

Not all systems of linear equations have solutions. Every system of linear equation has either no solution, exactly one solution or infinitely many solutions.

A system of equations that has no solutions is said to be **inconsistent**. If there is at least one solution, it is **consistent**.

The system (2) above is said to be homogeneous if the constants $b_1, b_2, \dots, b_m$ are all zero . i.e

$$a_{11}x_1 + a_{12}x_n \dots\dots + a_{1n}x_n = 0$$

$$a_{21}x_1 + a_{22}x_2 + \dots\dots + a_{2n}x_n = 0$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \qquad ..(3)$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$a_{m1}x_1 + a_{m2}x_2 + \dots\dots + a_{mn}x_n = 0$$

The above system (3) always has a solution, namely the zero n-tuple 0= (0,0...,0) called the zero or trivial solution .Any other solution, if it exists, is called a nonzero or nontrivial solution.

1.8 DEFINITION OF TERMS

ROUNDOFF ERROR

Since computers are limited in the number of decimal places they can carry, they roundoff or truncate most numerical quantities. A computer designed to store eight decimal places might record 2/3 either as .66666667 (roundedoff) or .666666 (Truncated). In either case, an error is introduced and is called roundoff error.

AUTOMATION

The oxford-advanced learners Dictionary define this as the use of automatic machines and equipments to do work previously done by people.

CHAPTER TWO

2.0 LITERATURE REVIEW

2.1 SOLUTION OF A SYSTEM OF LINEAR EQUATIONS.

As already stated, an arbitrary system of m linear equation in n unknowns will be written as

$$a_{11}x_1 + a_{12}x_2 + \ldots\ldots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \ldots\ldots\ldots + a_{2n}x_n = b_2$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad \ldots\ldots 2.1.1$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$a_{m1}x_1 + a_{m2}x_2 + \ldots\ldots + a_{mn}x_n = b_m$$

where x_1, x_2, ... x_n are the unknowns and the subscripted a's and b'sdenotes constants.

Considering the above systems of linear equation, the basic method for solving a system like this is to replace the given system by a new system that has the same solution set but which is easier to solve. This new system is generally obtained in a series of steps by applying the following three types of operations to eliminate unknowns systematically.

(1) Multiply an equation through by a nonzero constant

(2) Interchange two equations

(3) Add a multiple of one equation to another.

This procedure can also be stated as follows

To reduce the system of linear equation to a simpler system follow the following procedure

Step 1 interchange equations so that the first unknown x1 has a nonzero coefficient in the first equation, that is , so that $a_{11} \neq 0$

Step 2 for each $i > 1$, apply the operation

Thus the original system has been reduced to the following equivalent system

$$2x + 4y - z + 2v + 2w = 1$$
$$5z - 8v + 2w = -17$$
$$3z + v - 5w = 1$$

Observed that y has also been eliminated from the second and third equations . Here the unknown Z plays the role of the unknown X_{j2} above.

Note that the above equations, excluding the first , form a sub system which has fewer equations and fewer unknowns than the original system (equation 1).

We also note that

(1) If an equation $0x_1 + 0x_2 + \dots\dots\dots + 0x_n = b$, $b \neq 0$ occurs then the system is inconsistent and has no solution.

(2) If an equation $0x_1 + ox_2 + \dots\dots\dots\dots + 0x_n = 0$ occurs, then the equation can be deleted without affecting the solution.

Continuing the above process with each new "smaller" subsystem, we obtain by induction that the system (equation 1) is either inconsistent or is reducible to an equivalent system in the following form

$$
\begin{aligned}
& a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \dots\dots + a_{1n}x_n + = b_1 \\
& \quad a_{2j2}x_{j2} + a_{2ij2+1}x_{j2+1} + \dots\dots + a_{2n}x_n = b_2 \\
& \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \qquad (2.3) \\
& \quad a_{rjr}x_{jr} + a_{rj}j_{r+1} + \dots\dots + a_{rn}x_n = b_r
\end{aligned}
$$

where $1 < j_2 < \dots\dots < j_r$ and where the leading coefficients are not zero: $a_{11} \neq 0$, $a_{2j2} \neq 0 \dots a_{rjr} \neq 0$.

For notational convenience we use the same symbols a_{ij}, b_k in the system (2.3) as we used in the system (2.1) ,but clearly they may denote different scalars.

DEFINITION: The above system (2.3) is said to be in echelon form; the unknowns x_i which do not appear at the beginning of any equation $(i \neq 1, j_2, \dots\dots\dots, j_r)$ are termed free variables.

THEOREM: The solution of the system (2.3) in echelon form is as follows. There are two cases

(i) $r=n$ that is ,there are as many equations as unknowns. Then the system has a unique solution.

(ii) $r<n$. That is, there are fewer equations than unknowns. Then we can arbitrarily assign values to the $n-r$ free variables and obtain a solution of the system.

NOTE: The above theorem, in particular, implies that the system (2.3) and equivalent systems are consistent. Thus if the system (2.1) is consistent and reduces to case (ii) above, then we can assign many different values to the free variables and so obtain many solutions of the system. The following diagram illustrates this situation.

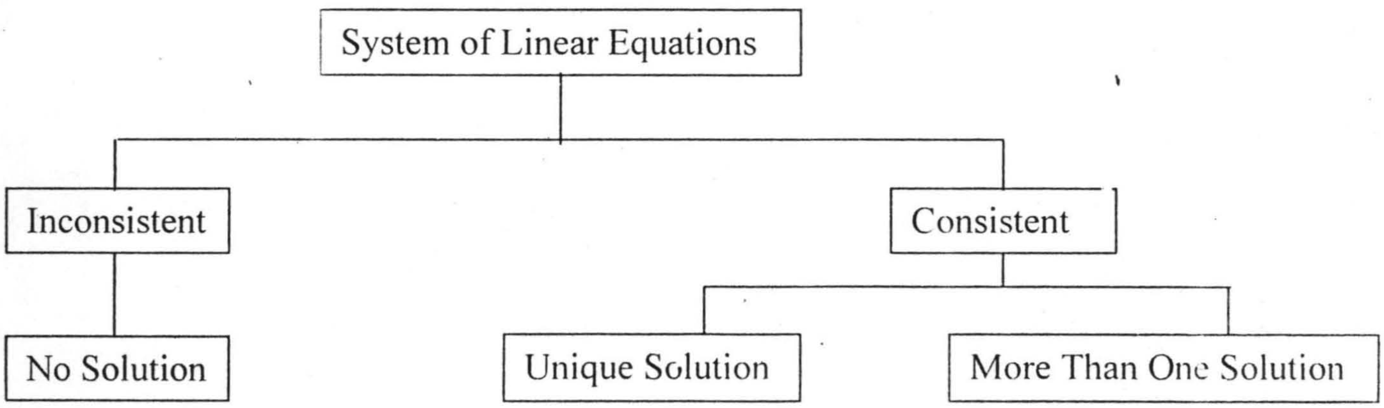

Figure 1

REMARK: We find the general solution of the system in the above example as follows:

