

**BRIGHT NICKEL PLATING OF METAL**

**BY**

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**A PROJECT SUBMITTED TO THE DEPARTMENT OF  
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FOR THE AWARD OF BACHELOR OF ENGINEERING  
(B. ENG) DEGREE IN CHEMICAL ENGINEERING**

**OCTOBER 2003**

**CERTIFICATION**

This is to certify that the project, bright nickel-plating of bottle-top opener is a work of SADIKU MUYINALOUWARANTE.

This work was carried out by her under my supervision and was approved for submission to the Department of Chemical Engineering, School of Engineering and Engineering Technology, Federal University Of Technology, Minna.

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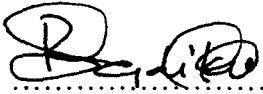
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## DECLARATION

I SADIKU MUYINAT OLUWARANTI declare that the project presented for the award of bachelor of engineering in chemical engineering department has not been presented for any other degree elsewhere.



SADIKU M.O

20/10/2003

DATE

## **DEDICATION**

This project is dedicated to God Almighty, the Giver of wisdom, who has never left me behind.

## ACKNOWLEDGEMENT

First of all, I wish to thank my parents for their continuous support and encouragement till; this very moment.

I will like to show my appreciation and gratitude to my supervisor, Dr. Duncan Aloko, who saw to it that I understood what I was doing and made sure that I followed the proper process, which made my work easier.

I will also show my appreciation to Rev Remi Adegoke in FIRRO who supervised during my practical.

My special thanks goes to my friends, Chinyere, Shola, Nihinlola, Uche and Tijani for their encouragement.

Lastly, I will like to thank my cousins, Bukola Oloruntoba and Bolaji Lewu.

## ABSTRACT

This research was aimed at experimenting on electroplating of bottle-top of opener using nickel to increase the strength and corrosion resistance. This can also serve as decorative and protective measure. The variables determined during the experiment are time (s), temperature (°C), and voltage in volts. The effects of these variables were observed. The result obtained is shown in Table 1. The temperature and the voltage of the process are within the range of operating condition. At 3.5 volts and at the temperature of 56 °C, highest output was obtained, i.e. the best-plated openers were obtained.

The surface area of the metal was calculated to be 18,7904cm<sup>3</sup> and was used to determine the amount of electrolyte used in covering the surface of the metal. The thickness of plated deposit is dependent upon the plating time. At 18 minutes, the surface appearance of the opener was very bright and at 27 minutes, the surface of the opener was not attractive, which indicates that the time was too long for the plating.

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## CHAPTER ONE

### 1.0 INTRODUCTION

This research project is a work aimed at the parametric evaluation on nickel-plating using watts formulation. The watts formulation consists of nickel sulphate ( $\text{NiSO}_4 \cdot \text{H}_2\text{O}$ ), nickel chloride ( $\text{NiCl}_2 \cdot 6\text{H}_2\text{O}$ ) and boric acid ( $\text{H}_3\text{PO}_4$ ), is by far the most widely used electrolyte. Modern-day organic additives produce brilliant deposits. These additives reverse the conventional nickel-plating in that they have virtually eliminated the laborious process of mechanical buffing. The more the corrosion resistance, sulphur-free, semi bright nickel deposits have also become important as under coating for bright nickel.

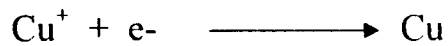
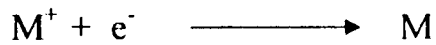
A nickel sulphamate bath is used for engineering application to produce low stress deposits. The bath is also useful in electro foaming and for plating parts that are susceptible to fatigue failure.

Nickel plating is widely used for decorative purposes. Bright nickel plate – especially in combination with a lower layer of sulphur-free, semi bright nickel and a much thinner upper layer of chromium is widely used over steel, brass, zinc due to coating and corrosion resistance to provide bright and corrosion resistance finish with a non tarnishing and wear-resistance surface.

