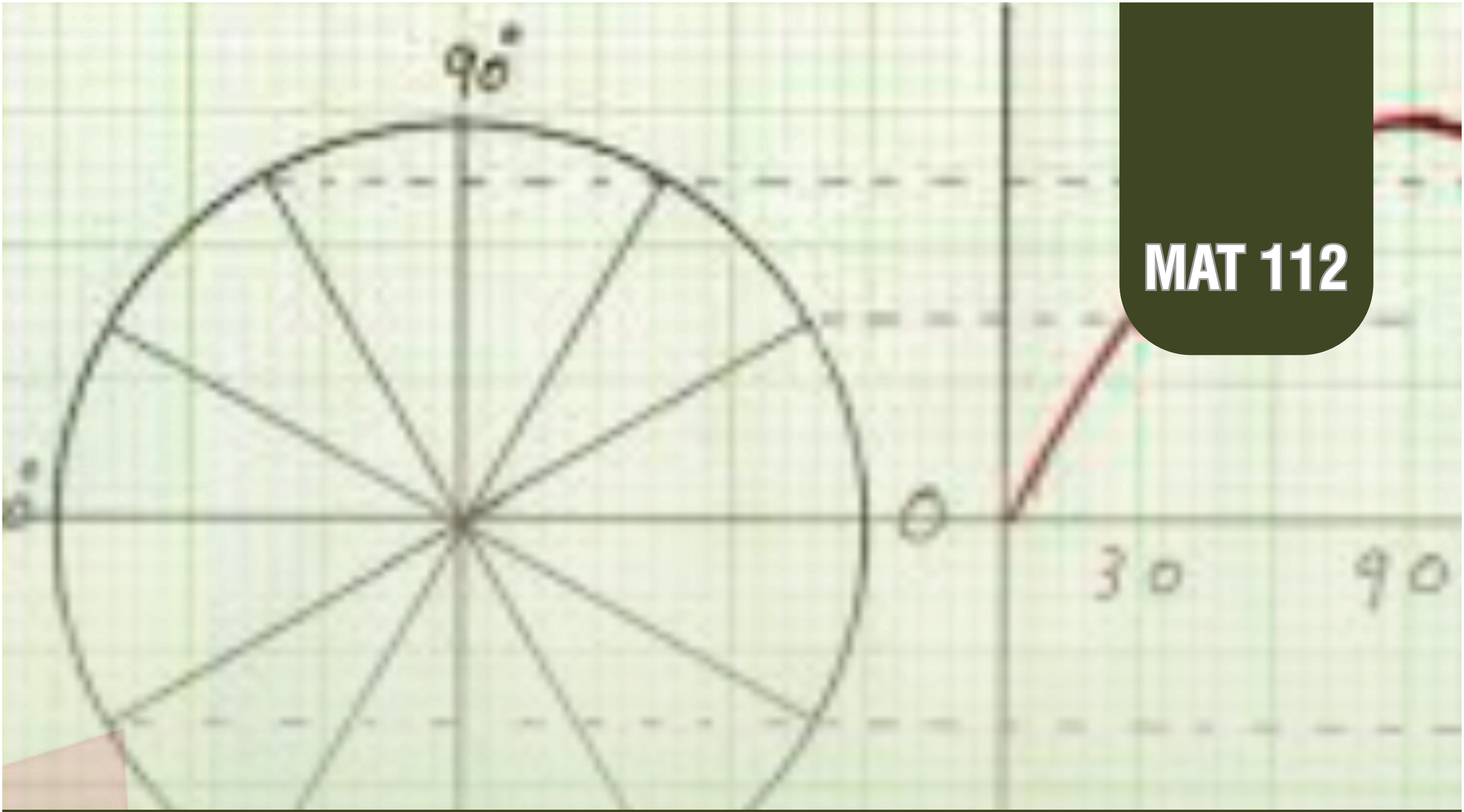

MAT 112

Coordinate Geometry And Trigonometry

FEDERAL UNIVERSITY OF TECHNOLOGY, MINNA
CODeL
CENTRE FOR OPEN DISTANCE AND e-LEARNING

FEDERAL UNIVERSITY OF TECHNOLOGY, MINNA
NIGER STATE, NIGERIA

CENTRE FOR OPEN DISTANCE AND e-LEARNING (CODeL)

B.TECH. COMPUTER SCIENCE PROGRAMME

COURSE TITLE

COORDINATE GEOMETRY AND TRIGONOMETRY

COURSE CODE

MTH 112

COURSE CODE

CPT 214

COURSE UNIT

2

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CPT 112: Coordinate Geometry and Trigonometry

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MAT 112 Study Guide

Introduction

MAT 112: Coordinate Geometry and Trigonometry is a 2- credit unit course for students studying towards acquiring a Bachelor of Science in any Science field. The course is divided into 4 modules and 15 study units. It will first introduce the slope of a straight line and equation of straight line. Next, Tangent and Normal through a Point on a Circle in detail. Thereafter, Equation of an ellipse as well as tangent and normal to an ellipse are discussed. This is followed by a discussion on equation of hyperbola and tangent and normal to hyperbola subsequently, an overview Trigonometry of angles is presented. Finally, the student is introduced to the Trigonometry of Identities.

The course guide therefore gives you an overview of what Mat 112 is all about, the textbooks and other materials to be referenced, what you expect to know in each unit, and how to work through the course material.

Recommended Study Time

This course is a 2-credit unit course having 15 study units. You are therefore enjoined to spend at least 2 hours in studying the content of each study unit.

What You Are About to Learn in This Course

The overall aim of this course, MAT 112 is to introduce you to basic concept of geometry and Trigonometry. At the end of this course you will be able to:

i. Obtain the slope and equation of a straight line
ii. Obtain the standard equation of a circle
iii. Obtain the standard equation of a parabola
iv. Obtain the standard equation of a hyperbola
v. Obtain the standard equation of an ellipse
vi. Obtain the equation tangent and normal to: circle, parabola, hyperbola and ellipse
vii. Solve problems involving Trigonometric ratios and Identity

Course Aims

This course aims to introduce students to the basics concept of geometry and trigonometry. It is expected that the knowledge will enable the reader to effectively use trigonometry to solve distance, angle and circle related problems

Course Objectives

It is important to note that each unit has specific objectives. Students should study them carefully before proceeding to subsequent units. Therefore, it may be useful to refer to these objectives in the course of your study of the unit to assess your progress. You should always look at the unit objectives after completing a unit. In this way, you can be sure that you have done what is required of you by the end of the unit.

However, below are overall objectives of this course. On completing this course, you should be able to:

i. Obtain the slope and equation of a straight line given the coordinate
ii. Obtain the mid – point of a straight line and dividing the straight line in the ratio m:n
iii. Get the angle between two straight lines.
iv. Determine when two lines are perpendicular
v. Obtain distance from a point (x_1, y_1) to the straight line $ax + by + c = 0$.
vi. State the proper definition of a conic section
vii. Obtain the standard equation of a circle
viii. Obtain the equation of a tangent and normal through a point to a circle
ix. Obtain the equation of a parabola as well as tangent and normal to parabola
x. Obtain the equation of an ellipse as well as tangent and normal to an ellipse
xi. Obtain the equation of a hyperbola as well as tangent and normal to an hyperbola
xii. Obtain vertex, focus, directrix, eccentricity and asymptotes as the case may be
xiii. Convert a measurement from radians to degrees (or vice versa)
xiv. Identify the Trigonometric functions.
xv. Identify the reference angle for the any given angle
xvi. Use Pythagoras theorem to identify the Pythagorean Identities
xvii. Identify the sum and difference identities of trigonometric ratios.
xviii. Solve problems involving double and half angle identities of trigonometric ratios.

Working Through This Course

In order to have a thorough understanding of the course units, you will need to read and understand the contents, practice the steps by solving problems involving length of Objects, determining position or coordinate on a circular object as well as solving parabolic and hyperbolic Problems.

This course is designed to cover approximately sixteen weeks, and it will require your devoted attention. You should do the exercises in the Tutor-Marked Assignments and submit to your tutors.

Course Materials

The major components of the course are:

1. Course Guide
2. Study Units
3. Text Books
4. Assignment File
5. Presentation Schedule

Study Units

There are 15 study units and 6 Modules in this course. They are:

Module One	**Coordinate Geometry and Trigonometry**	
	Unit 1	Slope of a Straight Line
	Unit 2	Equation of a Straight Line I
	Unit 3	Equation of a Straight Line II
Module Two	**Conic section**	
	Unit 1	Equation of a Circle
	Unit 2	Tangent and Normal through a Point on a Circle
	Unit 3	Equation of Parabola
	Unit 4	Tangent and Normal to a Parabola
Module Three	**Ellipse and Hyperbola**	
	Unit 1	Equation of an ellipse
	Unit 2	Tangent and normal to an ellipse
	Unit 3	Equation of a hyperbola
	Unit 4	Tangent and normal to a hyperbola
Module Four	**Trigonometry Angles and Identity**	
	Unit 1	Trigonometry (Angles I)
	Unit 2	Trigonometry (Angles II)
	Unit 3	Trigonometric Identity I
	Unit 4	Trigonometric Identity II

Recommended Texts

The following texts and Internet resource links will be of enormous benefit to you in learning this course:

Olabisi O. Ugbebor and Ninuola I. Akinwande (2000) Analytical Geometry and Mechanics, Ibadan: Y – Books.

Bostock, L. And Clandler, S. (1979) Pure Mathematics 2, UK: Stanley Thomes Ltd.

K. A. Stroud and Dexter J. Booth (2001) Engineering Mathematics Fifth Edition, Great Britain: Antony Rowe Ltd.

Stan Gibilisco (2003), Trigonometry Demystified, London: McGRAW – HILL.

Steven Butler (2002), Notes from Trigonometry, Brigham Young University.

Peggy Adamson and Jackie Nicholas (1998) Introduction to Trigonometric Functions: University of Sydney

Assignment File

The assignment file will be given to you in due course. In this file, you will find all the details of the work you must submit to your tutor for marking. The marks you obtain for these assignments will count towards the final mark for the course. Altogether, there are tutor marked assignments for this course.

Presentation Schedule

The presentation schedule included in this course guide provides you with important dates for completion of each tutor marked assignment. You should therefore endeavour to meet the deadlines.

Assessment

There are two aspects to the assessment of this course. First, there are tutor marked assignments; and second, the written examination. Therefore, you are expected to take note of the facts, information and problem solving gathered during the course. The tutor marked assignments must be submitted to your tutor for formal assessment, in accordance to the deadline given. The work submitted will count for 40% of your total course mark.

At the end of the course, you will need to sit for a final written examination. This examination will account for 60% of your total score. You will be required to submit some assignments by uploading them to MAT 112 pages on the u-learn portal.

Tutor-Marked Assignment (TMA)

There are TMAs in this course. You need to submit all the TMAs. The best 10 will therefore be counted. When you have completed each assignment, send them to your tutor as soon as possible and make certain that it gets to your tutor on or before the stipulated deadline. If for any reason you cannot complete your assignment on time, contact your tutor before the assignment is due to discuss the possibility of extension. Extension will not be granted after the deadline, unless on extraordinary cases.

Final Examination and Grading

The final examination for MAT 112 will last for a period of 2 hours and has a value of 60% of the total course grade. The examination will consist of questions which reflect the Self-Assessment Exercises and tutor marked assignments that you have previously encountered. Furthermore, all areas of the course will be examined. It would be better to use the time between finishing the last unit and sitting for the examination, to revise the entire course. You might find it useful to review your TMAs and comment on them before the examination. The final examination covers information from all parts of the course.

Practical Strategies for Working Through This Course

1. Read the course guide thoroughly

2. Organize a study schedule. Refer to the course overview for more details. Note the time you are expected to spend on each unit and how the assignment relates to the units. Important details, e.g. details of your tutorials and the date of the first day of the semester are available. You need to gather together all these information in one place such as a

diary, a wall chart calendar or an organizer. Whatever method you choose, you should decide on and write in your own dates for working on each unit.

3. Once you have created your own study schedule, do everything you can to stick to it. The major reason that students fail is that they get behind with their course works. If you get into difficulties with your schedule, please let your tutor know before it is too late for help.
4. Turn to Unit 1 and read the introduction and the learning outcomes for the unit.
5. Assemble the study materials. Information about what you need for a unit is given in the table of content at the beginning of each unit. You will almost always need both the study unit you are working on and one of the materials recommended for further readings, on your desk at the same time.
6. Work through the unit, the content of the unit itself has been arranged to provide a sequence for you to follow. As you work through the unit, you will be encouraged to read from your set books
7. Keep in mind that you will learn a lot by doing all your assignments carefully. They have been designed to help you meet the objectives of the course and will help you pass the examination.
8. Review the objectives of each study unit to confirm that you have achieved them.
9. If you are not certain about any of the objectives, review the study material and consult your tutor.
10. When you are confident that you have achieved a unit's objectives, you can start on the next unit. Proceed unit by unit through the course and try to pace your study so that you can keep yourself on schedule.
11. When you have submitted an assignment to your tutor for marking, do not wait for its return before starting on the next unit. Keep to your schedule. When the assignment is returned, pay particular attention to your tutor's comments, both on the tutor marked assignment form and also written on the assignment. Consult you tutor as soon as possible if you have any questions or problems.
12. After completing the last unit, review the course and prepare yourself for the final examination. Check that you have achieved the unit objectives (listed at the beginning of each unit) and the course objectives (listed in this course guide).

Tutors and Tutorials

There are few hours of tutorial provided in support of this course. You will be notified of the dates, time and location together with the name and phone number of your tutor as soon as you are allocated a tutorial group. Your tutor will mark and comment on your assignments, keep a close watch on your progress and on any difficulties, you might encounter and provide assistance to you during the course. You must mail your tutor marked assignment to your tutor well before the due date. At least two working days are required for this purpose. They will be marked by your tutor and returned to you as soon as possible.

Do not hesitate to contact your tutor by telephone, e-mail or discussion board if you need help. The following might be circumstances in which you would find help necessary: contact your tutor if:

- You do not understand any part of the study units or the assigned readings.
- You have difficulty with the self-test or exercise.
- You have questions or problems with an assignment, with your tutor's comments on an assignment or with the grading of an assignment.

You should endeavour to attend the tutorials. This is the only opportunity to have face to face contact with your tutor and ask questions which are answered instantly. You can raise any problem encountered in the course of your study. To gain the maximum benefit from the course tutorials, have some questions handy before attending them. You will learn a lot from participating actively in discussions.

GOODLUCK!

Table of Contents

Module 1

Coordinate Geometry and Trigonometry

Unit 1: Slope of a Straight Line

Unit 2: Equation of a Straight-Line I.

Unit 3: Equation of a Straight Line II.

Unit 1

Slope of a Straight Line

Contents

1.0 Introduction

Coordinate Geometry deal with study of lines and expressions involving coordinates. This is why you must learn the understanding of lines and coordinates in this unit in other to apply them in other units.

2.0 Learning Outcomes

At the end of this unit you should be able to:

1. Determine the Cartesian coordinates for two – dimensional.
2. Obtain the mid – point of a straight line and dividing the straight line in the ratio m:n.
3. Obtain the slope of a straight line given the coordinates.

3.0 Learning Content

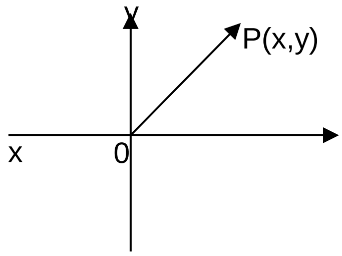
3.1 Definition

Coordinate Geometry deals with expressions and equations involving coordinates. We shall consider two coordinate systems

1. Cartesian Coordinates

2. Polar Coordinates

The usual symbol for Cartesian coordinates are (x, y) for two – dimensional. The plane is divided into four equal parts by two perpendicular lines denoted as x and y axes.