Let the free variables be assigned arbitrary values

say $y=a$, and $w=b$

Substituting $w=b$ into the second equation we obtain

$z= 1+ 2b$

Putting $y=a$, $z=1+2b$ and $w=b$ in the first equation,

we find $x= 4 -2a+b$

Thus the general solution of the system is

$X= 4 - 2a +b$

$Y =a$

$Z = 1+2b$

$W = b$

In other words, $(4 - 2a+b, a, 1+2b, b)$, where a and b are arbitrary numbers. Frequently the general solution is left in terms of the free variables y and w (instead of a and b) as follows;

$X=4 - 2y + w$ $\quad$ $Z = 1+2w$ or $(4 - 2y + w, y, 1+2w, w)$

2.2 SOLUTION OF A HOMOGENEOUS SYSTEM OF LINEAR EQUATION

A system of linear equations is said to be homogeneous if all the constant terms are zero: that is the system is of the form:

$$a_{11}x_1+a_{12}x_2+ \ldots\ldots\ldots +a_{1n}x_n=0$$

$$a_{21}x_1+a_{22}x_2+ \ldots\ldots\ldots +a_{2n}x_n=0$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$a_{m1}x_1+a_{m2}x_2+\ldots\ldots\ldots +a_{mn}x_n= 0$$

Every homogeneous system of linear equations is consistent, since all such system have $x_1=0, x_2=0 \ldots\ldots\ldots\ldots x_n=0$ as a solution. This solution is called the trivial solution; if there are other solutions, they are called nontrivial solution.

Since homogeneous systems of linear equation is clearly consistent, it can always be reduced to an equivalent homogeneous system in echelon form:

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots a_{1n}x_n = 0$$

$$a_{2j2}x_{j2} + a_{21j2}+1x_{j2+1} + \ldots + a_{2n}x_n=0$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$a_{rjr}x_{jr} + a_{r1jr+1}x_{r+1} + \ldots + a_{rn}x_n=0$$

Here we have the two possibilities.

(i) $r=n$. Then the system has only the zero solution (trivial solution)

(ii) $r < n$. Then the system has a non-zero solution (non-trivial solution)

If we begin with fewer equations than unknowns then, in echelon form $r<n$ and hence the system has a nonzero solution, that is ,

THEOREM: A homogeneous system of linear equations with more unknowns than equations has a non-zero solution.

Simply stated, for a homogeneous system of linear equations, exactly one of the following is true:

(i) The system has only the trivial solution

(ii) The system has infinitely many non-trivial solutions in addition to the trivial solution.

For example, the homogeneous system

$$x + 2y - 3z + w = 0$$
$$x - 3y + z - 2w = 0$$
$$2x + y - 3z + 5w = 0$$

has a non zero solution since there are four unknowns but only three equations.

2.3 GAUSSIAN ELIMINATION

2.3.1 AUGMENTED MATRIX

We shall consider the general case of the system of linear equation, thus,

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$
$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$
$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m$$

This system of m linear equations in n unknowns can be abbreviated by writing only the rectangular array of numbers

$$\begin{bmatrix} a_{11} & a_{12} \dots\dots a_{1n} & b_1 \\ a_{21} & a_{22} \dots\dots a_{2n} & b_2 \\ \dots & \dots\dots\dots & \dots \\ a_{m1} & a_{m2} \dots\dots a_{mn} & b_m \end{bmatrix}$$

This is called the augmented matrix for the system. (The term matrix is used in mathematics to denote a rectangular array of numbers)

For example, the augmented matrix for the system of equations

$$x_1 + x_2 + 2x_3 = 9$$
$$2x_1 + 4x_2 - 3x_3 = 1$$
$$3x_1 + 6x_2 - 5x_3 = 0$$

is

$$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 2 & 4 & -3 & 1 \\ 3 & 6 & -5 & 0 \end{bmatrix}$$

2.3.2 ELEMENTARY ROW OPERATIONS

As stated above, the basic method for solving a system of linear equations is to replace the given system by a new system that has the same solution set but which is easier to solve. This new system is generally obtained in a series of steps by applying the following three types of operations to eliminate unknowns systematically

(i) Multiply an equation through by a non-zero constant

(ii) Interchange two equations

(iii) Add a multiple of one equation to another.

Since the rows (horizontal lines) of an augmented matrix correspond to the equations in the associated system, these three operations correspond to the following operations on the rows of the augmented matrix.

(i) Multiply a row through by a non-zero constant

(ii) Interchange two rows

(iii) Add a multiple of one row to another.

These are called elementary row operations.

2.3.3 REDUCED ROW – ECHELON FORM

In this section, a systematic procedure for solving systems of linear equation is given. It is based on the idea of reducing the augmented matrix to a form that is simple enough that the system of equations can be solved by inspection.

Solution to the given linear system (2.3.1a) by reducing the augmented

(i) An elementary matrix of the FIRST KIND, is an n×n diagonal matrix Q, formed by taking the identity matrix I and replacing the ith diagonal element with a non zero constant q. for example, with n=4 and i=3

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & q & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Note that det. Q = q and that the inverse matrix Q^{-1} = diag. (1,1,1/q,1) is again like I, this time will 1/q in the ith diagonal form.

2. An elementary matrix of the SECOND KIND is an n×n matrix R, formed by interchanging any two rows i and j for I. For example, with n = 4, i = 1 and j =3

$$R= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Note that det. R = -1, and that R is self-inverse, that is, RR=1

3. An elementary matrix of the THIRD KIND is an n×n matrix S, formed by inserting a non-zero constant s into the I, j (i≠j) element of I. (This may also be construed as taking I and adding a multiple s of each element in row j to the corresponding element in row i) for example with n=4, i=3, j=1

$$S= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ s & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Note that det. S=1

Premultiplication of an arbitrary n×p matrix A by one of these elementary matrices produces an ELEMENTARY TRANSFORMATION of A, also termed an elementary row operation, on A. As example, we form the products QA, RA, and SA with n=3, i=2, j=3 and p= 4

1. $$QA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & q & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ qa_{21} & qa_{22} & qa_{23} & qa_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$$

2. $$RA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{21} & a_{22} & a_{23} & a_{24} \end{bmatrix}$$

3. $$SA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & s \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21}+sa_{31} & a_{22}+sa_{32} & a_{23}+sa_{33} & a_{24}+sa_{34} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$$

It is apparent that pre-multiplication by elementary matrices produces the following transformations of A

1 QA: Multiplication of all elements of one row by a scalar.

2 RA: Interchange of rows

3 SA: Addition of a scalar multiple of elements of one row to the corresponding elements of another row.

Observe that in each case the original elementary matrix can be formed from the identity matrix I by manipulating it exactly as we wish to have A manipulated.

Post-Multiplication of and arbitrary p×n matrix A by one of the elementary matrices is called an ELEMENTARY COLUMN OPERATION. The three types of operations produce the following results:

1 AQ: Multiplication of all elements of one column by a scalar.

2 AR: Interchange of two columns.

3 AS: Addition of a scalar Multiple of elements of one column to the corresponding elements of another column.

If A is any matrix and T is the matrix, resulting from elementary row or column operations on A. T and A are termed EQUIVALENT MATRICES. For the examples given above if A is a SQUARE matrix.