In the simplest terms, plating transfers metal from positive ANODE to the negative CATHODE following Faraday's Law. Series of objects are immersed into chemical baths or vats to change their surface condition. Every plating is unique and the number of terms and their chemical make-up differs based on desired results. Metals slabs or balls (anode) are placed in an electrolyte solution.

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Dissolved metal ions are driven by a d.c. electrical current of low voltage and high amperage (rectifier). During electroplating, pure metal dissolving from the anode is deposited on the cathode by an electron gain transfer method, which can be represented thus:



$M^+$  represents the metallic ion required, Cu and  $Ni^{2+}$  represent deposits of copper and Nickel. Object to be plated is placed on racked or perforated barrels and immersed in series of chemical solution to prepare them for plating. The object to be plated must be clean in order to be successfully plated.

## OBJECTIVES OF ELECTROPLATING

The objectives of electroplating are as follows:

- (i) To change the surface properties of metals and non-metals.
- (ii) To get improved appearance on the basis metal.
- (iii) To obtain improved resistance to corrosion, tarnish chemical attack and wear.
- (iv) To get increased hardness.

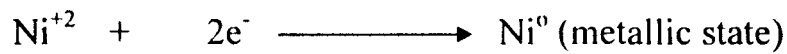
## 1.2 THEORY OF ELECTROPLATING

In electroplating, the work piece is made the cathode in a solution containing the ions of the metal being deposited. Direct current is passed between the anode and the work piece (cathode). The anode is usually constructed of the same material as the metal being plated,

although some plating processes use insoluble anodes. As the current flows, the metal ions gain electrons at the cathodic work piece and transform into a metal coating based on the following equation.



For nickel-plating, the following equation describes the reaction at the cathode



Proprietary additives (brighteners) are usually incorporated in the plating solution to alter the deposit in a desirable fashion. The additives brighten or level the deposit as well as to improve the uniformity of the deposits thickness over the entire work piece. The additives may also be used to alter physical properties such as hardness, ductility, internal stress and corrosion resistance.

## CHAPTER TWO

### 2.0 LITERATURE REVIEW

Electroplating is the covering of a thin layer of metal on a material using a metal in order to provide resistance to corrosion, special appearance such as colour or to increase the dimensions.

Ordinarily, metal wires are good conductors of electricity. This is due to easy availability of electron in a metal and they have freedom of movement. When an electric field is applied to a metal according to the direction of applied field, electrons starts moving from negative charged end of metal to its positive charge, this way current flow in metal. Substance conduct electricity when an electron or some other electrically charged the medium that carries the current by means of ions. The ability of a solvent especially H<sub>2</sub>O to ionize substance dissolved in it, in order to split them into two components which carry negatively and positively charge that enables electroplating possible.

The metal plates dipped in a solution through which the electrons are liberated or absorbed are called electrodes. Moreover, when an electric current is passed through a metallic conductor, according to the direction, so doing that, electron flows as a stream in a simple direction. Thus the flow of current in an electrolyte has two streams of cations and anions flowing mutually opposite directions. The total current strength is the sum of current strength due to the streams of cations and anions. A solution which is a good conductor of electricity produces these ions with the aid of the electrodes dipped into it when an electric field is applied, positively charged ion (cations) move to the negatively charged electrode and negatively charged ions (anions) move to the positively charged electrode.

## **2.1 ELECTROLYSIS**

A solution is a good conductor of electricity with cation and anions in it. These ions are normally produced from solute and are sometime produced by a reaction between the solute and solvent. When an electric current is flown in solution by dipping electrodes into it positively charged ions (cations) moves to the negatively charged electrode and negatively charged (anions) moves to the positively charged electrode. Thus, in an electrochemical reaction the electron chemical change involving combination with release of electrons takes place and ions are normally converted to the electrically neutral product, which are totally different from solution. This phenomenon is called ELECTROLYSIS. In other words, electrolysis is a process in which the anions and the cations present in dissolved state of a substance, are applying an electric field, these are transferred to their respective electrode (i.e. cathode and anode), and these loses their charges and are converted to a new substance.