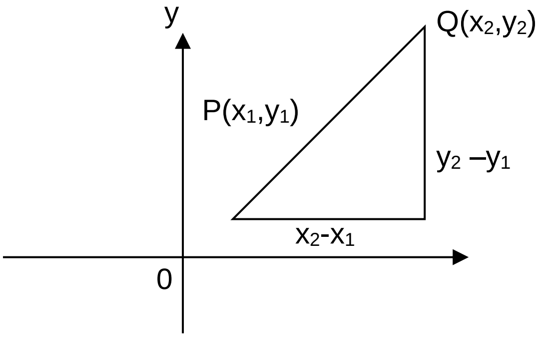
The point p in the plane has coordinate denoted by (x, y) where x is called the abscissa and y is called the ordinate. O which is the meeting point of the lines is called the origin and each of the four divisions of the plane is called a quadrant.

Self-Assessment Exercise(s) 1

1. What symbols are normally used to represent the two-dimensional Cartesian coordinates?
2. What is the x- and y-coordinate of the Cartesian plane called?

3.2 Distances between Two Points

Consider the diagram below



Let d be the distance between $P(x_1, y_1) and Q(x_2, y_2)$.

By Pythagoras theorem

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$\Rightarrow d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example

To find the distance between the points (2, 3) and (- 2, 5)

Solution

The distance is given as

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$x_1 = 2, \quad y_1 = 3, \quad x_2 = -2, \quad y_2 = 5$$

$$d = \sqrt{(-2 - 2)^2 + (5 - 3)^2}$$

$$d = \sqrt{(-4)^2 + (2)^2}$$

$$d = \sqrt{16 + 4}$$

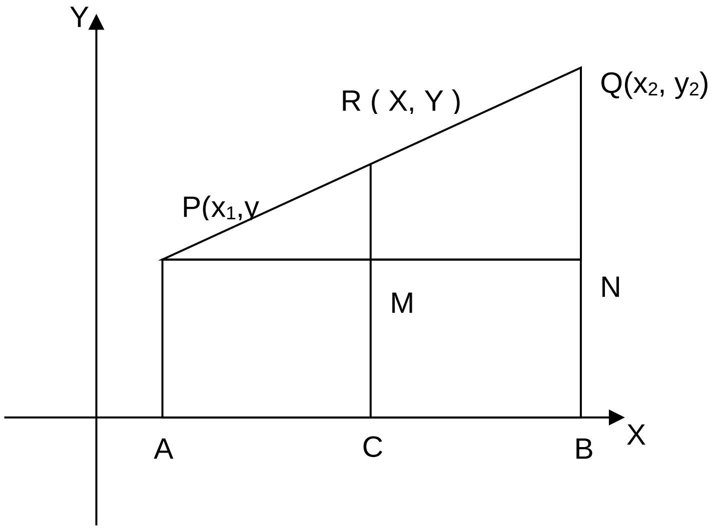
$$d = \sqrt{20} = 2\sqrt{5}$$

Self-Assessment Exercise(s) 2

1. Show that the distance between the points P (a, 2a) and Q (- 3a, - a) is 5a where “a” is a constant.
2. Find the distance between the points (2, 1) and (2, 5)

3.3 Dividing A Straight Line in the Ratio m : n

In the diagram below, R divides PQ internally in the ratio m : n



$$AC\colon CB = PM\colon MN = PR\colon RQ = m : n$$

$$since \quad AC\colon CB = m : n$$

$$we\ have \qquad \frac{AC}{CB} = \frac{m}{n}$$

$$ACn - CBm = 0$$

$$hance \quad n(X - x_1) - m(x_2 - X) = 0$$

$$since \quad AC = X - x_1 \text{ and } CB = x_2 - X$$

$$\therefore X = \frac{nx_1 + mx_2}{n + m}$$

Similarly,

$$Y = \frac{ny_1 + my_2}{n + m}$$

3.4 Mid – Point of a Straight Line

If the point $R(x, y)$ is the mid – point of the line joining the points $P(x_1, y_1)$ and $Q(x_2, y_2)$, then

$$m = n$$

$$\therefore X = \frac{nx_1 + nx_2}{n + n} = \frac{n(x_1 + x_2)}{2n} = \frac{x_1 + x_2}{2}$$

And

$$Y = \frac{ny_1 + ny_2}{n + n} = \frac{y_1 + y_2}{2}$$

Example

Given the points $A(1,-2)\ and\ B(4,1)$find the

i. Mid – point of the line AB

ii. Coordinates of the point that divides the line AB in the ratio 1 : 2.

Solution

i. The mid – point of the line AB is

$$X = \frac{x_1 + x_2}{2}$$

$$x_1 = 1, \qquad x_2 = 4$$

$$X = \frac{1+4}{2} = \frac{5}{2}$$

$$Y = \frac{y_1 + y_2}{2}$$

$$y_1 = -2, \qquad y_2 = 1$$

$$Y = \frac{-2+1}{2} = -\frac{1}{2}$$

∴ The mid – point of AB is $(X, Y) = \left(\frac{5}{2}, -\frac{1}{2}\right)$

ii. Coordinates of the point that divides the line AB in the ratio 1 : 2

$$m : n = 1 : 2$$

$$x_1 = 1, \qquad x_2 = 4$$

$$X = \frac{nx_1 + mx_2}{n+m}$$

$$X = \frac{2(1) + 1(4)}{2+1} = 2$$

Similarly

$$y_1 = -2, \qquad y_2 = 1$$

$$Y = \frac{ny_1 + my_2}{n+m}$$

$$Y = \frac{2(-2) + 1(1)}{2+1} = -1$$

∴ Coordinates of the point that divides the line AB in the ratio 1 : 2 is

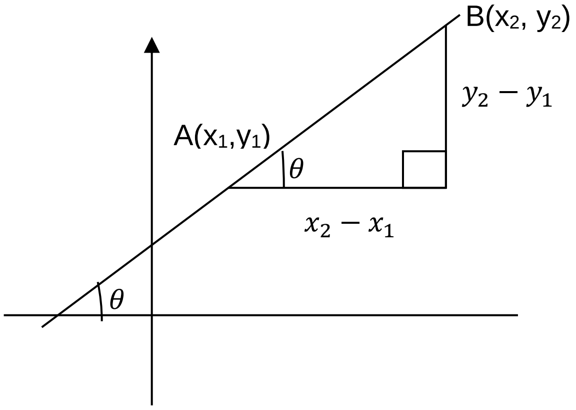
$$(X, Y) = (2, -1)$$

Self-Assessment Exercise 3

1. Find the mid-point of line XY. Given X (6,2) and Y (4,8)

3.5 The Slope of a Straight Line

The slope (or gradient) of a straight line is the value of the tangent of the angle which the line makes with the x – axis



Let m be the slope or gradient of the straight-line AB. Then,

$$m = tan\theta = \frac{y_2 - y_1}{x_2 - x_1}$$

Example

Obtain the slope of the line joining the point A (2, -1) and B (5, 2)

Solution

$$x_1 = 2, \quad x_2 = 5, \quad y_1 = -1, \quad y_2 = 2$$

$$m = tan\theta = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{2 - (-1)}{5 - 2} = 1$$

Self-Assessment Exercise 4

1. Find the slope of line CD. Given C (4,2) and D(1,6)

4.0 Conclusion

Two perpendicular lines divide the plane into four equal parts and that enable us to locate points on the plane using the x and y coordinates. Study of line and obtaining the slope becomes useful in further studies.

5.0 Summary

You have learnt in this unit the following:

1. To locate a point P on a plane with the Cartesian coordinates (x, y), where x is called the abscissa and y the ordinate.

2. To obtain the distance between two points $P(x_1, y_1) and Q(x_2, y_2)$. which is given as

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

3. To obtain the coordinate R (X, Y) that divides the straight line $P(x_1, y_1) and\ Q(x_2, y_2)$ in the ratio m : n, where $X = \frac{nx_1+mx_2}{n+m}$ and $Y = \frac{ny_1+my_2}{n+m}$

4. To obtain the slope (or gradient) m of a straight line $A(x_1, y_1) and\ B(x_2, y_2)$. i.e. $m = tan\theta = \frac{y_2-y_1}{x_2-x_1}$

6.0 Tutor-Marked Assignments

1. Find the distance between the two points
   i. A (0, 5) and B (-3, 8)
   ii. $A\left(-\frac{1}{2}, \frac{5}{8}\right)\ and\ B\left(-\frac{3}{2}, \frac{1}{4}\right)$
2. Obtain the point that divide the following lines in the ratio 2 : 3
   i. A (0, 5) and B (-3, 8)
   ii. $A\left(-\frac{1}{2}, \frac{5}{8}\right)\ and\ B\left(-\frac{3}{2}, \frac{1}{4}\right)$
3. Find the slope of the line joining the points $(-1, -5) and\ (-3, -2)$

7.0 References/Further Reading

Olabisi O. Ugbebor and Ninuola I. Akinwande (2000) Analytical Geometry and Mechanics, Ibadan: Y – Books.

Bostock, L. And Clandler, S. (1979) Pure Mathematics 2, UK: Stanley Thomes Ltd

Unit 2

Equation of a Straight Line I

Contents

1.0 Introduction

From unit 1 we are able to obtain the slope of straight line given two points on the line, this unit shall explain how to obtain the equation of the straight line given the a point on the straight line and the slope and also given two points on the straight line.

2.0 Learning Outcomes

At the end of this unit you should be able to:

1. Obtain equation of a straight line given the slope and a point on the line.
2. Obtain the equation of a straight line given two points on the line.
3. Determine different forms of straight lines.
4. Get the angle between two straight lines.

3.0 Learning Content

3.1 Equation of a Straight Line

The equation

$$ax + by + c = 0 \tag{1}$$

Is the equation of a straight-line?

If we put $a = 0$

We have

$$by + c = 0 \Rightarrow y = -\frac{c}{b} = d(say)$$

This gives the equation of a straight-line parallel to the x – axis.

If we put $b = 0$

We have

$$ax + c = 0 \Rightarrow x = -\frac{c}{a} = p(say)$$

This gives the equation of a straight-line parallel to the y – axis.

From equation (1)

If we put $a \neq 0\,, b \neq 0,\ then\ we\ have$

$$y + \frac{a}{b}x + \frac{c}{b} = 0$$

$$y = -\frac{a}{b}x - \frac{c}{b}$$

$$y = mx + c, \quad \left(m = -\tfrac{a}{b},\ \ c = -\tfrac{c}{b}\right) \tag{2}$$

m is called the slope of the line

c is called the y – intercept

If the point P$(x_1 y_1)$ lies on the line (2) above we have

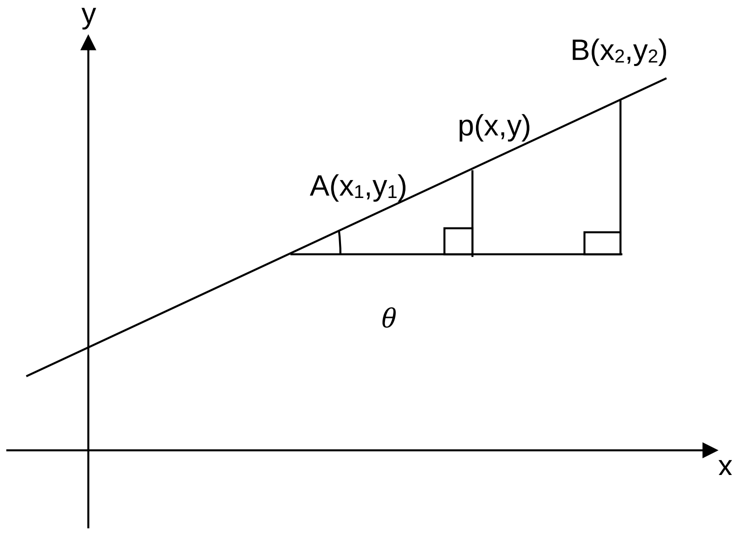
$$y_1 = mx_1 + c \tag{3}$$

Subtracting equation (3) from equation (2) gives

$$y - y_1 = m(x - x_1) \tag{4}$$

Equation (4) is a straight line with given slope m and passing through a given point $P(x_1, y_1)$.

Consider the diagram below



$$tan\theta = \frac{y_2 - y_1}{x_2 - x_1} = m$$

$$tan\theta = \frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1} = m$$

$$\Rightarrow \frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1} \tag{5}$$

Equation (5) is a line passing through two given points $(x_1,\ y_1)$ and $(x_2,\ y_2)$

Example

1. Find the equation of the line through (2, -3) whose slope is 3

Solution

$$y - y_1 = m(x - x_1)$$

$$x_1 = 2, \qquad y_1 = -3, \qquad m = 3$$

$$y - (-3) = 3(x - 2)$$

$$y = 3x - 9$$

2. Find the equation of the line through (2, -3) and (-1, 4)

Solution

$$\frac{y-y_1}{x-x_1}=\frac{y_2-y_1}{x_2-x_1}$$

$$x_1=2, \qquad y_1=-3, \qquad x_2=-1, \qquad y_2=4$$

$$\frac{y-(-3)}{x-2}=\frac{4-(-3)}{-1-2}$$

$$3y=-7x+5$$

3.2 Forms of the Equation of a Straight Line

If P (x, y) is an arbitrary point on the line, we have

i. The linear form

$$ax+by+c=0$$

Where a, b and c are constants.

ii. The slope intercept form

$$y=mx+c$$

Where m is the slope of the line and c is the y – intercept.

iii. The intercept forms

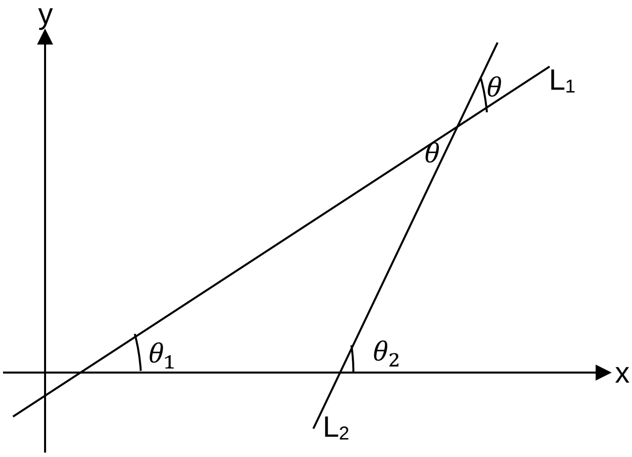
$$\frac{x}{a}+\frac{y}{b}=1$$

Where a and b are respectively the x and y intercept.

Self-Assessment Exercise 1

1. What are the three (3) forms of representing an arbitrary point on a line?

3.3 Angle between Two Straight Lines



Let the slope of l_1 be $tan\theta_1=m_1$

the slope of l_2 be $tan\theta_2 = m_2$

the angle between the two lines be θ.

From the diagram

$$\theta_2 = \theta_1 + \theta$$

$$\Rightarrow \theta = \theta_2 - \theta_1$$

$$tan\theta = \tan(\theta_2 - \theta_1)$$

$$= \frac{tan\theta_2 - tan\theta_1}{1 + tan\theta_2 tan\theta_1}$$

$$= \frac{m_2 - m_1}{1 + m_2 m_1}$$

Example

1. Find the tangent of the angle between the lines $y = 2x - 9 \;\; and \;\; 2y = 7x + 4$

Solution

Let the gradient of the line $\quad y = 2x - 9$ be

$$m_1 = 2$$

And the gradient of the line $\quad 2y = 7x + 4$ be

$$m_2 = \frac{7}{2}$$

$$tan\theta = \frac{m_2 - m_1}{1 + m_2 m_1}$$

$$tan\theta = \frac{\frac{7}{2} - 2}{1 + \frac{7}{2}(2)} = \frac{3}{16}$$

4.0 Conclusion

In further units, we shall study the tangent and normal to conic sections, tangent and normal are straight lines, therefore, studying the equation of straight line become necessary for further studies.