Det. (QA) = Det. Q x det A = q det. A

Det. (RA) = Det. R x det. A = - det A

Det. (SA) = Det. S x det. S x dat A = det. A

Thus, multiplication of all the elements of one row of a square matrix by a scalar also multiplies the determinant of the matrix by that scalar. Interchange of two rows changes the sign of the determinant (but not its magnitude) and addition of a scalar multiple of elements of one row to the corresponding elements of another row has no effect on the determinant.

Clearly, the product of elementary matrices is non-singular, for each component has an inverse. It is also true that every non-singular matrix can be written as a product of elementary matrices.

2.4.2 GAUSSIAN ELIMINATION

The direct methods of solving equations 2.4 are based on manipulations using the techniques expressed by the elementary matrices of 2.4.1 .We now describe one such method ,known as Gaussian elimination.

Consider a general system of three linear equations;

$$\begin{aligned} B_{11}x_1 + b_{12}x_2 + b_{13}x_3 &= u_1 \\ B_{21}x_1 + b_{22}x_2 + b_{23}x_3 &= u_2 \qquad \ldots\ldots\ldots\ldots 2.5 \\ B_{31}x_1 + b_{32}x_2 + b_{33}x_3 &= u_3 \end{aligned}$$

As a first step ,replace the second equation by the result of adding to it the first equation multiplied by $-b_{21}/b_{11}$. similarly ,replace the third equation by the result of adding to it the first equation multiplied by $-b_{31}/b_{11}$. The result is the system

$$\begin{aligned} b_{11}x_1 + b_{12}x_2 + b_{13}x_3 &= u_1 \\ b^1_{22}x_2 + b^1_{23}x_3 &= u^1_2 \qquad \ldots\ldots\ldots\ldots 2.5.1 \\ b^1_{32}x_2 + b^1_{33}x_3 &= u^1_3 \end{aligned}$$

In which the b^1 and u^1 are the new coefficients resulting from the above manipulations. Now multiply the second equation of (2.5.1) by $-b^1_{32}/b^1_{22}$, and add the result to the third equation of (2.5.1). the result is the triangular system.

$$\begin{aligned} b_{11}x_1 + b_{12}x_2 + b_{13}x_3 &= u_1 \\ b^1_{22}x_2 + b^1_{23}x_3 &= u^1_2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 2.52 \\ b^{11}_{33}x_3 &= u^{11}_3 \end{aligned}$$

In which b^{11}_{33} and u^{11}_3 result from the arithmetic operations. The system (2.5.2) is readily solved by the process of BACK SUBSTITUTION, in which x_3 is obtained from the last equation: this allows x_2 to be obtained from the second equation, and then x_1 can be found from the first equation.

The above method seems primitive at a first glance, but by the time it has been made suitable for implementation by automatic machines, it furnishes a powerful tool not only for solving equation (2.5), but also for finding the inverse of the related matrix of coefficients B, the determinant of B, the adjoint of B etc.

In so far as reaching (2.5.2) is concerned, all can be explained in terms of elementary matrices of the third kind. Note that matrices alone suffice, the presence of x_1, x_2, and x_3 bening superfluous. Define an AUGMENTED MATRIX C consiting of the original coefficient matrix B with the right- hand side vector U appended to it. That is

$$C = [B \vdots U] = \begin{bmatrix} b_{11} & b_{12} & b_{13} & u_1 \\ b_{21} & b_{22} & b_{23} & u_2 \\ b_{31} & b_{32} & b_{33} & u_3 \end{bmatrix}$$

In which the broken line denotes matrix partiting. Also define three elementary natrices of the third kind:

$$S_1 = \begin{bmatrix} 1 & 0 & 0 \\ -b_{22}/b_{11} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -b_{31}/b_{11} & 0 & 1 \end{bmatrix}$$

$$S_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -b^{1}_{32}/b^{1}_{22} & 1 \end{bmatrix}$$

The operations producing (2.5.2) from (2.5) can be then be expressed as

$$S_3S_2S_1\, C = \begin{bmatrix} b_{11} & b_{12} & b_{13} & u_1 \\ 0 & b^{1}_{22} & b^{1}_{23} & u_2^{1} \\ 0 & 0 & b^{11}_{33} & u^{11}_{3} \end{bmatrix}$$

The back-substitution is expressed in terms of premultiplication by elementary natrices of the first and third kinds. Let Q_1, Q_2 and Q_3 denote the three matrices of he first kind which are needed. For example,

$$Q1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/b^{11}_{33} \end{bmatrix}$$

Then, with three more matrices of the third kind, which we call S_4, S_5 and S_6, the complete sequence of operations results in

$$Q_3\, S_6\, S_5\, Q_2\, S_4\, Q_1\, S_3\, S_2\, S_1\, C = \begin{bmatrix} 1 & 0 & 0 & x_1 \\ 0 & 1 & 0 & x_2 \\ 0 & 0 & 1 & x_3 \end{bmatrix}$$

Let E denote the product of these nine elementary matrices. Then $EC = E[B \vdots U] = [I \vdots X]$, where $EB = I$ and $E = B^{-1}$. Hence, as a by product of solving equations such as (2.5) by elimination, we see that proper plaining cān proceduce B^{-1}. Clearly, we need not solve equations at all if only the inverse is needed, for in that event the column u s superfluous,

Since EB = I, det. (E) det. (B) = det. (I) = 1 from (2.4.1) the determinant of and S or third-kind elementary matrix is unity, wheras the determinant of a Q or first-kind matrix equals the value of that diagonal element which is usually not unity. Hence det (E) = det. (Q_3) x Det (Q_2) det (Q_1). That is det. (E) is the product of the diagonal elements (such as $1/b^{11}{}_{33}$) of the matrices Q_1, Q_2 and Q_3 used in the elimination process. This means that det. (B) is the product of their reciprocals.

The above arithmetic operations can be separated into two types:

(a) NORMALIZATION steps in which the diagonal elements are converted to unity.

(b) REDUCTION steps in which the off-diagonal elements are converted to zero.

Note that by augmenting the coefficient matrix with several right-hand side vectors, we can solve several sets of simulteneous equations, each having the same coefficient matrix, at little extra computatuional cost.