## **2.2 FUNDAMENTAL LAW OF ELECTROLYSIS**

Product of electrolysis only appears at the electrode surfaces and nowhere else in the solution because the electrolysis is the results of electron acceptance and electron shedding phenomena and this can occur only at electrode surface.

Electrolyte is the medium that carries the current by means of ions.

### **2.2.1 FARADAY'S FIRST LAW OF ELECTROLYSIS**

In 1832-1833, Michael Faraday observed that the amount of substance undergoing oxidation or reduction at each electrode during electrolysis is directly proportional to the amount of electricity that passes through the cell. That is, the mass of an ion liberated at an electrode is directly proportional to the quantity of electricity, the charge that passes through the electrolyte.

In any process of electrolysis, the deposit formed on an electrode is directly proportional to the quantity of electricity flown through the electrolyte. The quantity of electricity passed through electrolyte during the electrolysis is proportional to the current strength and for the time for which the current is passed.

If the quantity (Q) is measured in Coulombs and strength of the current flown (I) measured in Ampere and time (t) is measured in seconds. Hence, the quantity of electricity,

$$Q = I \times t$$

Weight of deposit on electrode:  $M \propto Q$

$$M \propto It$$

$$M = zIt$$

Where z is the constant of proportionality and it depends upon the characteristics of the ionic species discharged at the electrode. The constant, z, is called electrochemical equivalent of an ionic species and its unit is in gram per Coulomb (g/C).

2.2.15

### 2.2.2 FARADAY'S SECOND LAW OF ELECTROLYSIS

The second law of Faraday states that when the same quantity of electricity is passed through a number of electrolytes, the weight of corresponding deposit at different electrodes are exactly proportional to their chemical equivalents respectively.

Chemical equivalent is the ratio of atomic weight and valency. By second law of Faraday, when a current is passed through two electrolyte, if the deposit in two different electrodes  $W_1$  and  $W_2$  and their equivalent weight is  $M_1$  and  $M_2$ ,



then

$$\frac{W_1}{W_2} = \frac{M_1}{M_2}$$

In essence, the second law of Faraday shows that the quantity of electricity required to liberate one mole of any element is proportional to the charge number of its ion.

The charge number of an ion is the number of positive or negative charges, which the ion possesses.

### 2.2.3 MECHANISM OF ELECTRODEPOSITION

Although, the techniques of electroplating are well established and process can run very smoothly provided that the adequate control is exercised, the mechanism of ion reduction and accommodation in the lattice, which is extremely complex, and it is only comparatively recently that light has been shed on this subject.

The reaction steps involved are:

- (a) Transport of hydrated ions to the cathode diffusion layer.
- (b) Their passage across the diffusion layer and partial dehydration.
- (c) Adsorption of the partly dehydrated ions (anions) at the cathode surface and diffusion along this surface to low energy positions.
- (d) Complete dehydration, discharge and accommodation of these ions at appropriate lattice sites such as known as growth sites or active sites and in ideal cases are links in monatomic micro steps. At the site, the ion, having completely lost its molecules of hydration is no longer an anion and being in contact with one half of the number of atoms, it would be in contact within the bulk of the metal, it is said to be half crystal position.

Two reaction steps are involved in the deposition of an atom, namely;

- (a) Charge transfer across the double layer with formation of anions.
- (b) Surface diffusion of anions from their landing sites to monatomic steps.

Each of these reaction steps may control the overall rates of a reaction at a given electrode potential.

In the condition of Electro deposition, surface diffusion of anions is likely to be the controlling factor of the overall rate of deposition at low cathode over potentials, and the charge transfer should become rate controlling with increasing.

The growth of layer will largely be determined by the relative rates of processes and in a plating process, the current density is employed. At low current density, the surface diffusion is fast compared with the electron transfer and the atom is likely to end up in a favoured position in the lattice. At higher current density, the surface diffusion is no longer fast compared with electron transfer and further nuclei must formed the layer will be less ordered.