5.0 Summary

You have learnt in this unit the following:

1. Equation of a straight line to be $y - y_1 = m(x - x_1)$ given a point $P(x_1, y_1)$ and the slope m.
2. Equation of a straight line to be $\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$ given two points $P(x_1, y_1)$ and $Q(x_1, y_1)$
3. Three forms of equation of straight line

The linear form $ax + by + c = 0$

The slope intercept form $y = mx + c$

The intercept form $\frac{x}{a} + \frac{y}{b} = 1$

4. Angle between two straight lines l_1 and l_2 with the slopes m_1 and m_2 respectively to be $tan\theta = \frac{m_2 - m_1}{1 + m_2 m_1}$

6.0 Tutor-Marked Assignment

1. Find the equation of the line through (-1, -5) whose slope is 3.
2. Find the equation of the line through $(3\sqrt{5}, 2\sqrt{7})$and $(-\sqrt{5}, -3\sqrt{7})$. Hence write the equation in linear form, slope intercept form and intercept form.
3. Find the tangent of the angle between the lines $3y = 2x$ and $y = 7x - 4$

7.0 References/Further Reading

Olabisi O. Ugbebor and Ninuola I. Akinwande (2000) Analytical Geometry and Mechanics, Ibadan: Y – Books.

Bostock, L. And Clandler, S. (1979) Pure Mathematics 2, UK: Stanley Thomes Ltd

Unit 3

Equation of a Straight Line II

Contents

1.0 Introduction

From the previous unit, we are able to obtain the equation of a straight line; this unit will explain the distance from a straight line to the origin and to any other point.

2.0 Learning Outcomes

At the end of this unit you should be able to:

1. Determine when two lines are perpendicular
2. Obtain the distance from the origin to the straight line$ax + by + c = 0$.
3. Obtain distance from a point (x_1, y_1) to the straight line$ax + by + c = 0$.

3.0 Learning Content

3.1 Perpendicular Line

Two straight line $y = m_1x + c_1$ *and* $y = m_2x + c_2$ *are* said to be perpendicular if

$$1 + m_1m_2 = 0$$

$$\Rightarrow \quad m_1 = -\frac{1}{m_2},$$

and parallel if

$$m_1 - m_2 = 0$$

$$\Rightarrow m_1 = m_2$$

Example

Find the equation of the line through $(-1, 2)$ which is perpendicular to the line $6y - 5x - 2 = 0$

Solution

The gradient of the line $6y - 5x - 2 = 0$ is $m_1 = \frac{5}{6}$

And for the perpendicular line will be

$$m_2 = -\frac{1}{m_1} = -\frac{6}{5}$$

The equation of the perpendicular line is

$$y - y_1 = m(x - x_1)$$

$$x_1 = -1, \qquad y_1 = 2$$

$$y - 2 = -\frac{6}{5}(x - (-1))$$

$$5y + 6x - 4 = 0$$

3.2 Distance from the Origin to the Straight Line $ax + by + c = 0$

The distance d is given by

$$d = \left|\frac{c}{\sqrt{(a^2 + b^2)}}\right|$$

Example

Find the distance between the line $3y + 4x - 4 = 0$ and the origin

Solution

Comparing with the equation $ax + by + c = 0$

$$a = 4, \qquad b = 3, \qquad c = -4$$

$$d = \left|\frac{c}{\sqrt{(a^2 + b^2)}}\right|$$

$$d = \left|\frac{-4}{\sqrt{(4^2 + 3^2)}}\right|$$

$$d = \left|\frac{-4}{\sqrt{25}}\right| = \frac{4}{5}$$

3.3 Distance from a Point (x_1, y_1) to the Straight Line

$$\boldsymbol{ax + by + c = 0}$$

The distance d is given by

$$d = \left|\frac{ax_1 + by_1 + c}{\sqrt{(a^2 + b^2)}}\right|$$

Example

Find the distance between the point $(2, 3)$ the line $3y + 4x - 4 = 0$ and the origin

Solution

Comparing with the equation $ax + by + c = 0$

$$a = 4, \qquad b = 3, \qquad c = -4, \qquad x_1 = 2, \qquad y_1 = 3$$

$$d = \left|\frac{ax_1 + by_1 + c}{\sqrt{(a^2 + b^2)}}\right|$$

$$d = \left|\frac{4(2) + 3(3) - 4}{\sqrt{(4^2 + 3^2)}}\right|$$

$$d = \left|\frac{13}{\sqrt{25}}\right| = \frac{13}{5}$$

Self-Assessment Exercise 1

1. Find the slope of a line perpendicular to the line whose equation is 2*y* + 6*x* = 24.

4.0 Conclusion

Obtaining distance from the origin and any point on the plane to a straight line enable us to solve problems in plane figures, e.g. to obtain the area of a triangle where the height is not given but the vertex coordinates are given.

5.0 Summary

You have learnt in this unit the following:

1. Two straight line $y = m_1x + c_1 \; and \; y = m_2x + c_2 \; are$ said to be perpendicular if $m_1 = -\frac{1}{m_2}$ and parallel if $m_1 = m_2$

2. Distance from the origin to the straight line $ax + by + c = 0$ to be $d = \left|\frac{c}{\sqrt{(a^2+b^2)}}\right|$

3. Distance from a point (x_1, y_1) to the straight line $ax + by + c = 0$ to be $d = \left|\frac{ax_1+by_1+c}{\sqrt{(a^2+b^2)}}\right|$

6.0 Tutor-Marked Assignment

1. Given the points A (-2, 3) and B (4, 7), find the equation of the perpendicular bisector of the line AB.
2. The points A (3, 1), B (8, 2) and C (-1, 11) are the vertices of a triangle, if N is the foot of the perpendicular from A to BC, find the
   i. equation of BC
   ii. equation of AN
   iii. length AN.

7.0 References/Further Reading

Olabisi O. Ugbebor and Ninuola I. Akinwande (2000) Analytical Geometry and Mechanics, Ibadan: Y – Books.

Bostock, L. And Clandler, S. (1979) Pure Mathematics 2, UK: Stanley Thomes Ltd

Module 2

Conic Section

Unit 1

Equation of a Circle

Contents

1.0 Introduction

In previous units, we are able to study the behaviour of a straight line. Coordinate geometry is divided into two sections i.e. straight lines and conic sections. This unit will give the definition of conic section and the study of a circle.

2.0 Learning Outcome

At the end of this unit you should be able to:

1. Define a Conic Section
2. Obtain the standard equation of a circle

3.0 Learning Content

3.1 Conic Section

Conic Section are figures that can be formed by slicing a three-dimensional right circular cone with a plane. A conic section can be formally defined as a set or locus of a point that moves in the plane of a fixed point called the focus and the fixed line called the directrix. Circles, Parabolas, Ellipses and Hyperbolas are called Conic Sections, because they result from intersecting a cone with a plane.

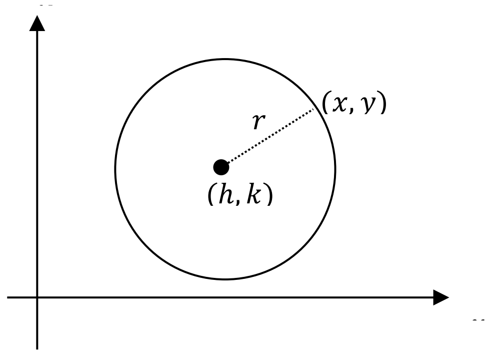
Self-Assessment Exercise(s) 1

1. What is a conic section?
2. How are conic sections formed?

3.2 Circles

A circle is defined as the set of all points in a plane that are equidistant from a fixed point called the center. The fixed distance from the center is called the radius and is denoted by r, where $r > 0$.

Suppose a circle is centered at the point (h, k) and has radius, r (Figure 11-3). The distance formula can be used to derive an equation of the circle.



Let (x, y) be any arbitrary point on the circle. Then, by definition, the distance between (h, k) and (x, y) must be r.

$$\sqrt{(x-h)^2 + (y-k)^2} = r$$

$$(x-h)^2 + (y-k)^2 = r^2$$

3.3 Standard Equation of a Circle

The standard equation of a circle, centered at (h, k) with radius r, is given by

$$(x-h)^2 + (y-k)^2 = r^2 \qquad where\ \ r > 0$$

Note: If a circle is centered at the origin (0,0), then the equation simplifies to $x^2 + y^2 = r^2$.

Example

Find the centre and radius of each of the following circles

1. $(x-4)^2 + (y+9)^2 = 36$
2. $x^2 + (y-\frac{7}{3})^2 = \frac{25}{4}$
3. $x^2 + y^2 = 5$
4. $x^2 + y^2 - 10x + 4y - 7 = 0$

Solution

1. $(x-4)^2 + (y+9)^2 = 36$

$(x-4)^2 + [y-(-9)]^2 = 6^2$

Compeering with $(x-h)^2 + (y-k)^2 = r^2$,
we have $h = 4\ , k = -9\ \ and\ r = 6$

∴ the centre is (4, -9) and the radius is 6

2. $x^2 + (y-\frac{7}{3})^2 = \frac{25}{4}$

$(x-0)^2 + \left(y-\frac{7}{3}\right)^2 = \left(\frac{5}{2}\right)^2$

The center is $\left(0,\ \ \frac{7}{3}\right)$ and the radius is $\frac{5}{2}$

3. $x^2 + y^2 = 5$

$(x-0)^2 + (y-0)^2 = \left(\sqrt{5}\right)^2$

The center is $(0,\ \ 0)$ and the radius is $\sqrt{5}$

4. $x^2 + y^2 - 10x + 4y - 7 = 0$

We write the equation in the form

(x−h)2+(y−k)2=r2 $\qquad (x-h)^2 + (y^2 + 4y) = \ \ 7$

From the equation, we group the x and y terms and move the constant to the right

$$(x^2 - 10x) + (y^2 + 4\mathrm{y}) = 7$$

Complete the square on x any by adding square of half the coefficients of both x and y to both sides of the equation.

$$(x^2 - 10x + 5^2) + (y^2 + 4\text{y} + 2^2) = 7 + 25 + 4$$

Factor and simplify

$$(x - 5)^2 + (y + 2)^2 = 36$$

$$\Rightarrow \ (x - 5)^2 + (y - (-2))^2 = 6^2$$

The centre is $(5,\ -2)$ and the radius is 6

Self-Assessment Exercise(s) 2

1. The point (-4, -2) lies on a circle. What is the length of the radius of this circle if the center is located at (-8, -10)?
2. Find the radius of a circle whose diameter has endpoints (-3, -2) and (7, 8).

4.0 Conclusion

Studying the conic section give a clue on how plane figures can be formed by slicing a right circular cone. Circle one of the conic sections is

5.0 Summary

You have learnt in this unit the following:

1. Define a Conic Section as locus of a point that moves in the plane of a fixed point called the focus and the fixed line called the directrix,
2. Definition of a circle and standard equation of a circle as $(x - h)^2 + (y - k)^2 = r^2$

6.0 Tutor-Marked Assignment

1. Identify the centre and the radius of the following circle

i. $(x - 4)^2 + (y + 2)^2 = 9$

ii. $x^2 + y^2 - 2x - 6y - 26 = 0$

2. Write an equation of a circle centered at (5, -1) with a diameter of 8.

7.0 References/Further Reading

Olabisi O. Ugbebor and Ninuola I. Akinwande (2000) Analytical Geometry and Mechanics, Ibadan: Y – Books.

Bostock, L. And Clandler, S. (1979) Pure Mathematics 2, UK: Stanley Thomes Ltd

Unit 2

Tangent and Normal Through A Point on A Circle

Contents

1.0 Introduction

The previous unit explain the equation of a circle. This unit will explain the equation of straight line that touches the circle at a point (tangent) and a line that is perpendicular to the tangent at that point (normal).

2.0 Learning Outcomes

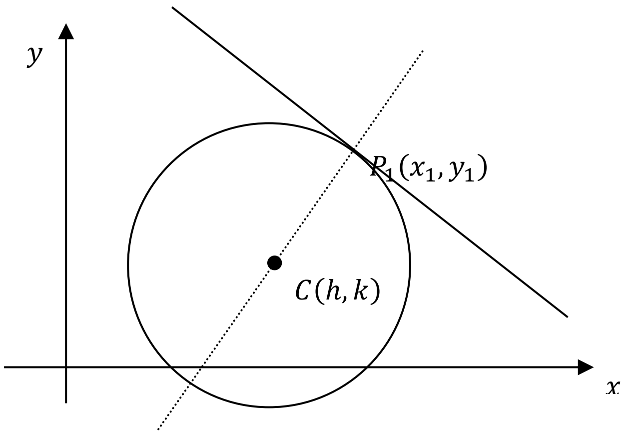
At the end of this unit you should be able to:

1. Obtain the equation of a tangent through a point to a circle.
2. Obtain the equation of normal through a point to a circle.

3.0 Learning Content

3.1 Tangent through a Point on a Circle

Let $(x-h)^2+(y-k)^2=r^2$ be a circle and $P_1(x_1,\ y_1)$ a point on it as shown in the figure below.



If the circle is centered at the origin, $x^2+y^2=r^2$, the equation of the tangent line at P_1 is

$$x_1x+y_1y=r^2$$

If the circle is given in general form,

$$x^2+y^2+ax+by+c=0$$

then the tangent line at P_1 is

$$xx_1+yy_1+\frac{a}{2}(x+x_1)+\frac{b}{2}(y+y_1)+c=0$$

3.2 Normal through a Point on a Circle

Since normal is perpendicular to the tangent, the slope to the normal will be equal to the negative inverse to the slope of the tangent

Let the slope to the tangent be m_1

Then the slope to the normal m_2 will be

$$m_2 = -\frac{1}{m_1}$$

Example

1. Find the equation of the tangent and normal to the circle $x^2 + y^2 = 4$ at the point P(2, 3)

Solution

Comparing with the equation of circle center (0, 0), we have r = 2.

At the point P (2, 3) gives $x_1 = 2$ and $y_1 = 3$

The equation of the tangent gives

$$2x + 3y = 2^2$$

$$\Rightarrow\ 2x + 3y = 4$$

Since normal is perpendicular to the tangent, the slope to the normal will be equal to the negative inverse to the slope of the tangent.

Let the slope to the tangent be m_1

$$i.e \quad m_1 = -\frac{2}{3}$$

Then slope to the normal will be m_2

$$m_2 = -\frac{1}{m_1} = \frac{3}{2}$$

∴ the equation of the normal is

$$y - y_1 = m_2(x - x_1)$$

$$\Rightarrow\ \ y - 3 = \frac{3}{2}(x - 2)$$

$$2y = 3x$$

2. Find the equation of the tangent and normal to the circle $x^2 + y^2 - 10x + 4y - 7 = 0$ at the point P (-2, 3).

Solution

Comparing with the general form of equation of a circle

$$x^2 + y^2 + ax + by + c = 0$$

$$a = -10, \qquad b = 4 \ \ and \ \ c = -7$$

With the point P (-2, 3)

$$x_1 = -2 \ \ and \ \ y_1 = 3$$

∴ the equation of the tangent gives

$$xx_1 + yy_1 + \frac{a}{2}(x + x_1) + \frac{b}{2}(y + y_1) + c = 0$$

$$\Rightarrow \quad -2x + 3y - \frac{10}{2}(x - 2) + \frac{4}{2}(y + 3) - 7 = 0$$

which simplify to

$$5y - 7x + 9 = 0$$

Let the slope to the tangent be m_1

$$m_1 = \frac{7}{5}$$

Then slope to the normal will be m_2

$$m_2 = -\frac{1}{m_1} = -\frac{5}{7}$$

∴ the equation of the normal is

$$y - y_1 = m_2(x - x_1)$$

$$\Rightarrow \ y - 3 = -\frac{5}{7}(x - (-2))$$

$$7y + 5x - 11 = 0$$

Self-Assessment Exercise 1

1. Determine the equation of the tangent to the circle $x^2+y^2-2y+6x-7 = 0$ at the point $F(-2;5)$.

4.0 Conclusion

Tangent and normal are straight lines that touches the curve at a point, studying the equation of tangent and normal to a circle will guide in understanding the concept of conic section. Since normal is perpendicular to tangent, we use the concept of perpendicular lines to obtain the slope of normal from the tangent.