Example: consider the system of equations

$$2x_1 - 7x_2 + 4x_3 = 9$$

$$x_1 + 9x_2 - 6x_3 = 1$$

$$-3x_1 + 8x_2 + 5x_3 = 6$$

for which the solution is $X_1 = 4$, $X_2 = 1$ and $X_3 = 2$. The augmented matrix

[B| U| I] will be formed and the Gaussian elimination procedure just described will be carried out, except that the nomalization steps will be introduced in a some what different order. Starting with the matrix

$$\begin{bmatrix} 2 & -7 & 4 & 9 & 1 & 0 & 0 \\ 1 & 9 & -6 & 1 & 0 & 1 & 0 \\ -3 & 8 & 5 & 6 & 0 & 0 & 1 \end{bmatrix}$$

We multiply the top row by ½ add –1 times the new first row to the second, and 3 times the new first row to the third row. The result is

$$\begin{bmatrix} 1 & -7/2 & 2 & 9/2 & ½ & 0 & 0 \\ 0 & 25/2 & -8 & -7/2 & -½ & 1 & 0 \\ 0 & -5/2 & 11 & 39/2 & 3/2 & 0 & 1 \end{bmatrix} \qquad 2.5.3$$

This is equivalent to having formed the equations

$$X_1 - 7/2x_2 + 2x_3 = 9/2$$

$$25/2x_2 - 8x_3 = -7/2$$

$$-5/2x_2 - 11x_3 = 39/2$$

Note that the operations performed are equivelent to the matrix multiplication

$$\begin{bmatrix} ½ & 0 & 0 \\ -½ & 1 & 0 \\ 3/2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & -7 & 4 & 9 & 1 & 0 & 0 \\ 1 & 9 & -6 & 1 & 0 & 1 & 0 \\ -3 & 8 & 5 & 6 & 0 & 0 & 1 \end{bmatrix}$$

which yields as a result

$$\begin{bmatrix} 1 & -7/2 & 2 & 9/2 & ½ & 0 & 0 \\ 0 & 25/2 & -8 & -7/2 & -½ & 1 & 0 \\ 0 & -2/5 & 11 & 39/2 & 3/2 & 0 & 1 \end{bmatrix}$$

Returning to 2.5.3 multiply the second row by 2/25 and then add 5/2 times the new second row to the third row. The result is

$$\begin{bmatrix} 1 & -7/2 & 2 & 9/2 & ½ & 0 & 0 \\ 0 & 1 & -16/25 & -7/25 & -1/25 & 2/25 & 0 \\ 0 & 0 & 47/5 & 94/5 & 7/5 & 1/5 & 1 \end{bmatrix}$$

The forward course has now been completed and correspounding to 2.:.2 we may write

$$x_1 - 7/2x_2 + \quad 2x_3 = \quad 9/2$$
$$x_2 \; -16/25x_3 = -7/25$$
$$47/5x_3 = \quad 94/5$$

To carry out the back subsitution, start by multiplying the last row by 5/47. Then multiply the new last row by 16/25 and add to the second row. Multiply this same last row by –2 and add to the first row. The result is

$$\begin{bmatrix} 1 & -7/2 & 0 & ½ & 19/94 & -2/47 & -10/47 \\ 0 & 1 & 0 & 1 & 13/235 & 22/235 & 16/235 \\ 0 & 0 & 1 & 2 & 7/47 & 1/47 & 5/47 \end{bmatrix}$$

Finally, Multiply the second row by 7/2 and add to the first. The result is

$$\begin{bmatrix} 1 & 0 & 0 & 4 & 93/235 & 67/235 & 6/235 \\ 0 & 1 & 0 & 1 & 13/235 & 12/235 & 16/235 \\ 0 & 0 & 1 & 2 & 7/47 & 1/47 & 5/47 \end{bmatrix}$$

This means, of course, that $x_1 = 4$, $x_2 = 1$, $x_3 = 2$ and the inverse of the matrix of coefficients is

$$\begin{bmatrix} 93/235 & 67/235 & 6/235 \\ 13/235 & 22/235 & 16/235 \\ 7/47 & 1/47 & 5/47 \end{bmatrix}$$

The determinant of the coeffienceint matrix B equals the product of the reciprocals of the diagonal elements appearing in the Q-type matrices involved in the above transformation.

Inspection shows that the relevant diagonal elements are simply the multiplying factors used in the normalization steps, so that

$$\text{Det. B} = \left[½ \times 2/25 \times 5/47\right]^{-1} = 235$$

iii. The method of transmission has been in practice for a very long time.

3.3 FEASIBILITY STUDY

In order to design a new system, the developer of the system must first of all embark on reasonable feasibility study. It is therefore, very paramount for this study.

Feasibility study focuses at the existing methods of solutions to linear systems or the system currently in use.It also highlight problems associated with the system and designing an alternative approach for the system . This is achieved by gathering and interpreting data in order to evolve proper understanding of the system, diagnose the problem associated with it and proffering solutions. This outcome is used to determine what must be done to solve the problems of the existing old system.

The existing system may be manual or partially automated. To this effect, other possibilities or alternatives may be outlined, compiled, with the cost – benefit analysis of the option and a recommendation of the solution to the management.

3.4 PROJECT FEASIBILITY.

For the purpose of this study, the existing system of solution to systems of linear equation and the automated version using pascal application packages was analysed on the basis of:

1. Operational Feasibility
2. Technical Feasibility and
3. Economic feasibility.

1. OPERATIONAL FEASIBILITY

This indicates that existing methods for solving systems of linear equations manually involves a lot of delay and are not directly applicable to the computer solution of the large-scale problems that arise in real world application. The proposed automated method in comparism to the manual method in solving systems of linear algebra on digital computers are minimizing the computer time (and thus cost) needed to obtain the solution, and minimizing inaccuracies due to roundoff errors.

2. TECHNICAL FEASIBILITY

The proposed alternative method to systems of linear equation should be carried out hand-in-hand with the manual methods of solutions. The aim is to enhance better understanding of the technicques and also to allow for certainty in solutions derived. However, the new system and its operation should be done by experienced personnel with both sound educational background and on the job experience in computer science. This will improve on the level of cohesion and understanding of the both procedures.

. ECONOMIC FEASIBILITY

This basically analyses the cost-benefit ratio of the system proposed i.e cost of implementing the system proposed with the associated benefit. It is viewed from three perspectives

. Development Cost

i Operational Cost and

ii Maintenance cost

. DEVELOPMENT COST

This involves the actual cost of installing the computer such as the cost of computer hardware and other associated cost of installing the software and its accessories. After a careful cost and Benefit analysis the estimated unit of the computer repaired is Ten, one scanner, one printer and one unterrupted power supply (UPS) and stabilizer and are all valued at about ₦2.8m

i OPERATONAL COST

After the installing of the new system it will have to be put in use or make operational otherwise, its purpose will be defeated. The cost of doing this is regarded as operational cost. And this involves the cost of employing at least one programmer and one analyst. There may be no need to employ supporting staff as those presently in the system are capable of operating the system after traning.

The estimated salary per annum of the programmer and the analyst will be in the region of ₦260,000.00 and ₦ 210,000.00 respectively. The cost of training 4 mathematics lecturer for two weeks is estimated at ₦160,000.00 as per details below.

Principal Lecturer	-	40,500.00
Senior Lecturer	-	33,000.00
Lecturer 1	-	30,000.00
Lecturer 2	-	30,000.00
1 Programmer	-	260,000.00
1 Analyst	-	210,000.00
		₦ 603,500.00

Wether the alternative method is introduced or not, the Salaries of exiting lecturers would be paid so it is a fixed cost not relevant for this estimate.

iii MAINTENANCE COST

Thiscost is the routine maintenance cost of the newly installed system. It may also be referred to as enhancement cost. The estimated cost of maintaing the new system will be relatively cheaper since the equipments are quite new. The estimated cost of doing this is in the region of ₦150,000.00 per annum, and it includes stationeries, electricity and servicing.

