On a Semi-Markov Model for Stock Exchange using Capital Assets

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Abstract
This study is aimed at develop a semi-Markov model for stock exchange by using stock values of the Capital Assets for both opening and closing price for the year 2014 to predict stock movement. In particular, we analyzed high frequency data from the Nigerian stock market from the first of January, 2014 until end of December, 2014 and we applied to it the semi-Markov chain model using exponential distribution. It was established that the closing prices were not independent and that the daily closing prices depends on the subsequent daily closing prices and that there is hope of recovering for Capital Assets after the experience of unprecedented decline in the stock value in the years past. Probability of the stock to be stable in the long run is higher than that of rising and falling respectively. This is an indication that there is a high tendency for stock price to be stable in the long run. The results of the analysis of long run behaviour of Capital Assets stock showed that most of these stocks have higher likelihood to increase in price. It was concluded that fifty percent of the stocks show a higher tendency of increase in price, twenty- five percent shows a higher likelihood of being stable and the remaining twenty- five percent show a higher likelihood of falling in the long run. This study recommends that government should make a clear economic policy framework to discourage investors from holding their money in cash or taking them out of the Nigerian market for safety, Central Bank of Nigeria should be able to establish foreign exchange management programme to control liquidity in the foreign exchange market and the model developed in this study should be implemented in predicting the long-term behaviour of the stock value price in Nigerian market.

Keywords: Bear market, stagnant market, bull market, blue chips, bonus, base price

Introduction

Forecasting the direction of future stock prices is a widely studied topic in many fields including trading, finance, statistics and computer science. The motivation for which is naturally to predict the direction of future prices such that stocks can be bought and sold at profitable positions. Professional traders typically use fundamental and/or technical analysis to analyze stocks and make investment decisions. Fundamental analysis is the traditional approach involving a study of company fundamentals such as revenues and expenses, market position, annual growth rates (Elliot and Kopp, 1999). The Nigerian Stock Exchange has been operating an Automated Trading System (ATS) since April 27, 1999, with dealers trading through a network of computers connected to a server. The ATS has facility for remote trading and surveillance. Consequently, many of the dealing members trade online, the exchange is in the process of establishing more branches for online real time trading. Trading on the exchange starts at 9.30 a.m. every business day and closes at 2.30 p.m. (Obodos, 2005). The principal role of a stock exchange in any economy is to mobile resources and directs them to the productive sectors of the economy. It offers relatively cheap source of capital for investment and working capital requirements compared to the traditional financial intermediaries (Abdulazeez, 2012).

The objective of this study is to develop a semi-Markov model for stock exchange by using stock values of the Capital Assets for both opening and closing price for the year 2014 to predict stock movement.

Literature Reviews

In modeling stock prices, two competing approaches exist: technical modeling and fundamental modeling. Technical models capture statistical phenomena observed directly in stock prices, while fundamental models consist of mathematical descriptions of the behaviour of economic agents that affect the stock price (Elliott and Kopp, 1999). The development and analysis of multi-agent fundamental models has been the subject of research in the Stochastic Systems Research Group of SZTAKI since 2006 (Fouque *et al.*, 2000). Semi-Markov Processes (SMP) are a wide class of stochastic processes which generalize at the same time both Markov chains and renewal processes. The main advantage of SMP is that they allow the use of whatever type of waiting time distribution for modeling the time to have a transition from one state to another one. On the contrary, Markovian models have constraints on the distribution of the waiting times in the states which should be necessarily represented by memory-less distributions (exponential or geometric for continuous and discrete time cases respectively (Delbaen and Schachermayer, 2008). The mathematical formulations on the trends of stock market are not new, other methods such as Markov models have been used to predict stock markets. However, the semi-Markov model developed in this study incorporates uncertainty and variability.

Methods and Materials

We examined a set of stock prices and use the probability method of Semi-Markov chains to predict the values of the stock prices in their immediate future. Thus, a data set of such prices was collected from Capital Assets examined, and then the probability method was applied. The process of stock market trends is considered as a semi-Markov process, three states of the market were specified. The states are Bull, Bear and Stagnant market states. Abubakar (1995) used semi-Markov modeling for the control of leprosy and found that the probability density function of the random variable T has the exponential distribution given. The study concluded that a variety of highly simplified assumptions are often needed to develop mathematical models. The complexity of analytical stochastic models makes them more difficult for the people who are supposed to use them. According to Pidd (1992), the communication gap could be minimized by the use of simulation and simple mathematics. It was also stressed that the semi-Markov model can be used as a predictive device for studying the health status of leprosy patients. The predictions are useful to doctors, hospital administrators, policy makers and the general public.