5.0 Summary

You have learnt in this unit the following:

1. Derived the equation of the tangent at the point $P_1(x_1, y_1)$ to the circle $x^2 + y^2 + ax + by + c = 0$ as $xx_1 + yy_1 + \frac{a}{2}(x + x_1) + \frac{b}{2}(y + y_1) + c = 0$

6.0 Tutor-Marked Assignment

Find the equation of tangent and normal to the circle $x^2 + y^2 + 2x + 4y - 4 = 0$ at the point $P(2, 3)$.

7.0 References/Further Reading

Olabisi O. Ugbebor and Ninuola I. Akinwande (2000) Analytical Geometry and Mechanics, Ibadan: Y – Books.

Bostock, L. And Clandler, S. (1979) Pure Mathematics 2, UK: Stanley Thomes Ltd

Unit 3

Equation of Parabola

Contents

1.0 Introduction

This unit explains the part of conic section formed by intersecting the plain through the cone and the top of the cone. Quadratic equations are like parabola, but parabola formed by conic section are little different.

2.0 Learning Outcomes

At the end of this unit you should be able to:

1. State the proper definition of a Parabola
2. Obtain the standard equation of a Parabola

3.0 Learning Contents

3.1 Definition

A **parabola** is the set of points in a plane that are equidistant from a fixed-point *F* (called the **focus**) and a fixed line (called the **directrix**). This definition is illustrated by Figure 1.

Notice that the point halfway between the focus and the directrix lies on the parabola; it is called the **vertex**. The line through the focus perpendicular to the directrix is called the **axis** of the parabola

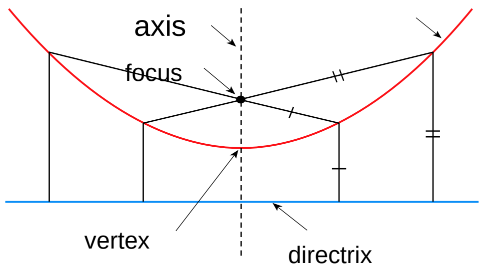
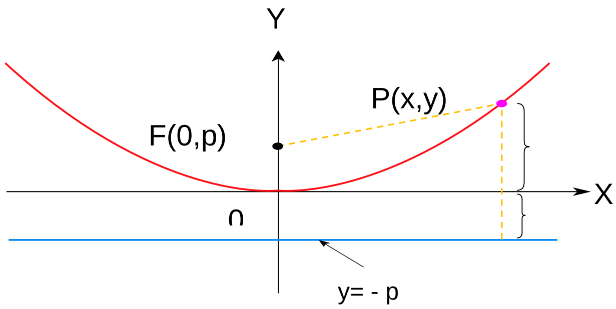


Figure1: Parabola

3.2 Equation of Parabola

We obtain a particularly simple equation for a parabola if we place its vertex at the origin *O* and its directrix parallel to the *x*-axis as in Figure 2. If the focus is the point (0, p) , then the directrix has the equation *y p*. If *P(x, y)* is any point on the parabola, then the distance from *P* to the focus is



$$|PF| = \sqrt{x^2 + (y-p)^2}$$

and the distance from *P* to the directrix is $|y+p|$. (Figure 2 illustrates the case where *p* >0.) The defining property of a parabola is that these distances are equal:

$$\sqrt{x^2 + (y-p)^2} = |y+p|$$

We get an equivalent equation by squaring and simplifying:

$$x^2 + (y-p)^2 = |y+p|^2 = (y+p)^2$$

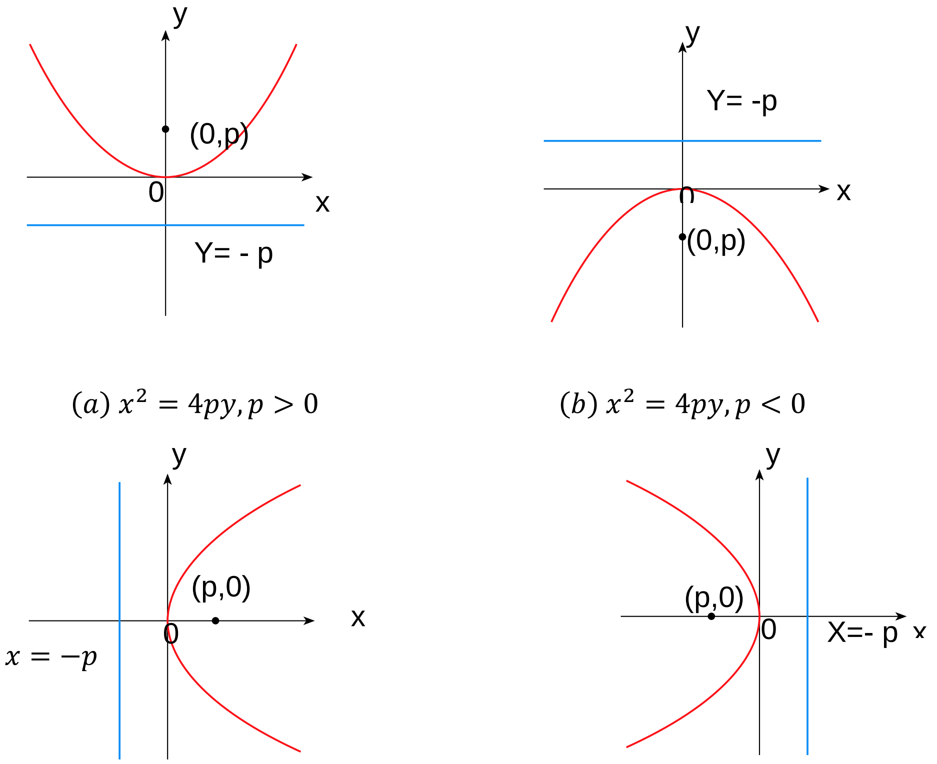
$$x^2 + y^2 - 2py + p^2 = y^2 + 2py + p^2$$

$$x^2 = 4py$$

An operation of the parabola with focus $(0, p)$ and directrix $y = -p\ is$

$$x^2 = 4py \qquad **$$

If we write $a = \frac{1}{(4p)}$, then the standard equation of a parabola (**) becomes $y = ax^2$ It opens upward if $p > 0$ and downward if $p < 0$ [see Figure 3, parts (a) and (b)]. The graph is symmetric with respect to the *y*-axis because (**) is unchanged when *x* is replaced by $-x$.



$(a)\ x^2 = 4py, p > 0$

$(b)\ x^2 = 4py, p < 0$

$(c)\ y^2 = 4px, p > 0$

$(d)\ y^2 = 4px, p < 0$

Figure 3

If we interchange *x* and *y* in (**), we obtain

$$y^2 = 4px$$

which is an equation of the parabola with focus (p, 0) and directrix $x = -p$. (Interchanging *x* and *y* amounts to reflecting about the diagonal line $y = x$. The parabola opens to the right if $p > 0$ and to the left if $p < 0$ [see Figure 3, parts (c) and (d)]. In both cases the graph is symmetric with respect to the *x*-axis, which is the axis of the parabola

Example

1. Find the vertex, focus and directrix of the parabola

$$x = 2y^2$$

$$\Rightarrow y^2 = \frac{1}{2}x,$$

$$4p = \frac{1}{2},$$

$$so\ p = \frac{1}{8}$$

$The\ vertex\ is\ (0,0), the\ focus\ is\ \left(\frac{1}{8},0\right), and\ the\ directrix\ is\ x = -\frac{1}{8}.$

1. Find the vertex, focus and directrix of the parabola

$$4y + x^2 = 0$$

$$\Rightarrow x^2 - 4y$$

$$4p = -4,$$

$$So,\ p = -1.$$

$The\ vertex\ is\ (0,0),\ the\ focus\ is\ (0,-1) and\ the\ directrix\ is\ y = 1$

2. Find the vertex, focus and directrix of the parabola

$$y^2 = 12x$$

$$4p = 12,$$

$$So, \quad p = 3.$$

$The\ vertex\ is\ (0,0), the\ focus\ is\ (3,0), and\ the\ directrix\ is\ x = -3.$

3. Find the vertex, focus and directrix of the parabola

$$(x+2)^2 = 8(y-3)$$

$$4p = 8$$

$$So,\ p = 2.$$

$The\ vertex\ is\ (-2,3), the\ focus\ is\ (-2,5), and\ the\ directrix\ is\ y = 1$

4. Find the vertex, focus and directrix of the parabola

$$y^2 + 2y + 12x + 25 = 0$$

$$\Rightarrow y^2 + 2y + 1 = -12x - 24$$

$$\Rightarrow (y+1)^2 = -12(x+2)$$

$$4p = -12,$$

$$So,\ p = -3.$$

The vertex is (-2, -1), the focus is (-5, -1), and the directrix is x = 1

Self-Assessment Exercise 1

1. Sketch the curve and find the equation of the parabola with focus (-2,0) and directrix $x = 2$

4.0 Conclusion

Studying the concept of parabola enable us to understand some real-life problems since it has numerous real-world applications e.g. a reflecting telescope has a mirror with the cross section in the shape of a parabola.

5.0 Summary

You have learnt in this unit the following:

1. Give the full description of a parabola as the set of points in a plane that are equidistant from a fixed-point *F* (called the **focus**) and a fixed line (called the **directrix**).
2. Obtain the equation of a parabola with focus $(0, p)$ and directrix $y = -p$ as $x^2 = 4py$ and focus $(p, 0)$ and directrix $x = -p$ to be $y^2 = 4px$

6.0 Tutor-Marked Assignment

Find the vertex, focus and directrix to the following parabolas

1. $4x^2 = -y$
2. $x - 1 = (y+5)^2$

7.0 References/Further Reading

Olabisi O. Ugbebor and Ninuola I. Akinwande (2000) Analytical Geometry and Mechanics, Ibadan: Y – Books.

Bostock, L. And Clandler, S. (1979) Pure Mathematics 2, UK: Stanley Thomes Ltd

Unit 4

Tangent and Normal to a Parabola

Contents

1.0 Introduction

Previous unit gives proper definition and description of parabola, in this unit we discuss the equation of tangent and normal to the parabola at a point.

2.0 Learning Outcome

At the end of this unit you should be able to:

1. Obtain the equation of tangent to parabola
2. Obtain the equation of normal to parabola

3.0 Learning Contents

3.1 Equation of Tangent to a Parabola

Differentiating the equation $y^2 = 4ax$ with respect to x gives

$$\frac{dy}{dx} = \frac{2a}{y}$$

So the slope of the tangent to the parabola at the point (x_1, y_1) is given by

$$m_1 = \frac{2a}{y_1}$$

The equation of the tangent is given as

$$y - y_1 = m_1(x - x_1)$$

$$\Rightarrow \quad yy_1 = 2a(x + x_1)$$

3.2 Equation of Normal to a Parabola

Since normal is perpendicular to the tangent, the slope of normal will be the negative inverse to that of the tangent.

i.e. the slope of the normal at the same point is given by

$$m_2 = -\frac{y_1}{2a}$$

$\therefore$ The equation of the normal is

$$y - y_1 = m_2(x - x_1)$$

$$\Rightarrow \quad y - y_1 = -\frac{y_1}{2a}(x - x_1)$$

Example

1. Find the equation of the tangent and normal to the parabola $y^2 = 6x$ at the (-2, 3).

Solution

From the equation of the parabola $y^2 = 6x$ and $y^2 = 4ax$

$$4a = 6 \Rightarrow a = {}^{3}/_{2}$$

From the point (-2, 3)

$$x_1 = -2 \;\; and \;\; y_1 = 3$$

The equation of the tangent

$$yy_1 = 2a(x + x_1)$$

$$3y = 2\left({}^{3}/_{2}\right)(x - 2)$$

$$y = x - 2$$

The equation of the normal

$$y - y_1 = -{}^{y_1}/_{2a}\,(x - x_1)$$

$$y - 3 = -{}^{3}/_{3}\,(x + 2)$$

$$y = -x + 1$$

2. Find the point of intersection of the line $x - 2y + 6 = 0$ and the parabola $y^2 = 6x$ and the equation of the tangent and normal to the parabola at the point of intersection.

Solution

From the equation $x - 2y + 6 = 0 \;\; \Rightarrow \;\; x = 2y - 6$

Substituting into the parabola gives

$$y^2 = 6x$$

$$= 6(2y - 6)$$

$$\Rightarrow \;\; y^2 - 12y + 36 = 0$$

Solving the quadratic equation gives

$$y = 6 \;\; twice$$

Substituting y into the equation of straight line gives

$$x = 6$$

∴ The point of intersection is $(x_1,\; y_1) = (6,\; 6)$

The tangent at this point is given by

$$yy_1 = 2a(x + x_1)$$

From $\quad y^2 = 6x = 4ax$

$$\Rightarrow \;\; 4a = 6$$

$$\therefore \;\; a = {}^{3}/_{2}$$

And the equation of the tangent at this point is given by

$$6y = 2\left({}^{3}/_{2}\right)(x + 6)$$

$$2y - x - 6 = 0$$

The equation of the normal is given by

$$y - y_1 = -\frac{y_1}{2a}(x - x_1)$$

$$y - 6 = -\frac{6}{2(^3/_2)}(x - 6)$$

$$y + 2x - 18 = 0$$

Self-Assessment Exercise

1. Find the equations of the tangents and normal to the parabola $y^2 = 16 \text{ x } x$ at the points (16,16) and (1, -4). The tangents intersect at the point $T$and the normal intersect at R. Prove that the line TR is parallel to the axis of the Parabola.

4.0 Conclusion

Getting the equation of a parabola alone is enough to give full description of parabola; we need the equation of tangent and normal to the parabola.

5.0 Summary

You have learnt in this unit the following:

1. Derived the equation of the tangent at the point $P(x_1,\ y_1)$ to the parabola $y^2 = 4ax$ as $yy_1 = 2a(x + x_1)$
2. Derived the equation of the normal at the point $P(x_1,\ y_1)$ to the parabola $y^2 = 4ax$ as $y - y_1 = -\frac{y_1}{2a}(x - x_1)$

6.0 Tutor-Marked Assignment

Find the equation of the tangent to the parabola $y^2 = 4ax$ which is parallel to the line $y + 2x = 0$

7.0 References/Further Reading

Olabisi O. Ugbebor and Ninuola I. Akinwande (2000) Analytical Geometry and Mechanics, Ibadan: Y – Books.

Bostock, L. And Clandler, S. (1979) Pure Mathematics 2, UK: Stanley Thomes Ltd

Module 3

Ellipse and Hyperbola

Unit 1: Equation of an ellipse

Unit 2: Tangent and normal to an ellipse

Unit 3: Equation of a hyperbola

Unit 4: Tangent and normal to a hyperbola

Unit 1

Equation of An Ellipse

Contents

1.0 Introduction

In unit 4 we discuss the formation of a circle by slicing a three-dimensional cone with a plane. In this unit, we discuss an ellipse which differ from a circle in that an ellipse does not have a constant radius. It has a radius that changes in between an x and y radius.

2.0 Learning Outcomes

At the end of this unit you should be able to:

1. State the proper definition of an Ellipse
2. Obtain the standard equation of an Ellipse

3.0 Learning Content

3.1 Definition

An **ellipse** is the set of points in a plane the sum of whose distances from two fixed points F_1 and F_2 is a constant (see Figure 1). These two fixed points are called the **foci** (plural of **focus**). One of Kepler's laws is that the orbits of the planets in the solar system are ellipses with the Sun at one focus.