Other advantage associated with the new system will includes reduction in : -

i Cost of Sationaries

ii Cost of Servicing

iii Timeliness.

3.5 OBJECTIVES GUIDING THE INVESTIGATION

In system analysis, the problem identification is the starting point of a system life-cycle. Ability to therefore identify this problem permits further analysis, once the problem is discovered the design of the new system can be carried out. On the basis of the above, the following objectives were used as a guide in this investigation

a. Reliability or Durrability of the system

b. Its Accurracy

c. Eliminates rigidity in the system.

The above objectives were used as a guide in the invertigation and were reflected in the design of the new system from the implememtation plan to the conversion stage.

3.6 THE CURRENT SYSTEM

The system will be partial automation using pascal-programming language as the language of Comminication.

Based on the principles of reasonable cost, flexibility ana reliabil.ty the new system solutions to linear algebra is a customize type that allows further intergrating of other aspects of automation into the system when the need arises.

The new systems have all the features of a user friendly system with the feature simplified for the operations and leacturers involved in the operation of the system..

3.7 REQUIREMENT SPECIFIENTION:

This is divided into two for easy identification:

i. Software Requirement Specification

ii. Hardward Requirement Specification.

i. SOFTWARE REQUIREMENT SPECIFICATION

Software involves the types of software that is to used. The software is in the internal structure of the computer and it includes:-

a. M.S-DOS Version 6.22

b. Pascal

RE

ii THE HARDWARE REQUIMENT SPECIFICATION

This is the physical part of the computer system and they are:-

a.1024KB RandomAccess Memory (RAM)

b.40 MB hard disk

. Printer 132 Colour line printer

. SVGA Monitor with colour capabilities.

.8 COST BENEFIT ANALYSIS

After detailed sampling of different hardware compnent available, the ollowing cost was estimated for the new proposed system under the following eadings.

.8.1SYSTEM COST

Development Cost

System analysis and design the reseacher and other volunteers who do not take ny monetory reward for their contributions did the job.

Software development and implementation was done by the reseacher and a rogrammer. Using the pascal compiler, it took the team about 120hours to complete ne task and the rate charged per hour was ₦50.00. This amount to ₦18,000.00

Equipment purchase hardware such as personal computer (PCS), printer, stabilizer cetara and the cost of installation amounted to about ₦2.5m (previously stated in e study)

personnel training for two weeks for lecturer on the operation of the new system ill cost ₦603,500.00 as previously calculated under operational cost.

System operation cost per annum using the following operational heading are

(a) Equipment Maintanance	45,000
(b) Program Maintanance	40,000
(c) Utilities- Electricity, Diesel	50,000
(d) Miscelleneous	25,000
	₦16,000.00

The total cost of operating the new system is summarized below:

(i) Software Development/ Implemetation	18,00.00
(ii) Acqusition of equipment	2,500,000.00
(iii) Personal Training	405,000.00

(iv) Operating cost 160,000.00

₦3,083,000.00

3.8.2 SYSTEM BENEFITS

(i) Saving from engaging the services of lecturers whose salaries are fixed and does not relate to the introdction of the system

(ii) Reduction in Overtime claim.

These saving will impact positivily on the organisations (school) Through

a. Better understanding of the method used.

b. Prevent carring too many paper work

c. Create awareness and challenges of our time

d. Cater for future expansion

Other benefits associated with the new system may not be quantified in monetary terms as they are qualitative. However, the change will impact immensely on the overall performance of the students and lecturers.

.9 INPUT SPECIFICATION

Input data are required to be entered by the user based on the problem at hand. he program is written in such a way that the user provides the integer number by iven a value which you are required to enter.

The input data is provided based on the problem at hand, which is to be solved.

.10 OUTPUT SPECIFICATION

The output is what is expected to be produced by the system. This could be iewed by displaying on the screen or printed out from the printer to obtain the hand opy.

The following are the type of output that can be operated from the system

X1=

X2=

X3=

X4=

CHAPTER FOUR

SYSTEM DEVELOPMENT AND IMPLMENTATION.

I.1 INTRODUCTION.

Software is a term used to describe all written programs, which are used in a ·articular computer installation. It is a program procedure or rules and any associated ucumentation pertaining to the operation of a computer system. Software evelopment entails series of activities or processes that should be carried out in the ause of developing a new system. The software development begins with the laid own structure in general design and detailed design of the automated system.

The design determines the approprite language for implemanting the system. owever, since system life cycle is all about systen development, the software evelopment is about a bye-product of the system development. When a system is on ound, the fool that automate the system is software. Hence, the stages for their evelopment are interwoven.

2 CHOICE OF LANGUAGE

Computer language is a means of communicating between programmers and e computer programmer that use the language ~~ot~~ to instruct computer on how certain sk or job should be done.

This choice of language depends on the following:

Type of task or job

The application of the task or job

. Volume of data element

. Complexity of the task or job

In this automation of the system, PASCAL programming language is the nguage of choice adopted for implementation. The choice of pascal is based on its gh speed and its ability to use structured English that tries to express verbal atement in a more logical framework. It uses natural language along with ideas of gic and block structuring used in a computer language. The technique is useful hen there are not too many conditions to be checked and the procedures are basically repetitive. It is simple to understand by its users and ideal software for instructional purposes, it is also very interactive (user friendly) with a simple procedure orientation.

The version of pascal used in this program is known as version 5.5 provided along with disk operating system (DOS) from Microsoft (MS- DOS). It is provided with the following features

Variables:

Variables in pascal like any language is defined to be name used to store data that can be changed, and is called an identifier in pascal

Data type:

Pascal has different variations for declaring an identifier to be a specific data- some of these are real, integer, Boolean, Array, Character etc.

Statements:

Statements are the executables in pascal language. They can also be termed expressions. The following are pascal statements: Assignment Statement, Arithmetic Statement, Relational Statement and Logical statement.

4.3 SOFTWARE DEVELOPMENT AND TESTING

In the course of developing this program or software (solutions to systems of linear equation) the following essential stages were followed:

1. UNDERSTANDING THE PROBLEM

Understanding a problem is part of the problem, hence, the programmer needs to know what exactly the program is to do and work from a program specification of the solution to systems of linear equations.

The specifications in this system are: -

(a) The need for an alternative procedure for solution to system of linear equations

(b) Perfect control of program and high restriction

Generally, program specification defines the inputs, processing and output. A good specification will define inputs, processing and output. It will normally specify what is needed by giving the exact relationship between output and inputs from which they are derived rather than prescribing how the program should be written.

(ii) PLANNING THE METHODS OF SOLUTION.

The methods of solution is prepared using

(a) An algorithm

b) A flow chart

For the design of the source program (see appendix (i) and (ii)). They are used to Generate the source programs. Pascal programming language uses what is called ompiler to translate source code to machine code.

TYPING THE INSTRUCTION IN A PROGRAMMING LANGUAGE.