In most electroplating process, this primary layer is thickening from a few atomic layers to the desired thickness. In this period the metal deposition reaction occurs effectively at an electrode of the electrodeposited. This does not necessarily mean that the electrode reaction is simple. In many cases, the electroplating reaction will involve reduction of a complexes metal species and then the electrode reaction may involve chemical predissociation steps. As the current density increases, surface diffusion is no longer fast compared with the discharge process and atoms no longer reach their equilibrium position in lattice. At the increased over potential, nucleation of additional growth centres remains more frequent event. Hence will be less ordered and macro scale features, steps, ridges and polycrystalline block growth, become more likely. With further increase in current density, outward growth of the layer becomes the increasing

importance and problems arising from mass transport control in solution can arise.

## **2.3 CHARACTERISTICS AND PROPERTIES OF NICKEL PLATING**

### **2.3.1 ELECTROPLATING PROCESS**

Nickel may be deposited to produce variety of finishes such as bright, dull white, black and grey, by modifying the composition of the bath, current density, temperature, the use of addition agents, agitation of the bath and occasionally other characteristics. Preliminary to the actual deposition of nickel in plating, the steel surface must be made clean and free from oxide.

A widely used electrolyte for producing a bright nickel plate is some type of the watts formulation which is approximately 200 – 300ml<sup>-1</sup> of NiSO<sub>4</sub>.7H<sub>2</sub>O, 60-70ml<sup>-1</sup> of NiCl<sub>2</sub>.6H<sub>2</sub>O, 30-40ml<sup>-1</sup> of N<sub>2</sub>BO<sub>3</sub>, 5-10mls/L of brightener, 50-60ml<sup>-1</sup> of anode (Ni).

### **2.3.2 CHARACTERISTICS OF NICKEL PLATING**

Bright nickel plate by the watts formulation is a soft plate with a hardness of about Vickers 140 (Rockwell 1375). By varying the pH and temperature of hard plating bath, a range of hardness can be obtained up to maximum of about Vickers 500. Nickel plate softens slowly under increasing temperature, until at 500°F. There is a loss in hardness of about 18%. Above 500°F, it softens rapidly, until at 1400 °F, it is thoroughly annealed. Ductility of the plating varies with hardness, and values from 35 to 1 per cent have been found.

Bright nickel plate is tightly adherent and free from porosity. The soft plating is coarse grained, but the hard plating is very fine grained. It has good throwing power readily takes a chromium plate nickel builds up much faster than chromium. At a current density of 100 Amp per sq. ft., nickel will plate at the rate of 0.005 in inch/hr as chromium, current density of ampere per sq. ft, will plate only at 0.0004in per hr. For this reason, nickel is often preferred where

heavy deposits are desired.

Nickel electrodeposits are under internal tensile stress. Therefore, when external tensile or bending loads are applied, the internal stresses induced by them are added to the existing residual tensile stresses. Hard nickel-plating like chromium plating contains numerous fine cracks.

### **2.3.3 CHEMICAL PROPERTIES OF NICKEL**

Nickel is only moderately reactive. It resists alkaline corrosion and does not burn in the massive state, although fine nickel wires can be ignited. Specially prepared nickel, consisting of very small, porous particles, burns spontaneously when exposed to the air. Nickel is above hydrogen in the electrochemical series, and it dissolves slowly in dilute acids, releasing hydrogen and forming the green deposit nickel ion  $Ni^{++}$  with solutions of oxidizing agents, including strong nitric acid, nickel becomes passive and resists attack. In metallic form nickel is a moderately strong reducing agent. Through physiochemical processes, nickel can take up considerable amount of hydrogen, as can palladium and platinum, especially when the metals are heated and in the finely divided state. Release of the hydrogen to other substances for chemical reaction is one reason for the catalytic action of these metals.

### **2.3.4 NICKEL ALLOYS**

Nickel alloys are combination of nickel with other metals. Nickel has been used in electroplating since 1843 and as an alloying addition to steel since about 1889. It was 1<sup>st</sup> used as a base for alloys, with the introduction of nickel-copper alloy in about 1905.