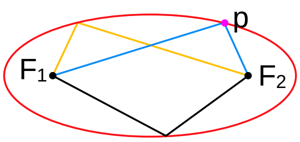


Figure 1

3.2 Equation of an Ellipse

In order to obtain the simplest equation for an ellipse, we place the foci on the x-axis at the points, $(-c, 0)$ and $(c, 0)$ as in Figure 2 so that the origin is halfway between the foci. Let the sum of the distances from a point on the ellipse to the foci be $2a > 0$. Then $p(x, y)$ is a point on the ellipse when

$$|PF_1| + |PF_2| = 2a$$

That is $$\sqrt{(x+c)^2 + y^2} + \sqrt{(x-c)^2 + y^2} = 2a$$

or $$\sqrt{(x-c)^2 + y^2} = 2a - \sqrt{(x+c)^2 + y^2}$$

Squaring both sides, we have

$$x^2 - 2cx + c^2 + y^2 = 4a^2 - 4a\sqrt{(x+c)^2 + y^2} + x^2 + 2cx + c^2 + y^2$$

which simplifies to $$a\sqrt{(x+c)^2 + y^2} = a^2 + cx$$

we square again:

$$a^2(x^2 + 2cx + c^2 + y^2) = a^4 + 2a^2cx + c^2x^2$$

which becomes $$(a^2 - c^2)x^2 + a^2y^2 = a^2(a^2 - c^2)$$

From triangle F_1F_2P in Figure 2 we see that $2c < 2a,\ so\ c < a$ and, therefore $a^2 - c^2 > 0$. For convenience, let $b^2 = a^2 - c^2$. Then the equation of the ellipse becomes $b^2x^2 + a^2y^2 = a^2b^2$ or, if both sides are divided by b^2a^2,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad 3.1$$

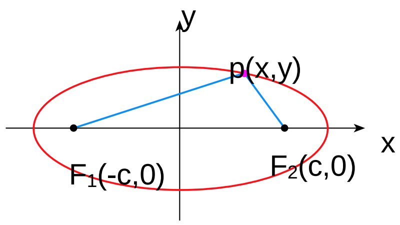


Figure 2

Since $b^2 = a^2 - c^2 < a^2, it\ follows\ that\ b < a$. The x-intercepts are found by setting $y = 0.\ Then\ \frac{x^2}{a^2} = 1, or\ x^2 = a^2, x = \pm a$. The corresponding points$(a, 0)$ and $(-a, 0)$ are called the **vertices** of the ellipse and the line segment joining the vertices is called the **major axis**. To find the y-intercepts we set $x = 0\ and\ obtain\ y^2 = b^2, so\ y = \pm b$. Equation 3.1 is unchanged if x is replaced by $-x\ or\ y\ is\ replaced\ by - y$, so the ellipse is symmetric about both axes. Notice that if the foci coincide, then $c = 0,\ so\ a = b\ and\ the\ ellipse\ \ becomes\ a\ circle\ with\ radious\ r = a = b$

We summarize this discussion as follows (see also Figure 3).

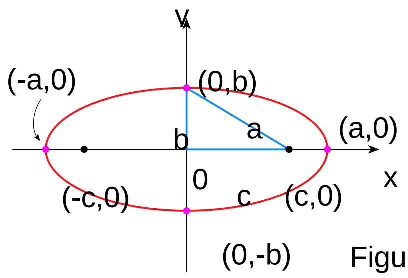


Figure 3

$The\ ellipse$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad a \geq b > 0 \qquad 3.2$$

$$has\ foci\ (\pm c, 0), where\ c^2 = a^2 - b^2. vertices\ (\pm a, 0).$$

If the foci of an ellipse are located on the y-axis at $(0, \pm c)$, then we can find its equation by interchanging x and y in (3.2). (See Figure 4.)

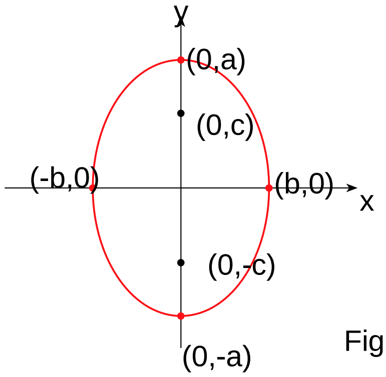


Figure 4

The ellipse

$$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1 \quad a \geq b > 0$$

has foci $(0, \pm c,)$, *where* $c^2 = a^2 - b^2$, *and vertices* $(0, \pm a)$.

Example

1. Find the vertices and foci of the ellipse

$$\frac{x^2}{9}+\frac{y^2}{5}=1$$

$$\Rightarrow a = \sqrt{9} = 3,$$

$$b = \sqrt{5},$$

$$c = \sqrt{a^2 - b^2} = \sqrt{9-5} = 2.$$

The ellipse is centered at $(0,0)$, *with vertices at* $(\pm 3, 0)$. *The foci are* $(\pm 2, 0)$.

2. Find the vertices and foci of the ellipse

$$\frac{x^2}{64}+\frac{y^2}{100}=1$$

$$\Rightarrow a = \sqrt{100} = 10,$$

$$b = \sqrt{64} = 8$$

$$c = \sqrt{a^2 - b^2} = \sqrt{100-64} = 6.$$

The ellipse is centered at $(0,0)$, *with vertices at* $(0, \ \pm 10)$. *The foci are* $(0, \ \pm 6)$.3.

3. Find the vertices and foci of the ellipse

$$9x^2 - 18x + 4y^2 = 27$$

$$\Rightarrow 9(x^2 - 2x + 1) + 4y^2 = 27 + 9$$

$$\Rightarrow 9(x-1)^2 + 4y^2 = 36$$

$$\Rightarrow \frac{(x-1)^2}{4} + \frac{y^2}{9} = 1$$

$$\Rightarrow a = 3, b = 2$$

$$c = \sqrt{5} \Rightarrow center(1,0), vertices(1, \pm 3), foci\left(1, \pm\sqrt{5}\right)$$

3. Find the vertices and foci of the ellipse

$$x^2 - 6x + 2y^2 + 4y = -7$$

$$\Leftrightarrow x^2 - 6x + 9 + 2(y^2 + 2y + 1) = -7 + 9 + 2$$

$$\Leftrightarrow (x-3)^2 + 2(y+1)^2 = 4$$

$$\Leftrightarrow \frac{(x-3)^2}{4} + \frac{(y+1)^2}{2} = 1$$

$$\Rightarrow a = 2, b = \sqrt{2} = c$$

$$\Rightarrow center(3,-1), vertices(1,-1) and (5,-1), foci\left(3 \pm \sqrt{2}, -1\right)$$

Self-Assessment Exercise 1

1. An ellipse is given by the equation $8x^2 + 2y^2 = 32$. Find
   a) the major axis and the minor axis of the ellipse and their lengths,
   b) the vertices of the ellipse,
   c) and the foci of this ellipse.

4.0 Conclusion

In parabola, we have only one fixed point called the focus while in ellipse we have two fixed points called foci that make

5.0 Summary

You have learnt in this unit the following:

1. Give the full description of an Ellipse as the set of points in a plane the sum of whose distances from two fixed points F_1 and F_2 is a constant. These two fixed points are called the **foci** (plural of **focus**).

2. Obtain the equation of an Ellipse with $foci\ (\pm c, 0)$ and vertices $vertices\ (\pm a, 0)$ as $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad a \geq b > 0$

6.0 Tutor-Marked Assignment

Find the vertices and foci of the following ellipses

1. $\frac{x^2}{36} + \frac{y^2}{4} = 1$
2. $x^2 + 25y^2 - 25 = 0$

7.0 References/Further Reading

Olabisi O. Ugbebor and Ninuola I. Akinwande (2000) Analytical Geometry and Mechanics, Ibadan: Y – Books.

Bostock, L. And Clandler, S. (1979) Pure Mathematics 2, UK: Stanley Thomes Ltd

Unit 2

Tangent and Normal to An Ellipse

Contents

1.0 Introduction

The previous unit gives full description of the equation of an ellipse. In this unit we study the behaviour and obtain the equation of a line that touches an ellipse at a point (tangent) and the equation of a line perpendicular to the tangent at that point.

2.0 Learning Outcomes

At the end of this unit you should be able to:

1. Obtain the equation of tangent to an Ellipse
2. Obtain the equation of normal to an Ellipse

3.0 Learning Content

3.1 Equation of Tangent to an Ellipse

Take the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad a \geq b > 0$$

To obtain the slope of the tangent line we differentiate implicitly

$$\frac{2x}{a^2} + \frac{2yy^1}{b^2} = 0$$

Solving for y^1, we obtain

$$y^1 = -\frac{b^2x}{a^2y}$$

Say $P(x_1\,, y_1)$ is a fixed point of the ellipse.

The slope of the tangent line in point P is

$$y^1 = m = -\frac{b^2x_1}{a^2y_1}$$

The equation of the tangent line is

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow \quad y - y_1 = -\frac{b^2x_1}{a^2y_1}(x - x_1)$$

$$\Rightarrow \frac{xx_1}{a^2} + \frac{yy_1}{b^2} = \frac{x_1^2}{a^2} + \frac{y_1^2}{b^2}$$

Since $P(x_1\,, y_1)$ is a point on the ellipse, we have the equation of the ellipse at point P to be

$$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$$

3.2 Equation of Normal to an Ellipse

Since normal is perpendicular to the tangent, the slope of normal will be the negative inverse to that of the tangent.

i.e. the slope of the tangent at point P is

$$m_1 = -\frac{b^2 x_1}{a^2 y_1}$$

The slope of the normal at the same point will be

$$m_2 = \frac{a^2 y_1}{b^2 x_1}$$

∴ The equation of the normal is

$$y - y_1 = m_2(x - x_1)$$

$$\Rightarrow \quad y - y_1 = \frac{a^2 y_1}{b^2 x_1}(x - x_1)$$

Example

1. Find the equation of tangent and normal to the ellipse $9x^2 + 16y^2 = 144$ at the point (2, 3)

Solution

Dividing each term of the equation by 144 we obtain

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

Comparing with the equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

We have $\qquad a^2 = 16, \; b^2 = 9$

At the point (2, 3), we $x_1 = 2, y_1 = 3$

Substituting into the equation of the tangent

$$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$$

We have

$$\frac{2x}{16} + \frac{3y}{9} = 1$$

$$\Rightarrow \quad 3x + 8y = 24$$

To obtain the equation of the normal we substitute into the equation

$$y - y_1 = \frac{a^2 y_1}{b^2 x_1}(x - x_1)$$

We have

$$y - 3 = \frac{16(3)}{9(2)}(x - 2)$$

Simplifying gives

$$3y - 8x + 7 = 0$$

Self-Assessment Exercise

1. Find the equation of the Tangent and Normal to the Ellipse $5x^2 + 3y^2 = 137$ at the point in the first quadrant whose ordinate is 2.

4.0 Conclusion

Getting the equation of an ellipse alone is not enough give full description of an ellipse, but we need to get its relationship with the lines that touch the ellipse at a point.

5.0 Summary

You have learnt in this unit the following:

1. Derived the equation of the tangent at the point $P(x_1,\ y_1)$ to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ as $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$

2. Derived the equation of the normal at the point $P(x_1,\ y_1)$ to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ as $y - y_1 = \frac{a^2 y_1}{b^2 x_1}(x - x_1)$

6.0 Tutor-Marked Assignment

Find the equation of the tangent and normal to the ellipse $4x^2 + 25y^2 = 25$ at the point (-2, 3)

7.0 References/Further Reading

Olabisi O. Ugbebor and Ninuola I. Akinwande (2000) Analytical Geometry and Mechanics, Ibadan: Y – Books.

Bostock, L. And Clandler, S. (1979) Pure Mathematics 2, UK: Stanley Thomes Ltd

Unit 3

Equation of a Hyperbola

Contents

1.0 Introduction

In unit 4 we discuss the formation of a circle by slicing a three-dimensional cone with a plain. This unit will discuss the formation of hyperbola which is formed by slicing the top and bottom section of a cone.

2.0 Learning Outcomes

At the end of this unit you should be able to:

1. State the proper definition of a hyperbola
2. Obtain the standard equation of a hyperbola

3.0 Learning Content

3.1 Definition

A hyperbola is the set of all points in a plane the difference of whose distances from two fixed point $F_1 and\ F_2$(the foci) is a constant. This definition is illustrated in figure 1.

Hyperbolas occur frequently as graphs of equations in chemistry, physics, biology and economics (Boyle's Law Ohm's Law, supply and demand curves). A particularly significant application of hyperbolas is found in the navigation systems developed in world wars I and II

3.2 Equation of a Hyperbola

Notice that the definition of hyperbola is similar to that of an ellipse. The only change is that the sum of distances has become a difference. In fact, the derivation of the equation of hyperbola is also similar to the one given earlier for an ellipse. It is left as an exercise to show that when the foci are on the x – axis at $(\pm c, 0)$ and the difference of distance is $|PF_1| - |PF_2| = \pm 2a$, then the equation of the hyperbola is

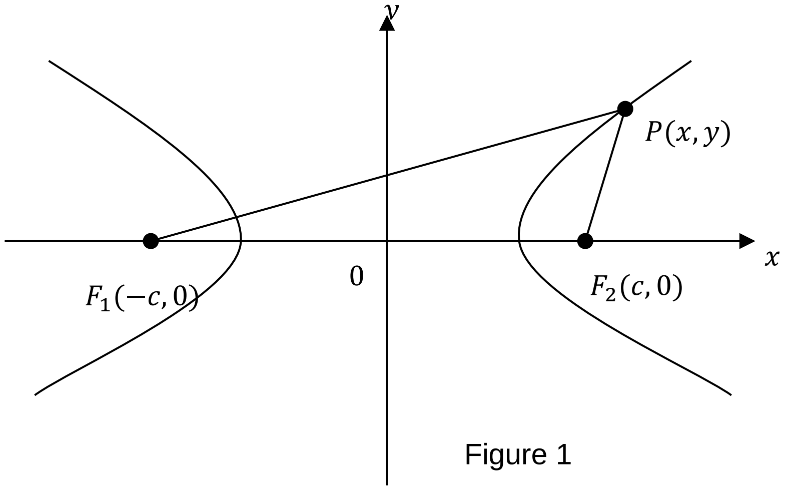


Figure 1

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad (1)$$

where $c^2 = a^2 + b^2$. $Notice\ that$ the x – intercepts are again $\pm a$ and the point $(a, 0)$ and $(-a, 0)$ are the vertices of the hyperbola. But if we put $x = 0$ in equation 1 we get $y^2 = -b^2$, which is impossible, so there is no y – intercept. The hyperbola is symmetric with respect to both axes.

To analyse the hyperbola further, we look at equation 1 and obtain

$$\frac{x^2}{a^2} = 1 + \frac{y^2}{b^2} \geq 1$$

This shows that $x^2 \geq a^2,\ so\ |x| = \sqrt{x^2} \geq a$. Therefore, we have

$x \geq a\ or\ x \leq -a$. This means that the hyperbola consists of two parts, called its branches.

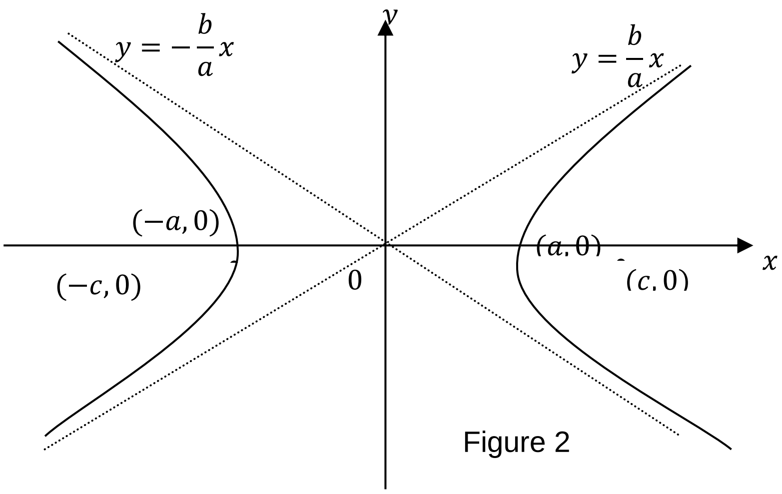


Figure 2

When we draw a hyperbola, it is useful to first draw its asymptotes, which are the dashed line $y = \left(\frac{b}{a}\right) x\ and\ y = -\left(\frac{b}{a}\right) x\ \ shows\ in\ fiqure\ 2$. Both branches of the hyperbola approach the asymptotes; that is, they come arbitrarily close to the asymptotes.