This is the last step to step-wise refinement .The instrucion design in a flow hart are converted to a programming language called pascal.

v) TESTING THE PROGRAM

Once a program is written, it has to be subjected to various tests that have been ritten out and transcribes correctly. These tests reveal errors, which are immediately orrected.

reas tested include

) Integrating testing

) System testing

) User Acceptance testing

This program has been tested for A and b above and C is left for the user to xecute.

4 SYSTEM TESTING

This testing ensures that the individual programs have been written correctly nd that the system as a whole will work with the link between the programs in a suit. here must be coordination with clerical procedure involved. To this end the sytem ust provide the necessary list data as follows:

(1) PROGRAM TESTING

Test data is supplied to ensure that all possible contingencies (as specified in the system specification) have been catered for by the programmer. Expected results of the test is worked out for compansion purposes.

(2) PROCEDURE TESTING

This ensures that the whole system fits together as planned. This involves the clerical procedure which preceeds input and output procedure. Over timing and ability of staff/students to handle the anticipated volumes will be under scrutiny.

4.5 CHANGE-OVER PROCEDURE

Change over procedure is the process of executing the new system vis-à-vis the old system This may be achieved in the following ways:

(a) Parallel

(b) Direct

(b) Pilot

(1) PARALLEL

It is the process of running old and new system concurrently using the same inputs. The outputs are compared and reasons for differences resolved. Output from the old system continues to be distributed until the new system has proved satisfactory. At this stage the system is maintained and the new system is kept alongside the old system (in this work)

(ii) DIRECT

the old system is discontinued altogether and the new system becomes operational immediately. However, for this reseach work the old system is not discontinued because the aim is to proffer Altenative method to solution by system of linear equation, which is.

(1) More reliable and effective

(2) Prompt solution to complex problems

(3) the calibre of staff/students involve do not require any much further training

(iii) PILOT

This involves the changing of over of the part of the system at a time either as parallel or direct. That is a variation of either of the two methods previously discussed.

4.6 STARTING THE SYSTEM

Carrying out the following steps can start the system.

(i) Boot the (system) computer

(ii) At windows envioment, click start

(iii) Highlight program

(iv) from the Pop-menu Click Ms Dos

(v) A the c-promt, change drive A: i.e type C:\> A: and press enter key.

(vi) Type A:\> turbo and press enter key.

This takes you to the IDE enviroment of pascal.

(vii) Press ALT-F to open or press F3 .

(viii) Use tab key to to select Gauss J.pas

(ix) To compile program –ALT F9

(x) To run the program use Ctrl –F9

CHAPTER FIVE

5.0 SUMMARY AND CONCLUSION

An attempt is made by developing an algorithm for the above procedure, which is called Gauss-Jordan elimination.

Let the starting array be the n x (n +m) augmented matrix A. Consisting of an n x n coefficient matrix with m appended columns:

$$\begin{bmatrix} a_{11} & a_{12} & \cdots\cdots & a_{1n} & a_{1,n+1} & a_{1,n+2} & \cdots & a_{1,n+m} \\ a_{21} & a_{22} & \cdots\cdots & a_{2n} & a_{2,n+1} & a_{2,n+2} & \cdots & a_{2,n+m} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} & a_{n,n+1} & a_{n,n+2} & \cdots & a_{n,n+m} \end{bmatrix}$$

Let k = 1,2,.....n be the pivot counter, so that a_{kk} is the pivot element for the k th pass of the reduction. It is understood that the values of the elements of A will be modified during computation.

The algorithm is

$$\left.\begin{array}{l} \text{Normalization} \\ a_{kj} \leftarrow a_{kj}/a_{kk} \quad J = n+m,\ n+m-1, \ldots\ldots K \\ \\ \\ \text{Reduction} \\ \left.\begin{array}{ll} A_{ij} \leftarrow a_{ij} - a_{ik}\, a_{kj} \quad I = 1,2\ldots.n & \\ \quad j = n+m,\ n+m-1 \ldots.k & i \neq k \end{array}\right\} \end{array}\right\} K = 1,2\ldots.n$$

NOTE: The left arrow is intended to mean substitution in place.

5.1.1 DISCUSSION OF RESULT.

(a) Since no non zero elements appear to the left of a_{kk} in the kth row at the beginning of the kth pass it is unnecessary to normalize a_{kj} for $j<k_i - 1$

(b) In order to avoid premature modification of elements in the pivot column, the column counter j is always decremented from its highest value (n + m) until the pivot ... is reached

Thus far, elementary matrices of the second kind have not been used. Neither has mention been made of the fact that at some stage, say the first,a potential divisor or pivot, such as b_{11}, may be zero. In this event, we can think of interchanging rows which is expressible of course, in terms of elementary row operations of the second kind. A related problem is that of maintaining sufficient accuracy during intermediate calcutions in order to achieve specified accuracy in the final results. This might be expected for a nearly sigular system. It can also happen when the magnitude of one of the pivot elements is relatively small. Consider for instance, the system

$0.0003x_1 + 3.0000\, x_2 = 2.0001$

$1.0000x_1 + 1.0000x_2 = 1,0000$

Which has the exact slution $x_1 = 1/3, x_2 = 2/3$.if the equations are solved using pivots on the matrix diagonal as indicated in the previous example there results.

$1.0000x_1 + 10000x_2 = 6667$

$x_2 = 6666/9999$

If x_2 from the second equation is taken to be 0.6667,then from the first equation x_1 = 0.0000;for x_2 = 0.66667,x_1=0.30000; for x_2=0.666667, x1=0.330000 etc. the solution depends highly on the number of figures retained; if the equations are solved in reverser order, that is, by interchanging the two rows and proceeding as before, then x_2 is found to be 1.9998/2.9997 =0.66667, while x_1=0.33333. This example indicates the advisability of choosing as the pivot the coefficient of largest absolute value in a column, rather than merely the first in the line.

5.1.2 Computer output

Considering the following example

$2x_1 - 7x_2 + 4x_3 = 9$

$x_1 + 9x_2 - 6x_3 = 1$

$-3x_1 + 8x_2 + 5x_3 = 6$

The computer program will solve as follows

It demand the following

Result

m=n=3

Input the augmented matrix

2 -7 4

1 9 -6

-3 8 5

Input the column vector [916]t

Input the identity matrix

1 0 0

0 1 0

0 0 1

output result (solution vector, rows)

$x_1 = 4$

$x_2 = 1$

$x_3 = 2$

Since the aim of introducing automated alternative method in solving systems of linear equation is to augment or rather add to the existing manual methods. It can therefore be interesting when you see an improvement in the system, particularly in this computer age.

The analysis and design of the automated solutions to system of linear equation has undergone a computer life cycle.This analysis was through with the aim of identifying associated problems,the feasibility studies carried out was to determine how feasible or viable the system would be.The cost and benefit analysis done was aimed at the benefit of the new (approach) system to the enhancement of students performance.

The emergence of this automation will help in eliminating or reducing minimally most of the problems or difficulties associated with the existing methods as well as improving its comprehension. Even though the new system has obvious advantages over the existing method, it is not without its own constraints, such as, administrative bottlenecks in getting approval for the new approach from both the administrators and policy formulators especially in designing the curricular. It is capital intensive and the finance of the automation may be slowed or delayed. It may even be difficult to carry out.