A Markov process is described by the "state" of a system and state "transition" as a process that runs in time. At any given point in time, the process is in a given state and could possibly make a transition to another state after a period of time. A Markov process in continuous time and continuous state is called a Markov chain (Ghysels *et al.*, 1996). Markov process is a sequence ($x_0, x_1, x_2, \ldots, x_n$) of discrete random variables with the property that the conditional probability distribution of; x_{n+1} given $x_0, x_1, x_2, \ldots, x_n$ depend only on the value of x_n but not further on $x_0, x_1, x_2, \ldots, x_{n-1}$. That is for any set of values, $h, i, \ldots, j$ in the discrete state space given in equation (1);

$$P(x_{n+1} = j / x_0 = h \ldots x_n = j) = p(x_{n+1} = j / x_n = I) = p_{ij}; \qquad i, j = 1,2,3, \ldots \qquad (1)$$

The matrix P whose entries are the p_{ij} `s is called the transition probability matrix for the process. In this process, the probability of making transition to a future state does not depend on the previous state and only depend on the present state. In other words, the probability of making a transition to a future state does not depend on the past history. The matrix P and the initial state transition probabilities completely specify the process. If the transition probabilities depend on time, then the Markov chain is non-homogenous, otherwise it is homogenous.

The stochastic process occurring in most real life situations are such that for a discrete set of parameters;

$t_1, t_2, t_3, \ldots, t_n;\ t \in T$, the random variable;

$X(t_1), X(t_2), X(t_3),... X(t_n)$; exhibit a semi-Markov process.
and transition probabilities for; $i, j \in X$, $s, t \geq 0$
We then refer to the process given in equation (2);

$$J(t) = J_{N(t)}, t \geq 0 \quad (2)$$

as a semi – Markov process with

X = state space which is the set of the possible values of $i \, and \, j$

P_{ij} = probability that the process whenever in state i, moves into state j.

Model Assumptions

The following assumptions are made:

(i) The movement of the stock-prices of the Capital Assets is considered between the three states as a random variable indexed by time parameter.

(ii) In a unit of time, if the stock-price profile is in bull market state the probability that a transition is made to a bear market state or stagnant market is dependent on the present state (bull market).

(iii) Unit of time is one day

(iv) The following are the assumed states for the process

(a) Bull market state: A state at which stock price value changes upward (rising) from the previous state.

(b) Stagnant market state: A state at which stock price value does not change (stable) from the previous state

(c) Bear market state: A state where stock price value changes downward (dropping) from the previous state.

A possible transitions between the states is given in Figure 1

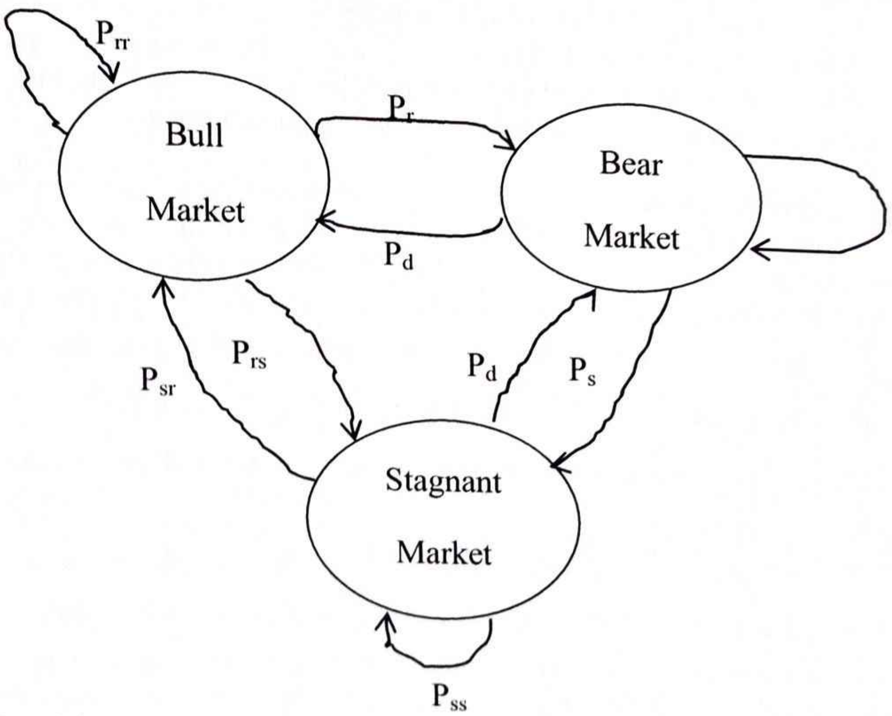

Figure 1: Transition diagram for movement of the stock-prices

where;