T$he\ hyperbola$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$has\ foci\ (\pm c, 0), where\ c^2 = a^2 + b^2, vertices\ (\pm a, 0), and\ asymtotes$

$$y = \pm \left(\frac{b}{a}\right) x.$$

If the foci of a hyperbola are on the $y - axis$, then by reversing the roles of x and y we obtain the following information.

T$he\ hyperbola$

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

$has\ foci\ (0, \pm c,), where\ c^2 = a^2 + b^2, vertices\ (0, \pm a), and\ asymptotes$

$$y = \pm\ \left(\frac{a}{b}\right) x.$$

Example

1. Find the vertices, foci and asymptotes of the hyperbola

$$\frac{x^2}{144} - \frac{y^2}{25} = 1$$

$$\Rightarrow a = 12, b = 5, c = \sqrt{144 + 25} = 13$$

$$\Rightarrow center(0,0), vertice\ (\pm 12, 0),\ \ foci\ (\pm 13, 0), asymptotes\ \ y = \pm\frac{5}{12}x.$$

2. Find the vertices, foci and asymptotes of the hyperbola

$$\frac{y^2}{16} - \frac{x^2}{36} = 1$$

$$\Rightarrow a = 4, b = 6, c = \sqrt{16 + 36} = \sqrt{52} = 2\sqrt{13}$$

$$\Rightarrow center(0,0), vertice\ (0, \quad \pm 4),$$
$$foci\ \left(0, \quad \pm 2\sqrt{13}\right), asymptotes\ \ are\ the\ lines\ y = \pm\frac{a}{b}x = \pm\frac{2}{3}x.$$

3. Find the vertices, foci and asymptotes of the hyperbola

$$9x^2 - 4y^2 = 36$$

$$\frac{x^2}{4} - \frac{y^2}{9} = 1$$

$$\Rightarrow a = 2, b = 3, c = \sqrt{4 + 9} = \sqrt{13}$$

$$\Rightarrow center(0, 0), vertice\ (\pm 2, \quad 0),$$
$$foci\ \left(\pm\sqrt{13}, \ \ 0\right), asymptotes\ \ are\ the\ lines\ y = \pm\frac{a}{b}x = \pm\frac{3}{2}x$$

4. Find the vertices, foci and asymptotes of the hyperbola

$$16x^2 + 64x - 9y^2 - 90y = 305$$

$$16(x^2 + 4x + 4) - 9(y^2 + 10y + 25) = 305 + 64 - 225$$

$$16(x + 2)^2 - 9(y + 5)^2 = 144$$

$$\frac{(x+2)^2}{9} - \frac{(y+5)^2}{16} = 1$$

$$\Rightarrow a = 3, b = 4, c = \sqrt{9 + 16} = \sqrt{25} = 5$$

$$\Rightarrow center(-2, \ -5), vertice\ (-5, -5)\ and\ (1, -5),$$
$$foci\ (-7, -5)\ and\ (3, -5), asymptotes\ \ are\ the\ lines\ y + 5 = \pm\frac{4}{3}(x + 2)$$

Self-Assessment Exercise 1

1. Find the equation of a hyperbola with foci at (-2, 0) and (2 , 0) and asymptotes given by the equation y = x and y = -x.

4.0 Conclusion

By sampling replacing b^2 in equation of an ellipse with $-b^2$ give us the equation of hyperbola.

5.0 Summary

You have learnt in this unit the following:

1. Give the full description of a hyperbola as the set of points in a plane the sum of whose distances from two fixed points F_1 and F_2 is a constant. These two fixed points are called the **foci** (plural of **focus**).

2. Obtain the equation of a hyperbola as $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

   $with\ foci\ (\pm c, 0), where\ c^2 = a^2 + b^2, vertices\ (\pm a, 0), and\ asymtotes$

   $y = \pm\left(\frac{b}{a}\right)x.$

6.0 Tutor-Marked Assignment

Find the vertices, foci and asymptotes of the following hyperbolas

1. $\frac{x^2}{12} - \frac{y^2}{9} = 1$
2. $\frac{y^2}{10} - \frac{x^2}{4} = 1$
3. $x^2 - 25y^2 - 25 = 0$

7.0 References/Further Reading

Olabisi O. Ugbebor and Ninuola I. Akinwande (2000) Analytical Geometry and Mechanics, Ibadan: Y – Books.

Bostock, L. And Clandler, S. (1979) Pure Mathematics 2, UK: Stanley Thomes Ltd

Unit 4

Tangent and Normal to a Hyperbola

Contents

1.0 Introduction

If you compare equation of ellipse in unit 8 with equation of hyperbola in unit 10, we discover equation of hyperbola is obtain by replacing b^2 in equation of an ellipse by $-b^2$ to obtain equation of a hyperbola. In this unit, we obtain tangent and normal to hyperbola by replacing b^2 by $-b^2$.

2.0 Learning Outcomes

At the end of this unit you should be able to:

1. Obtain the equation of tangent to a Hyperbola
2. Obtain the equation of normal to a Hyperbola

3.0 Learning Content

3.1 Equation of Tangent to a Hyperbola

Equation of an ellipse is given as

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

By replacing b^2 by $-b^2$, we obtain

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad 1$$

Equation 1 gives the equation of hyperbola.

The equation of tangent to an ellipse at the point (x_1, y_1)

$$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1 \qquad 2$$

By replacing b^2 by $-b^2$ in equation 2, we have

$$\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1 \qquad 3$$

Equation 3 gives the equation of tangent to hyperbola at the point (x_1, y_1).

Self-Assessment Exercise 1

Prove that an equation of the tangent line to the graph of the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

At the Point $P(x_o, y_o)$ is

$$\frac{xx_0}{a^2} - \frac{yy_0}{b^2} = 1$$

3.2 Equation of Normal to a Hyperbola

From the equation of normal to an ellipse

$$y - y_1 = \frac{a^2 y_1}{b^2 x_1}(x - x_1)$$

By replacing b^2 by $-b^2$, we obtain

$$y - y_1 = -\frac{a^2 y_1}{b^2 x_1}(x - x_1)$$

Simplifying we have

$$a^2 y_1(x - x_1) + b^2 x_1(y - y_1) = 0$$

Example

Find the equation of tangent and normal to the hyperbola $16x^2 - 25y^2 = 400$ at the point (3, 2).

Solution

Dividing each term of the equation by 400, we obtain

$$\frac{x^2}{25} - \frac{y^2}{16} = 1$$

Comparing with the equation

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

We have $a^2 = 25,\ \ b^2 = 16$

at the point (3, 2) we have $x_1 = 3,\ \ y_1 = 2$

substituting into the equation of the tangent

$$\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$$

We have

$$\frac{3x}{25} - \frac{2y}{16} = 1$$

$$\frac{3x}{25} - \frac{y}{8} = 1$$

$$24x - 25y = 200$$

To obtain the equation of the normal we substitute into the equation

$$a^2 y_1(x - x_1) + b^2 x_1(y - y_1) = 0$$

We have

$$25(2)(x - 3) + 16(3)(y - 2) = 0$$

$$25(x - 3) + 24(y - 2) = 0$$

$$25x - 75 + 24y - 48 = 0$$

$$25x + 24y = 123$$

Self-Assessment Exercise 2

1. Find the equation of normal to the hyperbola $x^2/a^2 - y^2/b^2 = 1$ at (x_1, y_1)

4.0 Conclusion

It is possible for us to differentiate the equation of hyperbola to obtain the slope, we now use the slope with equation of a straight line given the slope and a point to obtain the equation of the tangent and normal to the hyperbola, but it become easier by just replacing b^2 by $-b^2$ in the ellipse to obtain that of hyperbola.

5.0 Summary

You have learnt in this unit the following:

1. Derived the equation of the tangent at the point $P(x_1, y_1)$ to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ as $\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$
2. Derived the equation of the normal at the point $P(x_1, y_1)$ to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ as $a^2y_1(x - x_1) + b^2x_1(y - y_1) = 0$

6.0 Tutor-Marked Assignment

1. Find the equation of the tangent and normal to the hyperbola $2x^2 - y^2 = -10$ at the point (2, 3).
2. Find the normal to the hyperbola $3x^2 - 4y^2 = 12$ which is parallel to the line $-x + y = 0$.

7.0 References/Further Reading

Olabisi O. Ugbebor and Ninuola I. Akinwande (2000) Analytical Geometry and Mechanics, Ibadan: Y – Books.

Bostock, L. And Clandler, S. (1979) Pure Mathematics 2, UK: Stanley Thomes Ltd

Module 4

Trigonometry Angles and Identity

Unit 1: Trigonometry (Angles I).

Unit 2: Trigonometry (Angles II).

Unit 3: Trigonometric Identity I

Unit 4: Trigonometric Identity II

Unit 1

Trigonometry (Angles I)

Contents

1.0 Introduction

This course is divided into two parts coordinate geometry and trigonometry. This unit will introduce the concept of trigonometry by identifying the trigonometric functions and conversion of a measurement from radians to degrees (or vice versa).

2.0 Learning Outcomes

At the end of this unit you should be able to:

1. Convert a measurement from radians to degrees (or vice versa)
2. Identify the Trigonometric functions.

3.0 Learning Content

3.1 Angles

An angle can be thought of as the amount of rotation required to take one straight line to another line with a common point. Angles are often labelled with Greek letters, for example θ.

The classical concept of trigonometry deals with the relationship between the angles and sides of triangles. Angles can be expressed in degrees or radius. To convert a measurement from radians to degrees (or vice versa) we use the following relationship.

$$\pi rad = 180^0$$

The relationship gives the following two equations.

$$1\, rad = \left(\frac{180}{\pi}\right)^0, \qquad 1^0 = \frac{\pi}{180} rad$$

Example

1. Convert $\frac{\pi}{3}$ radians to degrees

Solution

$$\frac{\pi}{3} rad = \frac{\pi}{3} . \left(\frac{180}{\pi}\right)^0 = 60^0$$

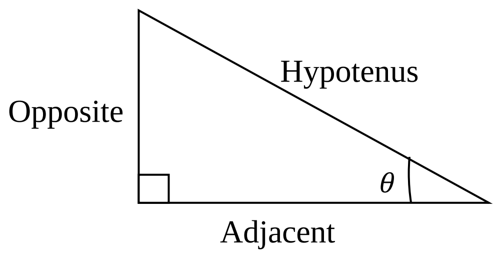
2. Convert 270^0 *to radians*

Solution

$$270^0 = 270^0 \left(\frac{\pi}{180^0} rad\right) = \frac{3\pi}{2} rad.$$

3.2 Trigonometric Functions

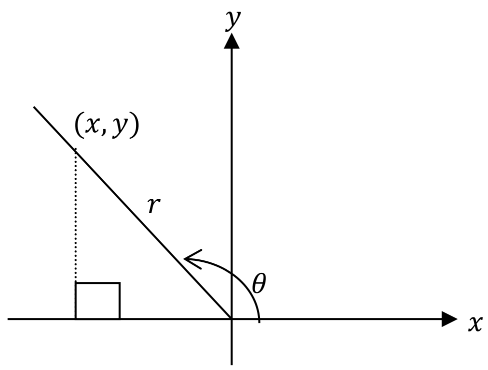
Consider a right-angle triangle below



The angle θ $must$ be acute (angle is less then 90^0), then

$$\sin\theta = \frac{opp}{hyp}, \qquad cos\theta = \frac{adj}{hyp}, \qquad \tan\theta = \frac{opp}{adj}$$

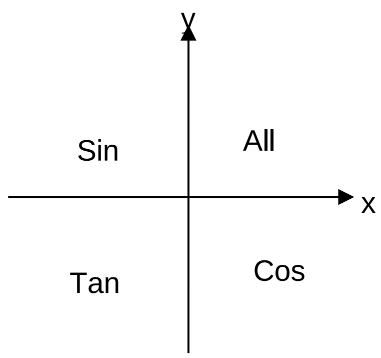
For angle that are obtuse (angle is greater than 90^0) or negative, we consider the diagram below with x and y the values of the x and y coordinates resp.



Then

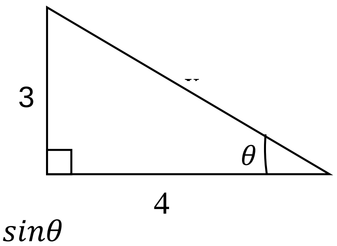
$$\sin\theta = \frac{y}{r}, \quad \cos\theta = \frac{x}{r}, \qquad \tan\theta = \frac{y}{x}$$

The CAST graph below helps in remembering the signs of trigonometric functions for different angles. The function will be negative in all quadrants except those that indicate that the function is positive.



Example

Given the right-angle triangle below, obtain



i. $sin\theta$

ii. $cos\theta$

iii. $tan\theta$

Solution

By Pythagoras theorem

$$x^2 = 3^2 + 4^2$$

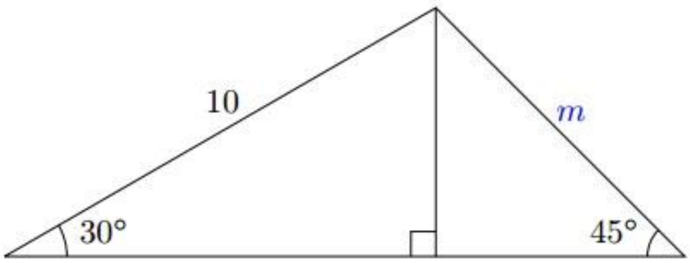
$$x^2 = 25$$

$$\Rightarrow \quad x = 5$$

$$\sin\theta = \frac{y}{r} = \frac{3}{5}, \quad \cos\theta = \frac{x}{r} = \frac{4}{5}, \qquad \tan\theta = \frac{y}{x} = \frac{3}{4}$$

Self-Assessment Exercise 1

Find the value of m in the triangle below?



4.0 Conclusion

Angles are measure either in radian or in degree. There are three primary trigonometric function, the sine, cosine and tangent. They are abbreviated by sin, cos, tan, other trigonometric functions can be obtained by taken the reciprocal of these three.

5.0 Summary

You have learnt in this unit the following:

1. Convert a measurement from radians to degrees (or vice versa), i.e.

$1\,rad = \left(\frac{180}{\pi}\right)^0$, $1^0 = \frac{\pi}{180} rad$

2. Identify the Trigonometric functions, i.e $\sin\theta = \frac{opp}{hyp}$, $cos\theta = \frac{adj}{hyp}$, $\tan\theta = \frac{opp}{adj}$

6.0 Tutor-Marked Assignment

1. Convert $\frac{2\pi}{3}$ radians to degrees
2. Convert 300^0 $to\ radians$
3. Given $cos\theta = \frac{4}{7}$, find $sin\theta$ and $tan\theta$

7.0 References/Further Reading

K. A. Stroud and Dexter J. Booth (2001) Engineering Mathematics Fifth Edition, Great Britain: Antony Rowe Ltd.

Stan Gibilisco (2003), Trigonometry Demystified, London: McGRAW – HILL.

Steven Butler (2002), Notes from Trigonometry, Brigham Young University.

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Unit 2

Trigonometry (Angles II)

Contents

1.0 Introduction

The previous unit introduce the concept of trigonometry which deals with angles. This unit will identify the trigonometric ratio of special angles i.e. angles we can easily obtain their trig values without using any calculating devices.

2.0 Learning Outcomes

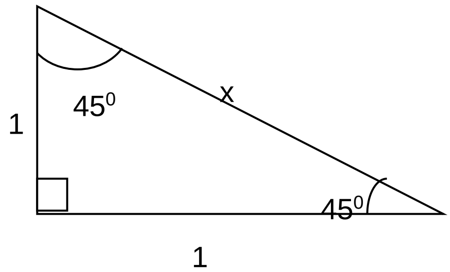
At the end of this unit you should be able to:

1. Identify the special angles trig ratio
2. Identify the reference angle for the any given angle

3.0 Learning Content

3.1 Special Angles

Consider the right angle Isosceles triangle below.