Logistic constraints and power failure can frustrate the implementation plan of the new approach.

5.2 LIMITATION

There are unavoidable constraints that limits this project.The limitations range from that of time to financial.The limitation of the project can be summed as follows:

1) The project is designed to handle automation of solutions of systems of linear equation. Some of the reports generated by this design for effective and accurate performance of the procedure includes:

. Elementary Transformations of matrices.

i. Direct methods based on manipulations using the technique (s) discussed above.

These in themselves are not exhaustive and it becomes limitation.

2) Most of the entries are done manually. This could result to errors due to human limitation.

3) The alternative method can only be implemented on stand alone personal computers.

4) The alternative method is designed using pascal-programming language. That is, the alternative method most be implemented were there is a pascal compiler (programming language).

CONCLUSION

The incorporation of computer-based alternative to solving problems together with manual methods in modern days has become a common phenomenom or worldwide affair. This is not unconnected to the relevance of computer in virtually all aspects of human endeavour. This interest is, however, intensified by the capacity of computer in performing a given set of procedures with all the necessary accuracy. It is not subjected to commiting errors, and has the ability to accomplish any task with high speed.

It is therefore, rational to introduce an improved application package like pascal in the automation of systems of linear equations in order to enhance the overall performance of the academia.

5.4 RECOMMENDATIONS:

Based on the researcher's finding, it was discovered that the solution of systems of linear eqation using the direct method exposes students ability to cope with automation procedures, which is largely based on the methods of elementary matrices. The new alternative approach to be incorporated along with the direct method has carefully looked at the advantageous way by which solutions to the systems of linear equations can be achieved more accurately, timely and efficiently with reduced roundoff errors.

It is therefore recommended that curriculum planners, school administrators and others, should introduce the designed alternative into their existing methods of solving linear equation problems, for accuracy and correct output, so as to improve in the students perception and understanding of the systems of linear equations generally.

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APPENDIX 1

Gauss-Jordan Elimination Method
Algorithm.

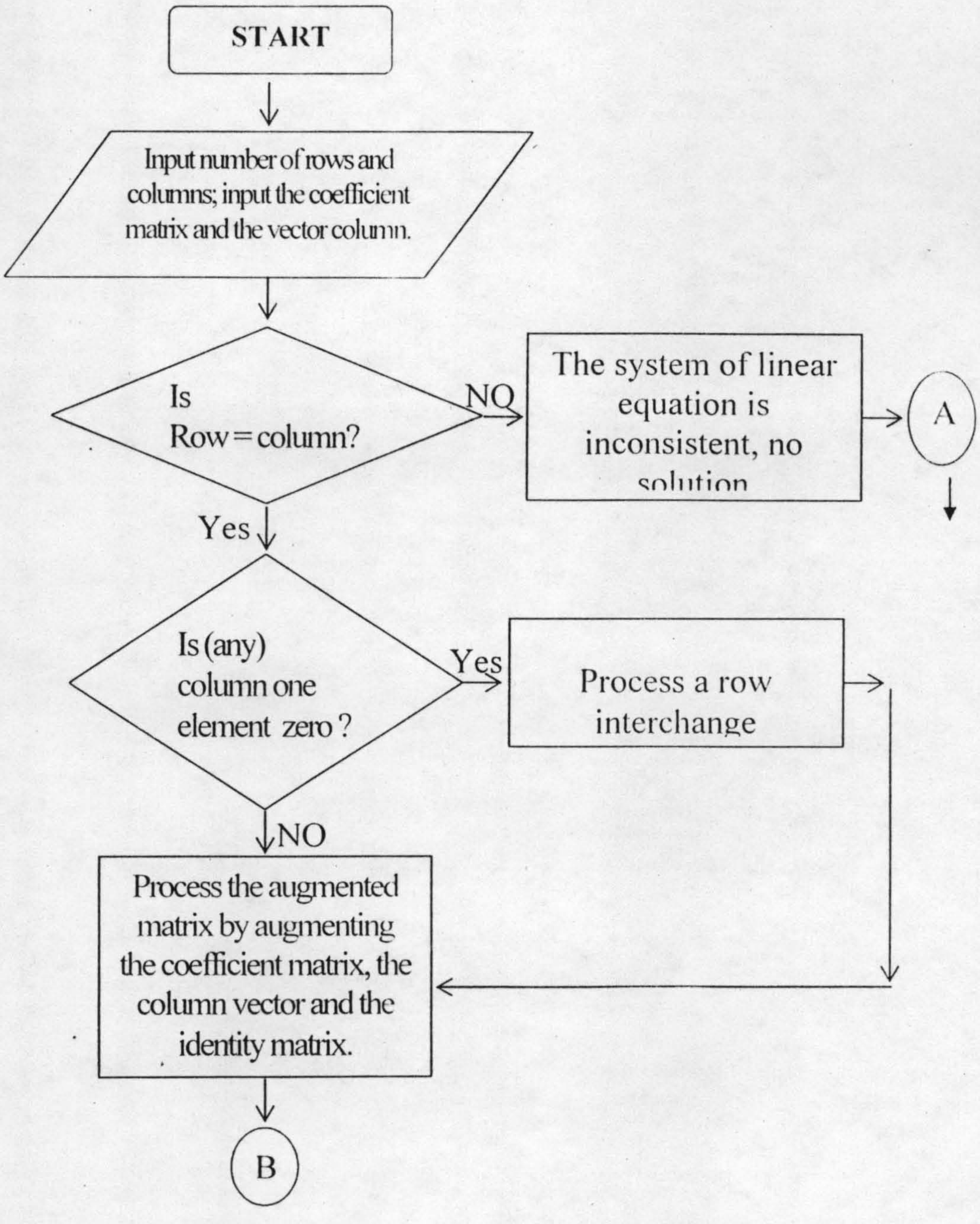
1. Starting array $n \times (n + m)$
2. Pivot Counter $K = 1, 2, \ldots, n$
   Pivot Element a_{kk}