P_{rr} = Probability of stock prices rising after previous rise (Bull to Bull state)

P_{rd} = Probability the stock price dropping after rises in the previous day (Bull to Bear state)

P_{rs} = Probability that stock prices remain stable after rising in the previous day (Bull to Stagnant state)
P_{sr} = Probability of stock price rising after being stable in the previous day (Stagnant to Bull state)
P_{sd} = Probability of the stock price dropping after remaining stable in the preceding day (Stagnant to Bear state)
P_{ss} = Probability that stock remains stable after remaining stable in previous day (Stagnant to Stagnant state)
P_{dr} = Probability that stock price rises after dropping in the previous day (Bear to Bull state).
P_{ds} = Probability that stock prices remains stable after dropping in previous day (Bear to Stagnant state)
P_{dd} = Probability that the stock price drop after dropping the previous day (Bear to Bear state)

The transition probabilities for the process is given as matrix (3).

$$P = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} \quad (3)$$

Probability Matrices " P" for the Processes

We recorded the transition probability matrices " P"" for the process as equations 4-12

$$P_{rr} = \frac{n(rr)}{n(rr)+n(rd)+n(rs)} \quad (4)$$

$$P_{rd} = \frac{n(rd)}{n(rr)+n(rd)+n(rs)} \quad (5)$$

$$P_{rs} = 1-[P(rr)+(P(rs))] \quad (6)$$

$$P_{dr} = \frac{n(dr)}{n(dr)+n(dd)+n(ds)} \quad (7)$$

$$P_{dd} = \frac{n(dd)}{n(dr)+n(dd)+n(ds)} \quad (8)$$

$$P_{ds} = 1-[P(dr)+(P(dd))] \quad (9)$$

$$P_{sr} = \frac{n(sr)}{n(sr)+n(sd)+n(ss)} \quad (10)$$

$$P_{sd} = \frac{n(sd)}{n(sr)+n(sd)+n(ss)} \quad (11)$$

$$P_{ss} = 1-[P(sr)+(P(sd))] \quad (12)$$

Then, the transition probability matrices were then arranged as shown in matrix (13);

$$P = \begin{bmatrix} p_{rr} & p_{rd} & p_{rs} \\ p_{dr} & p_{dd} & p_{ds} \\ p_{sr} & p_{sd} & p_{ss} \end{bmatrix} \quad (13)$$

This long term or steady state probability is therefore;

$$[R_L, S_L, D_L]$$

where

R_L = probability of the stock rising in the long run.

The transition probabilities are estimated using relative frequency given in Table 2 and we have matrix P;

$$P = \begin{bmatrix} 0.97 & 0.03 & 0 \\ 0.02 & 0.96 & 0.02 \\ 0 & 0.10 & 0.90 \end{bmatrix}$$

Table 3: Interval Transition Probabilities

n	Q_{12}	Q_{21}	Q_{32}	Q_{32}
1	0.02624333	0.018892778	0.018892778	0.07381718
2	0.02630925	0.01893568	0.01893568	0.07481718
3	0.02637517	0.01894357	0.01894357	0.07581718
4	0.02644109	0.01895147	0.01895147	0.07681718
5	0.02650701	0.01895936	0.01895936	0.07781718
6	0.02657292	0.01896725	0.01896725	0.07848171
7	0.02663884	0.01897515	0.01897515	0.07981718
8	0.02670476	0.01898304	0.01898304	0.08081718
9	0.02677068	0.01899094	0.01899094	0.08181718
10	0.0268366	0.01899883	0.01899883	0.08281718
11	0.02690251	0.01900673	0.01900673	0.08381718
12	0.02696843	0.01901462	0.01901462	0.08481718
13	0.02703435	0.01902252	0.01902252	0.08581718
14	0.02710027	0.01903041	0.01903041	0.08681718
15	0.02716619	0.0190383	0.0190383	0.08781718

Table 4: Mean Holding Time

State 1	State 2	State 3
21.3333333	50.3333333	10.0000000

Table 5: Test for Independence of daily closing price

States	χ^2	p - value
State 1	162.40	0.001
State 2	77.13	0.000
State 3	163.24	0.001