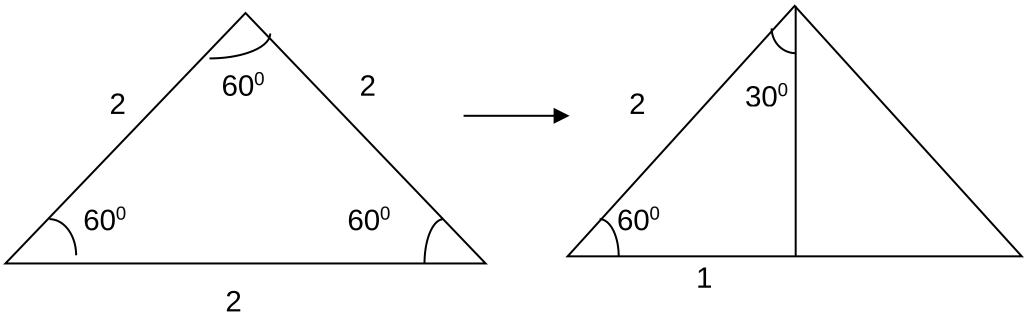


by Pythagoras's theorem

$$x = \sqrt{2}$$

$$\sin 45^0 = \frac{opp}{hyp} = \frac{1}{\sqrt{2}}, \; cos45^0 = \frac{Adj}{hyp} = \frac{1}{\sqrt{2}},$$

Consider the equilateral triangle with side lengths of 2

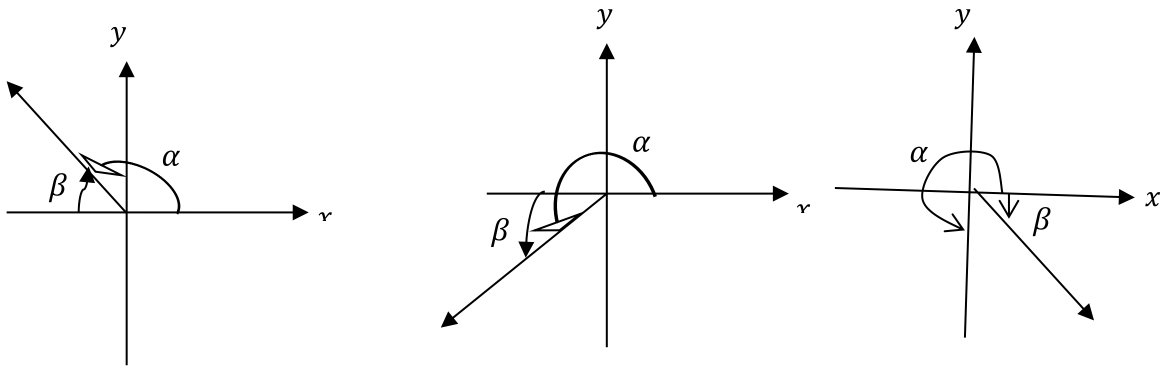


$$\therefore sin30^0 = \frac{opp}{hyp} = \frac{1}{2}, \qquad cos30^0 = \frac{adj}{hyp} = \frac{\sqrt{3}}{2}$$

$$sin60^0 = \frac{\sqrt{3}}{2}, \qquad cos60^0 = \frac{1}{2}$$

3.2 Reference Angle

The reference angle for the angle $\propto$ is the positive acute angle formed by the terminal side of $\propto$ and the $x - axis$. The following diagrams shows the reference angle for an angle $\propto$ whose terminal side lies in the second or fourth quadrants.



Example

1. Find the exact value of $sin120^0$

Solution

The terminal side of the angle whose measure is 120^0 lies in the $2nd$ quadrant, so $180^0 - 120^0 = 60^0 is$ the reference angle. Also, sin is positive in the 2nd quadrant.

Thus, $sin120^0 = sin60^0 = \frac{\sqrt{3}}{2}$

2. Find the exact value of $cos210^0$

This lies in 3rd quadrant $210^0 - 180^0 = 30^0$ is the reference angle. Also cos is negative in the 3rd quadrant.

Thus, $cos210^0 = -cos30^0 = -\frac{\sqrt{3}}{2}$

3. Find the exact value of $\sin 315^0$

Solution

$360^0 - 315^0 = 45^0$

$thus \sin 315^0 = -\sin 45^0 = -\frac{1}{\sqrt{2}}$

Self-Assessment Exercise 1

1. Find the Exact Angels of Cos 150^0 And Sin 150^0

4.0 Conclusion

Some angles are special just because we can easily obtain their trig function without consulting any calculating devices. Angles fall in quadrants other than the first quadrant can also have their value of the trigonometric function by using the reference angle.

5.0 Summary

You have learnt in this unit the following:

1. To identify the special angles by right angle Isosceles triangle and equilateral triangle.
2. To identify the reference angle for the angle $\propto$ as the positive acute angle formed by the terminal side of $\propto$ and the $x - axis$.

6.0 Tutor-Marked Assignment

Find the exact value of

1. $sin270^0$
2. $cos270^0$
3. $tan270^0$

7.0 References/Further Reading

K. A. Stroud and Dexter J. Booth (2001) Engineering Mathematics Fifth Edition, Great Britain: Antony Rowe Ltd.

Stan Gibilisco (2003), Trigonometry Demystified, London: McGRAW – HILL.

Steven Butler (2002), Notes from Trigonometry, Brigham Young University.

Peggy Adamson and Jackie Nicholas (1998) Introduction to Trigonometric Functions: University of Sydney.

Unit 3

Trigonometric Identity I

Contents

1.0 Introduction

Having understand the basic concept of trigonometric function from the previous unit, this unit will use the concept of quotient, reciprocal and Pythagorean identities to obtain some trig identities.

2.0 Learning Outcomes

At the end of this unit you should be able to:

1. Identify the quotient and reciprocal identities of trigonometric ratios.
2. Use Pythagoras theorem to identify the Pythagorean Identities

3.0 Learning Content

3.1 Quotient and Reciprocal Identities

Quotient identity

$$tan\theta = \frac{sin\theta}{cos\theta}$$

Reciprocal identities

$$sec\theta = \frac{1}{cos\theta}, \qquad csc\theta = \frac{1}{sin\theta}, \qquad cot\theta = \frac{1}{tan\theta}$$

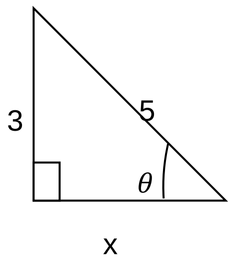
Example

1. Suppose that $sin\theta = \frac{3}{5} and\ \theta\ is\ acute.$

$Find\ the\ other\ trigonometric\ ratios\ for\ \theta$

Solution

$$sin\theta = \frac{3}{5} = \frac{opp}{hyp}$$



$\therefore x = 4$

Thus $cos\theta = \frac{4}{5}$, $tan\theta = \frac{3}{4}$, $sec\theta = \frac{1}{cos\theta} = \frac{5}{4}$

$csc\theta = \frac{1}{sin\theta} = \frac{5}{3}$, $cot\theta = \frac{1}{tan\theta} = \frac{4}{3}$

2. Suppose that $cos\alpha = -\frac{3}{5}$ $and\ \alpha\ lies\ in\ quadrant\ II,$ Find the other trigonometric ratios for α

Solution

$$cos\alpha = -\frac{3}{5} = \frac{x}{r}$$

$$\Rightarrow since\ r > 0, x = -3 and\ r = 5$$

To find y

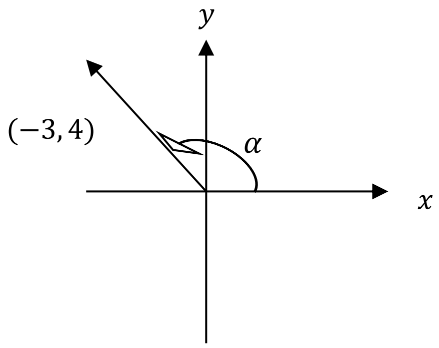
$$r = \sqrt{x^2 + y^2}$$

$$\Rightarrow 5 = \sqrt{(-3)^2 + (y)^2}$$

$$\Rightarrow y = \pm 4.$$

Since α lies in quadrant II,

$$y > 0\ \Rightarrow y = 4$$



$sin\alpha = \frac{4}{5}, csc\alpha = \frac{1}{sin\propto} = \frac{5}{4}$

$cos\alpha = -\frac{3}{5}, sec\alpha \frac{1}{cos\alpha} = -\frac{5}{3}$

$tan\alpha\ = \frac{4}{5}, cot\alpha\ = \frac{1}{tan\propto} = -\frac{3}{4}$

3. Suppose that $tan\theta = \frac{2}{3}$ $and\ sin\theta < 0. Find\ the\ other\ trigonometric\ ratios\ for\ \theta$

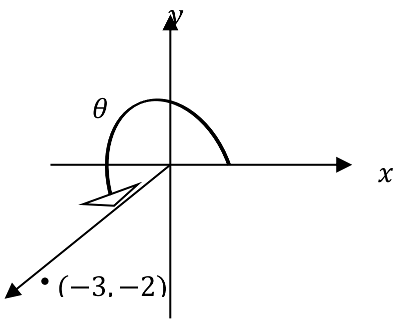
Solution

Since $sin\theta < 0, than\ y = 0$

$\Rightarrow tan\theta = \frac{2}{3} = \frac{-2}{-3} = \frac{y}{x}$

$\Rightarrow x = -3\ and\ y = -2$

$thus, r = \sqrt{x^2 + y^2} = \sqrt{(-2)^2 + (-3)^2} = \sqrt{13}$



$$sin\theta = \frac{-2}{\sqrt{13}}, \quad csc = \frac{1}{sin\theta} = \frac{\sqrt{13}}{-2}$$

$$cos\theta = \frac{-3}{\sqrt{13}}, \quad sec = \frac{1}{cos\theta} = \frac{\sqrt{13}}{-3}$$

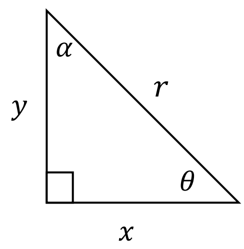
$$tan\theta = \frac{2}{3}, \quad cot\theta = \frac{1}{tan\theta} = \frac{3}{2}$$

Self-Assessment Exercise 1

1. Prove that Tan y/sin y=sec y

3.2 Pythagorean Identities

Consider the diagram below



By Pythagoras theorem

$$x^2 + y^2 = r^2$$

$$\Rightarrow \frac{x^2+y^2}{r^2} = 1$$

$$or \left(\frac{x}{r}\right)^2 + \left(\frac{y}{r}\right)^2 = 1$$

From the diagram

$$sin\theta = \frac{y}{r}, \quad cos\theta = \frac{x}{r}$$

$$\therefore \ sin^2\theta + cos^2\theta = 1 \qquad (1)$$

Divide each term of equation (1) by $cos^2\theta$

$$\frac{sin^2\theta}{cos^2\theta}+\frac{cos^2\theta}{cos^2\theta}=\frac{1}{cos^2\theta}\rightarrow tan^2\theta+1=sin^2\theta \qquad (2)$$

Divide each term of equation (1) by $sin^2\theta$

$$\frac{sin^2\theta}{sin^2\theta}+\frac{cos^2\theta}{sin^2\theta}=\frac{1}{sin^2\theta}\rightarrow 1+cot^2\theta=csc^2\theta$$

$$\Rightarrow \quad 1+\cot^2\theta=\csc^2\theta \qquad (3)$$

Self-Assessment Exercise 2

1. Prove that $tanx+\frac{\cos x}{1+sinx}=\frac{1}{\cos x}$

4.0 Conclusion

The only three primary trigonometric functions are sine, cosine and tangent. Quotient, reciprocal and Pythagorean identities are obtained by simple arithmetic with sine, cosine and tangent.

5.0 Summary

You have learnt in this unit the following:

1. To identify the quotient and reciprocal identities of trigonometric ratios, i.e.
$$tan\theta=\frac{sin\theta}{cos\theta},\qquad sec\theta=\frac{1}{cos\theta},\qquad csc\theta=\frac{1}{sin\theta},\qquad cot\theta=\frac{1}{tan\theta}$$
2. Use Pythagoras theorem to identify the Pythagorean Identities, i.e.
$$\therefore\ sin^2\theta+cos^2\theta=1,\qquad tan^2\theta+1=sin^2\theta,\qquad 1+\cot^2\theta=\csc^2\theta$$

6.0 Tutor-Marked Assignment

Suppose that $cos\alpha=-\frac{5}{7}$ $and\ \alpha\ lies\ in\ quadrant\ II,$ Find the other trigonometric ratios for α

7.0 References/Further Reading

K. A. Stroud and Dexter J. Booth (2001) Engineering Mathematics Fifth Edition, Great Britain: Antony Rowe Ltd.

Stan Gibilisco (2003), Trigonometry Demystified, London: McGRAW – HILL.

Steven Butler (2002), Notes from Trigonometry, Brigham Young University.

Peggy Adamson and Jackie Nicholas (1998) Introduction to Trigonometric Functions: University of Sydney.

Unit 4

Trigonometric Identity II

Contents

1.0 Introduction

We can still obtain the trig values of some angles that are not special angles without using any calculating devices by splitting the angle into set of special angles. This unit uses the concept of sum and different identities to obtain the trig value of some angles.

2.0 Learning Outcomes

At the end of this unit you should be able to:

1. Identify the sum and difference identities of trigonometric ratios.
2. Solve problems involving double and half angle identities of trigonometric ratios.