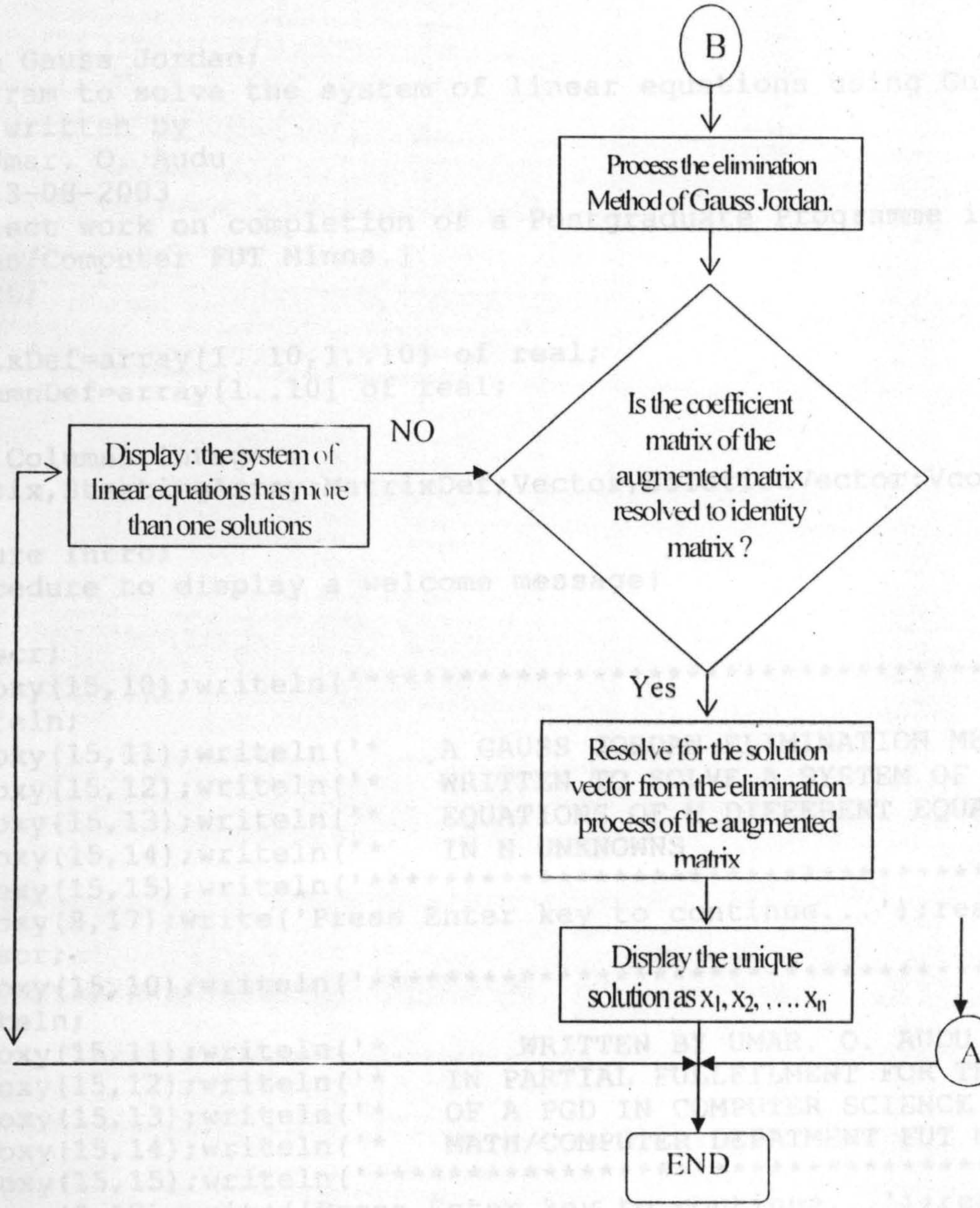
$$
\left.
\begin{array}{ll}
\text{3. Normalization} & a_{kj} \longleftarrow a_{kj}/a_{kk} \quad j = n + m, n + m - 1 \ldots\ldots k \\[6pt]
\text{4. Reduction} & \left. \begin{array}{l} a_{ij} \longleftarrow a_{ij} - a_{ik}\, a_{kj} \\ j = n+m, n+m-1 \ldots k \end{array} \right\} \quad \begin{array}{l} i = 1, 2 \ldots n \\ (i \neq k) \end{array}
\end{array}
\right\} \quad k = 1, \ldots\ldots n
$$

APPENDIX 2

Gauss Jordan Elimination Method

FLOW CHART




B
Process the elimination Method of Gauss Jordan.
Is the coefficient matrix of the augmented matrix resolved to identity matrix ?
NO
Display : the system of linear equations has more than one solutions
Yes
Resolve for the solution vector from the elimination process of the augmented matrix
Display the unique solution as $x_1, x_2, \ldots x_n$
A
END

```
      for m:=1 to Rows do
        for n:=Columns+2 to 2*Columns+1 do
           StartingArray[m,n]:=Ident[m,n-(Columns+1)];
nd;{end of Augemented Matrix}


rocedure RowInterchange(var GMatrix:MatrixDef;Rows,Columns:integer);
A procedure to process row interchange in case if first element of
the first column entries is zero}
ar m,n:integer;
    tempValue:real;
egin
    for m:=1 to Rows-1 do
      if GMatrix[m,1]=0.0 then
         for n:=1 to Columns do
           begin
              tempValue:=GMatrix[m,n];
              GMatrix[m,n]:=GMatrix[m+1,n];
              GMatrix[m+1,n]:=tempValue
           end
end;{end of Row Interchange}

procedure EliminationProcess(var SolutionVector:VcolumnDef;StartingArray:Ma
ixDef;Rows,
Columns:integer);
{A procedure that caters for Normalisation and Reduction process defined by
auss Jordan Algorithm}
var i,j,k:integer;
begin
  {Elimination process start here}
  for k:=1 to Rows do
    begin
      {Normalization process of the augemented matrix here}
      for j:=(Rows+Columns)+1 downto k do
         StartingArray[k,j]:=StartingArray[k,j]/StartingArray[k,k];{pivot E
ment}
      {Reduction process of the augemented matrix here}
      for i:=1 to Rows do
         if i<>k then
           for j:=(Rows+Columns)+1 downto k do
             StartingArray[i,j]:=StartingArray[i,j]-StartingArray[i,k]*Star
ngArray[k,j];
  end;{end of Elimination process}
  {Assigning the final solution of computation i.e the solution vector}
  for i:=1 to Rows do
     SolutionVector[i]:=StartingArray[i,Columns+1];
end;{end of Elimination process}

procedure Displayresult(SolutionVector:VcolumnDef;Rows:integer);
{A procedure to display the computational result on the screen}
var i:integer;
begin
```

Gaussj.pas

```
 Clrscr;
 Writeln ('THE UNIQUE SOLUTION');
 For I: =1 to rows do
       Writeln ('value x ', I, '=', Solutionvector [ I ]:3:4);
 Readln
End; {end of display procedure}

{Main body of the program}
 begin
 intro;
 clrscr;
 inputMatrix (Gmatrix, Vector, Rows, Columns);
 if consistencyTest (Rows, Columns ) then
  begin
       Rowinterchange (Gmatrix,Rows,Columns);
       AugmentedM (StartingArray,Gmatrix,Vector<Rows,Columns);
       Eliminationprocess (SolutionVector,StartingArray,Rows,Columns);
       Displayresult (solutionVector,Rows);
  End
Else
       If Rows>Columns then
         Begin
             Writeln (The system of linear equations is inconsistent and
undetermined');
                Writeln ('hence No Solution!');
                Readln;
            End
       Else
            Begin
                Writeln (,The solution of the linear system is non-unique`):
                Writeln ('having series of solutions.');
                Readln;
           End;
       Writeln;writeln ('THANKS TO GUASS JORDAN');readln
End.{end of guass program}
```

AN AUTOMATED SOLUTION FOR LINEAR ALGEBRAIC SYSTEMS

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