### 2.3.5 ELECTRO POLISHING

Electro polishing is used when a thin bright deposit is to be produced. In this case the deposit essentially copies the substrate surface contour. The substrate surface therefore must have a bright finish that can be attained by electro polishing.

### 2.3.6 NICKEL ELECTROPLATING

Nickel plating is carried out extensively. Nickel coatings covered with chromium provide corrosion resistant and decorative finishes for steel brass and zinc base die coating. The most widely used plating solution is the watt bath, which contains nickel sulphate, nickel chloride, and boric acid. All chloride, sulphurate, and borate plating solution are also used. There are a number of compounds, mostly organic, which can be added to nickel-plating baths. When especially bright nickel is plated, at least one from each of two groups of compounds called Class I and Class II brighteners must be added to the plating solution. Class I compounds, for example, benzene and naphthalene disulphonic acids, contain sulphur. Class II brightener for example butynediol, do not contain sulphur. Class II brighteners can also cause levelling and, if added to the plating bath without the sulphur-containing agents, results in semi bright nickel deposits. For improved corrosion protection, a duplex nickel deposit, that is, a relatively thick, semi bright, sulphur free plate covered with a thinner bright coating is frequently used. A thin chromium layer is plated over the bright nickel. If corrosion begins through a pore or crack in the chromium, it will penetrate to the more semi bright nickel and then spread laterally. Thus unsightly rust spots, which result from penetration to the steel, are less likely to develop.

## 2.3.7 NICKEL AS ANODE

The development of all aspect of nickel deposition was assisted by the introduction of better anodes. The importance of their high purity becomes increasing recognized. However, the purer the anode becomes the more difficult they are to dissolve. In 1929, the depolarised anode was accepted and was a great advance since it dissolves smoothly all condition.