3.0 Learning Contents

3.1 Sum and Difference Identities

$$sin(A \pm B) = sinAcosB \pm cosAsinB$$

$$cos(A \pm B) = cosAcosB \mp sinAsinB$$

$$tan(A \pm B) = \frac{tanA \pm tanB}{1 \mp tanAtanB}$$

Example

We recall that

$$\sin 45^0 = \frac{opp}{hyp} = \frac{1}{\sqrt{2}},$$

$$cos45^0 = \frac{Adj}{hyp} = \frac{1}{\sqrt{2}},$$

$$Tan45^0 = 1$$

$$sin30^0 = \frac{1}{2}, \qquad cos30^0 = \frac{\sqrt{3}}{2}$$

$$sin60^0 = \frac{\sqrt{3}}{2}, \qquad cos = 60^0 = \frac{1}{2}$$

$$tan30^0 = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}, \quad tan60^0 = \sqrt{3}$$

Evaluate each of the following in simple surd forms.

a. $sin75^0$

b. $cos75^0$

c. $sin15^0$

d. $cos15^0$

e. $sin105^0$

f. $tan75^0$

g. $tan15^0$

h. $sin255^0$

Solution

a. $sin75^0 = sin(45^0 + 30^0) = sin45^0cos30^0 + sin30^0cos45^0$

$$= \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{2 \times 3}}{4} + \frac{\sqrt{2}}{4}$$

$$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$$

$$= \frac{\sqrt{6} + \sqrt{2}}{4}$$

b. $cos75^0 = cos(45^0 + 30^0)$

$$= cos45^0cos30^0 - sin45^0sin30^0$$

$$= \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \times \frac{1}{2}$$

$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$

$$= \frac{\sqrt{6} - \sqrt{2}}{4}$$

c. $sin15^0 = sin(45^0 - 30^0) = sin45^0cos30^0 - sin30^0cos45^0$

$$= \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$

$$= \frac{1}{4}(\sqrt{6} - \sqrt{2})$$

d. $\cos 15^0 = cos(45^0 - 30^0) = cos45^0cos30^0 + sin45^0sin30^0$

$$= \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \times \frac{1}{2}$$

$$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$$

e. $sin105^0 = sin(60^0 + 45^0) = sin60^0cos45^0 + sin45^0cos60^0$

$$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \times \frac{1}{2}$$

$$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$$

f. $tan75^0 = tan(45^0 + 30^0) = \frac{tan45^0 + tan30^0}{1 - tan45^0tan30^0}$

$$= \frac{1 + \frac{\sqrt{3}}{3}}{1 - 1 \times \frac{\sqrt{3}}{3}}$$

$$= \frac{\frac{3+\sqrt{3}}{3}}{\frac{3-\sqrt{3}}{3}} = \frac{3+\sqrt{3}}{3-\sqrt{3}}.$$

g. $tan15^0 = \tan(45^0 - 30^0)$

$$= \frac{tan45^0 - tan30^0}{1 + tan45^0 tan30^0}$$

$$= \frac{1 - \frac{\sqrt{3}}{3}}{1 + 1 \times \frac{\sqrt{3}}{3}}$$

$$= \frac{\frac{3-\sqrt{3}}{3}}{\frac{3+\sqrt{3}}{3}}$$

$$= \frac{3-\sqrt{3}}{3+\sqrt{3}}$$

h. $sin345^0 = sin(360^0 - 15^0)$

$$= sin360^0 cos15^0 - sin15^0 cos360^0$$

$$= 0 - \frac{1}{4}(\sqrt{6} - \sqrt{2})$$

Self-Assessment Exercise 1

1. If $\sin A = \frac{3}{5}$ *and* $\cos B = \frac{-12}{13}$ where $0 \le A \le \frac{\pi}{2}$ and $\pi \le B \le \frac{3\pi}{2}$ find

a. Cos(A+B)

b. Tan (A-B)

3.2 Double Angle Identities

1. $\sin 2A = \sin(A + A) = sinAcosA + sinAcosA = 2sinAcosA.$

2. $cos2A = cos(A + A) = cosAcosA - sinAsinA$

$$= cos^2A - sin^2A \qquad (1)$$

$$= 1 - sin^2A - sin^2A \ \ (Since\ \cos^2 A = 1 - \sin^2 A)$$

$$= 1 - 2sin^2A$$

$$from(1)$$

$$cos^2A - sin^2A = cos^2A - (1 - cos^2A)\ (Since\ \ \sin^2 A = 1 - \cos^2 A)$$

$$= cos^2A - 1 + cos^2A$$

$$= 2cos^2A - 1$$

3. $tan2A = tan(A + A)$

$$= \frac{tanA + tanA}{1 - tanAtanA}$$

$$= \frac{2tanA}{1 - tan^2A}$$

Example 1

Find the value of $2sin15^0cos15^0$.

Solution

$$sin2A = 2sinAcosA.$$

$$= 2sin15^0cos15^0 = sin2(15^0) = sin30^0 = \frac{1}{2}$$

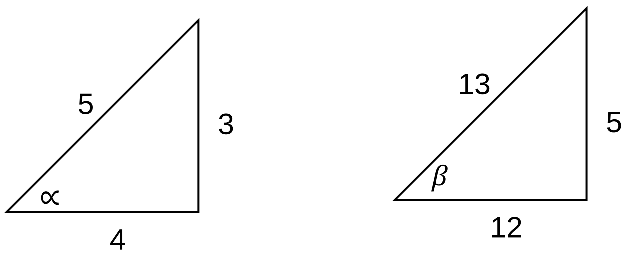
Example 2

If $\propto$ and β are acute angle such that $sin \propto = \frac{3}{5} and\ tan\beta = \frac{5}{12}$,

$$find\ without\ using\ table$$

1. $\sin(\propto + \beta)$
2. $sin2 \propto$
3. $cos2\beta$
4. $tan2 \propto$

Solution



$sin \propto = \frac{3}{5}$, $\quad sin\beta = \frac{5}{13}$

$$cos \propto = \frac{4}{5}\ , \quad cos\beta = \frac{12}{13}$$

$$tan\alpha = \frac{3}{4}\ , \quad tan\beta = \frac{5}{12}$$

a. $\quad \sin(\alpha + \beta) = sin\alpha cos\beta + sin\beta cos\alpha$

$$= \frac{3}{5} \times \frac{12}{13} + \frac{5}{13} \times \frac{4}{5}$$

$$= \frac{36}{65} + \frac{20}{85} = \frac{56}{65}$$

b. $sin2\alpha = 2sin\alpha cos\alpha$

$$= 2 \times \frac{3}{5} \times \frac{4}{5} = \frac{24}{25}$$

c. $cos2\beta = cos^2\beta - sin^2\beta$

$$= (cos\beta)^2 - (\sin \mathrm{\beta})^2$$

$$= \left(\frac{12}{13}\right)^2 + \left(\frac{5}{13}\right)^2$$

$$= \frac{144}{169} - \frac{25}{189}$$

$$= \frac{144 - 25}{169}$$

$$= \frac{119}{169}$$

d. $tan2 \propto = \frac{2tan\propto}{1-tan^2\propto}$

$$= \frac{2 \times \frac{3}{4}}{1 - \left(\frac{3}{4}\right)^2}$$

$$= \frac{\frac{3}{2}}{\frac{16-9}{16}} = \frac{\frac{3}{2}}{\frac{7}{16}} = \frac{3}{2} \div \frac{7}{16}$$

$$= \frac{3}{2} \times \frac{16}{7}$$

$$\frac{24}{7}$$

Self-Assessment Exercise 2

1. Find the exact value of $\cos 2x$if $\sin x = -\frac{12}{13}$ (in quadrant III)

3.3 Half Angle Identities

1. $cosA = \cos\left(\frac{A}{2} + \frac{A}{2}\right)$

$$= cos^2\frac{A}{2} - sin^2\frac{A}{2}$$

$$= 1 - sin^2\frac{A}{2} - sin^2\frac{A}{2}$$

$$= 1 - 2sin^2\frac{A}{2}$$

$$\therefore\ cosA = 1 - 2sin^2\frac{A}{2}$$

2. $2sin^2\frac{A}{2} = 1 - cosA$

$$sin^2\frac{A}{2} = \frac{1 - cosA}{2}$$

$$cosA = cos^2\frac{A}{2} - sin^2\frac{A}{2}$$

$$= cos^2\frac{A}{2} - \left(1 - cos^2\frac{A}{2} -\right)$$

$$= cos^2\frac{A}{2} - 1 + cos^2\frac{A}{2}$$

$$= 2cos^2\frac{A}{2} - 1$$

3. $2cos^2\frac{A}{2} = cosA + 1$

$$cos^2\frac{A}{2} = \frac{1 + cosA}{2}$$

$$tan\frac{A}{2} = \frac{sin\,{}^{A}/_{2}}{cos\,{}^{A}/_{2}}$$

$$\tan^2\frac{A}{2} = \frac{\sin^2\frac{A}{2}}{\cos^2\frac{A}{2}}$$

$$\tan^2\frac{A}{2} = \frac{1 - cosA}{1 + cosA}$$

Self-Assessment Exercise 3

1. Using the half angle formula, find the exact value of cos 15^0

4.0 Conclusion

An angle can be split into different special angles by the sum or different, that enables us to obtain the value of the trigonometric ratio of the angle without using calculator.

5.0 Summary

You have learnt in this unit the following:

the sum and difference identities, i.e $sin(A \pm B) = sinAcosB \pm cosAsinB,$

$$cos(A \pm B) = cosAcosB \mp sinAsinB, \; tan(A \pm B) = \frac{tanA \pm tanB}{1 \mp tanAtanB}$$

Solve problems involving double and half angle, i.e. $\sin 2A = 2sinAcosA$, $cos2A = 1 - 2sin^2A = 2cos^2A - 1, tan2A = \frac{2tanA}{1-tan^2A}$

$sin^2\frac{A}{2} = \frac{1-cosA}{2}$, $cos^2\frac{A}{2} = \frac{1+cosA}{2}$, $\tan^2\frac{A}{2} = \frac{1-cosA}{1+cosA}$

6.0 Tutor-Marked Assignment

1. Evaluate each of the following in simple surd forms.

$$sin255^0$$

$$cos345^0$$

2. Find the value of $2sin75^0cos75^0$

7.0 References/Further Reading

K. A. Stroud and Dexter J. Booth (2001) Engineering Mathematics Fifth Edition, Great Britain: Antony Rowe Ltd.

Stan Gibilisco (2003), Trigonometry Demystified, London: McGRAW – HILL.

Steven Butler (2002), Notes from Trigonometry, Brigham Young University.

Peggy Adamson and Jackie Nicholas (1998) Introduction to Trigonometric Functions: University of Sydney.

Answers to Self-Assessment Exercises

Module 1: Coordinate Geometry and Trigonometry

Unit 1: Slope of a Straight Line

SAE 1

1. x, and y 2. Abscissa and Ordinate
2. Abscissa and Ordinate

SAE 2

1. 5a

2. 4

SAE 3

1. The Mid-Point of XY is (5, 5)

SAE 4

1. m = -3

Unit 2: Equation of a Straight Line I

SAE 1

1. Linear, Slope and Intercept Form

Unit 3: Equation of a Straight Line II

SAE 1

1. 1/3

Module 2: Conic Section

Unit 1: Equation of a Circle

SAE 1

1. A Set of Points that move in a plane of a fixed point and a fixed line.
2. Conic Sections are formed by slicing a Three-Dimensional Right Circular Cone with a Plane.

SAE 2

1. $\sqrt{(80)}$

2. $5\sqrt{2}$

Unit 2: Tangent and Normal Through A Point on A Circle

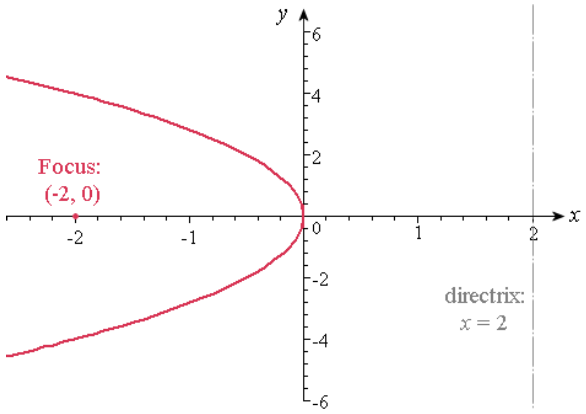
SAE 1

1. The equation of the tangent to the circle of F is y = 1/4x+9/2

Unit 3: Equation of Parabola

SAE 1

1. The curve looks like this:



After sketching, we can see that the equation required is in the following form, since we have a horizontal axis:

$y^2 = 4px$

Since p = -2 (from the question), we can directly write the equation of the parabola:

$y^2 = -8x$

Unit 4: Tangent and Normal to a Parabola

SAE 1

The equation for the tangent and normal are:

$16y = 8(x + 16)\ \ and\ \ y - 16 = -\frac{16}{8}(16 - 8)$ respectively. And the point is (21:6)

Module 3: Ellipse and Hyperbola

Unit 1: Equation of an Ellipse

SAE 1

a. The major axis is 8 and the minor axis is 4
b. The vertices are on the major axis at the points (0, 4) and (0, -4)
c. The foci are at the points (0, 2√3) and (0, -2√3)

Unit 2: Tangent and Normal to an Ellipse

SAE 1

1. $\frac{3x}{137/5} + \frac{2y}{137/3} = 1$

The equation of the Normal is given by the equation

$$\frac{x-5}{5/(\frac{137}{5})}+\frac{y-2}{2/(\frac{137}{3})}$$

Unit 3: Equation of a Hyperbola

SAE 1

1. The equation of the hyperbola is given by: $x^2/2 - y^2/2 = 1$

Unit 4: Tangent and Normal to a Hyperbola

SAE 1

SAE 2

1. $a^2 y_1(x - x_1) + b^2 x_1(y - y_1) = 0$

Module 4: Trigonometry Angles and Identity

Unit 1: Trigonometry Angle I

SAE 1

1. m = $5\sqrt{2}$

Unit 2: Trigonometry Angle II

SAE 1

1. Cos $150^o = \frac{\sqrt{3}}{2}$
   Sin 150o = 1/2

Unit 3: Trigonometry Identity I

SAE 1

SAE 2

Unit 4: Trigonometry Identity II

SAE 1

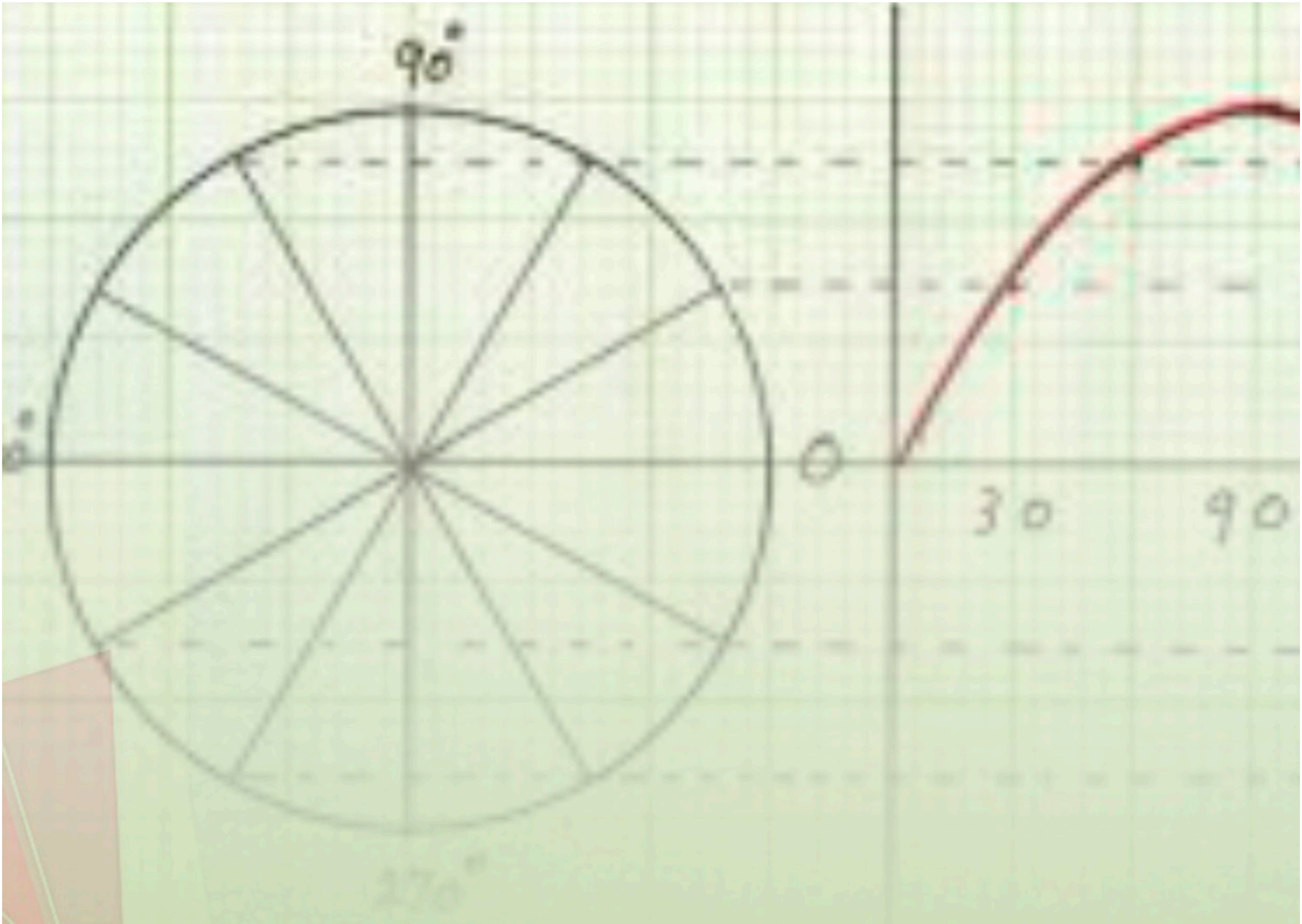
a. -33/65 b. 16/63

SAE 2

1. $-\frac{119}{169}$

SAE 3

1. $+\frac{\sqrt{2+\sqrt{3}}}{2}$



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