Discussion of Results

Results presented in Tables 1- 5 revealed that the daily closing prices for all the three states stocks were not independent (p_value < 0.05). This means that the daily closing prices depends on the subsequent daily closing prices and that there is hope of recovering for Capital Assets after the experience of unprecedented decline in the stock value in the years past. Probability of the stock to be stable in the long run is higher than that of rising and falling respectively. This is an indication that there is a high tendency for stock price to be stable in the long run. The results of the analysis of long run behaviour of capital assets stock shows that most of these stocks have higher likelihood to increase in price In summary, fifty percent of the stocks show a higher tendency of increase in price, twenty- five percent shows a higher likelihood of being stable and the remaining twenty- five percent show a higher likelihood of falling in the long run. This is in line with the results found by Ngaloru *et al.* (2015).The results presented show that if the past volatility is used as an exponentially weighted index, the model is able to reproduce more correctly the behavior of market returns. The returns generated by the model are uncorrelated while the square of returns presents a long range correlation very similar to that of real data. We have also shown by analyzing different stocks from different markets that results do not depend on the particular stock chosen for the analysis even if the value of the weights may depends on the stock. This is in agreement with the findings by Abdulazeez (2012).

The stock value of the Capital Assets for both opening and closing price for the year 2014 (02/01/2014 – (31/12/2014) revealed that the stock market does not operate on weekend and public holidays. The closing price is been grouped according to the three states and that the holding time in each state before making a transition to another state follows the exponential distribution with parameter λ. The mean holding time are 21days, 50days and 10 respectively for state 1, state2 and state3 respectively. The findings here corroborates the results obtained by Obodos (2005)

Conclusions

This study established that if there are no transactions in the minute, the price remains unchanged (even in the case the title is suspended and reopened in the same day). The model returns as a semi-Markov chain with the discretized state space chosen is symmetrical with respect to returns. The study also concludes that returns were uncorrelated and shows an independently and identically distribution with the model tested to reproduce such behavior. The semi-Markov model starts at the same value but the persistence is very short and after few time steps the autocorrelation decreases to zero. A very interesting behavior is instead shown by the semi-Markov models with memory index. This is because short memories are not enough to identify in which volatility status is the market, too long memories mix together different status and then much of the information is lost in the average. Therefore, if the past volatility is used as an exponentially weighted index, the model is able to reproduce more correctly the behavior of market returns. The returns generated by the model are uncorrelated while the square of returns presents a long range correlation. The study further concludes that different stocks from different markets that results do not depend on the particular stock chosen for the analysis even if the value of the weights depends on the stock.

Recommendations

Basing on the conclusions from this study, the following recommendations were made:

(i) There should be clear economic policy framework by government to discourage investors from holding their money in cash or taking them out of the Nigerian market for safety.

(ii) Central Bank of Nigeria should be able to establish foreign exchange management programme to control liquidity in the foreign exchange market.

(iii) The model developed in this study should be implemented in predicting the long-term behaviour of the stock value price in Nigerian market.

References

Abdulazeez, H. (2012). Markov Chain Model and the Application to Foreign Debt Management in Nigeria". Unpublished Masters Dissertation, Federal University of Technology, Minna, Nigeria.

Abubakar, U. Y. (1995). "Semi-Markov Modelling for the Control of Leprosy". Unpublished Masters Dissertation, University of Jos, Jos, Nigeria.

Ahmed, H. O. (2016). "Developing a Semi-Markov Model for Stock Exchange Using Capital Assets". Unpublished Masters Dissertation, Federal University of Technology, Minna, Nigeria.

Delbaen, F. and Schachermayer, W. (2008). "The Fundamental Theorem of Asset Pricing for Unbounded Stochastic Processes". *Math. Ann*. 3(2): 215–250.

Elliott, R. J. and Kopp, E. (1999). *Mathematics of Financial Markets*. Springer, Berlin Heidelberg.

Fouque, J. P., Papanicolaou, G. and Sircar, K. R (2000). *Derivatives in Financial Markets with Stochastic Volatility*. Cambridge University Press, Cambridge.

Ghysels, E., Harvey, A. C. and Renault, E. (1996). *Stochastic volatility*. Elsevier, Amsterdam.

Jarrow, R., Bhargava, T. N. and Tsantas, N. (1997). "A Markov Model for the Term Structure of Credit Risk Spreads". *The Review of Financial Studies*, 10(1): 481-523.

Ngaloru, S. N., Abdulazeez, H., Ebuk, L. E., Adiukwu, R. and Nafiu, L. A. (2015). "Application of Markov Chain Model to Foreign Debt Management in Nigerian Economy". *The American Journal of Innovative Research and Applied Sciences*. 1(2): 59 – 66.

Obodos, E. (2005). "Predicting Stock Market Prices in Nigeria: A Preliminary Investigation". MBA Dissertation, University of Benin, Benin City, Nigeria.

Pidd, M. S. (1992). *Computer Simulation in Management Science*. Third edition. John Wiley, New York.