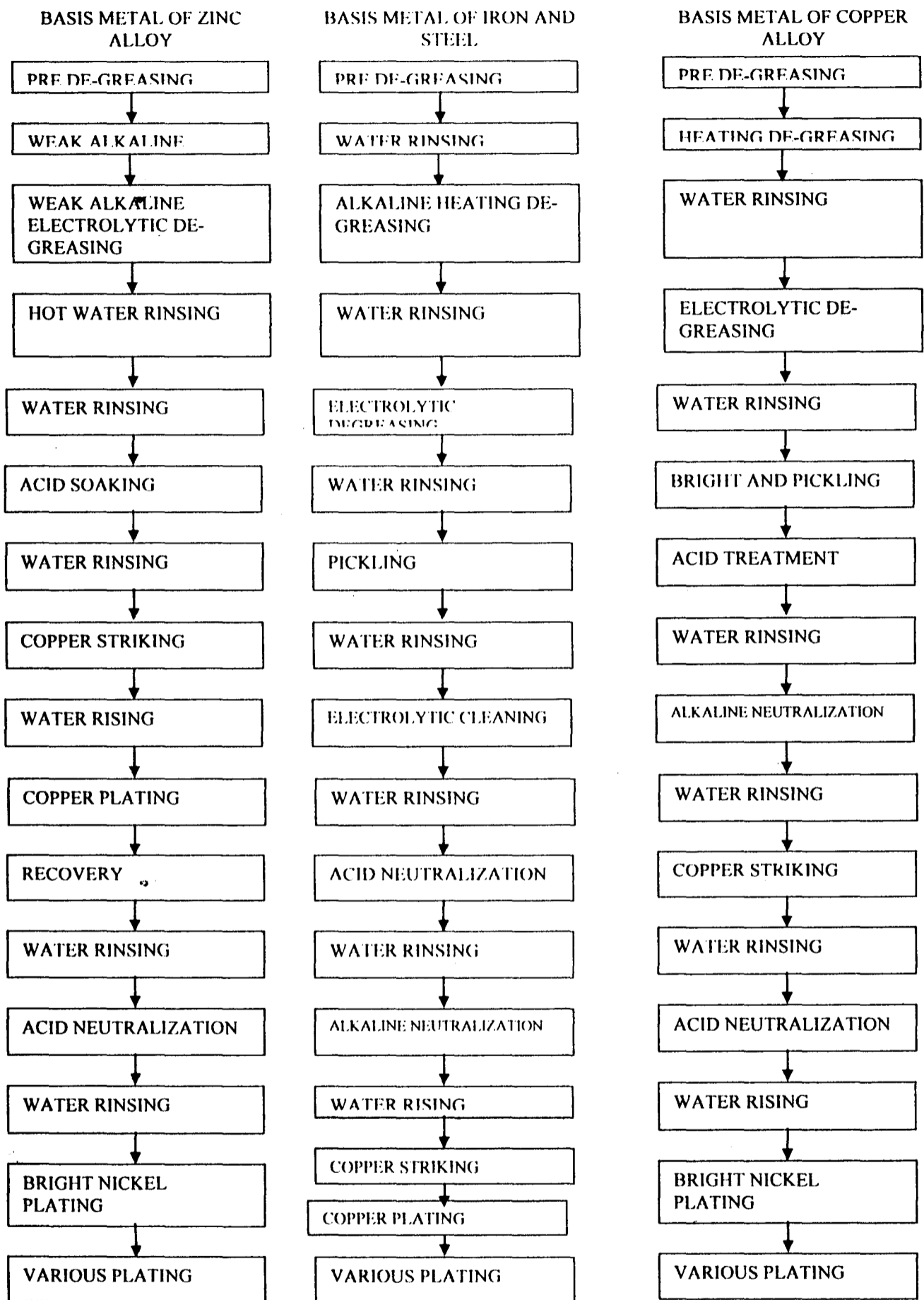


Fig1: Standard plating processes for various basis metals.

## CHAPTER THREE

### 3.0 EXPERIMENTAL PROCEDURE

#### 3.1 MATERIALS

##### 3.1.1 PLATING MATERIALS AND SOLUTION

Nickel formulation and solutions are

- (a) Nickel Sulphate 200-300g/l ( $\text{NiSO}_4 \cdot 6 \text{H}_2\text{O}$ )
- (b) Nickel Chloride 50-60g/l ( $\text{NiCl}_2 \cdot 6\text{H}_2\text{O}$ )
- (c) Boric acid 30-40g/l ( $\text{H}_3\text{PO}_4$ )
- (d) Brightener  $0.1-1 \text{ml}^{-1}$

##### 3.1.2 ELECTRODES

- (a) Anode
- (b) Cathode used were steel openers

##### 3.1.3 APPARATUS AND EQUIPMENT

The apparatus used include:

- (a) Plating bath
- (b) Rectifier
- (c) Racks
- (d) Heater
- (e) Cold swill tank
- (f) Thermometer, used to take the temperature readings of the electrolyte during heating
- (g) Stop watch used for timing the plating process

#### 3.2 EQUIPMENT

The equipment used for carrying out the experiments are:



### 3.2.1 CHEMICAL BALANCE

Chemical balance was used to obtain the weight of the various salts samples of salts that made up the electrolyte.

### 3.2.2 DIRECT CURRENT SUPPLIER

A rectifier was used as D.C. source to provide current to the electrolytic arrangement (electrolytic cells).

### 3.2.3 ELECTRIC HEATER

Electric heater was used to raise the temperature of the bath.

## 3.3 PROCEDURE

### 3.3.1 PREPARATION OF THE PLATING BATH

The plating bath was thoroughly clean to remove all the solid particles and dirt, which was done by sweeping and wiping. It was examined to determine what further cleaning is required. The electrolyte was prepared by weighing each salt of the electrolyte composition on a chemical balance to get the mass needed. Small amount of distilled water was added to the salts to make the solution of it. More water was later added to make it up to 100ml, that is, 1 litre.

### 3.3.2 PRETREATMENT

Each of the openers were soaked and washed in detergent to remove every element of oil impurities from their surface, that is, the degreasing process was done after which they were thoroughly rinsed in water until it had been satisfied that the openers were free from the oil.

All the openers were pickled in a concentrated solution of hydrochloric acid. The pickling process was done for about 15 minutes to ensure the complete

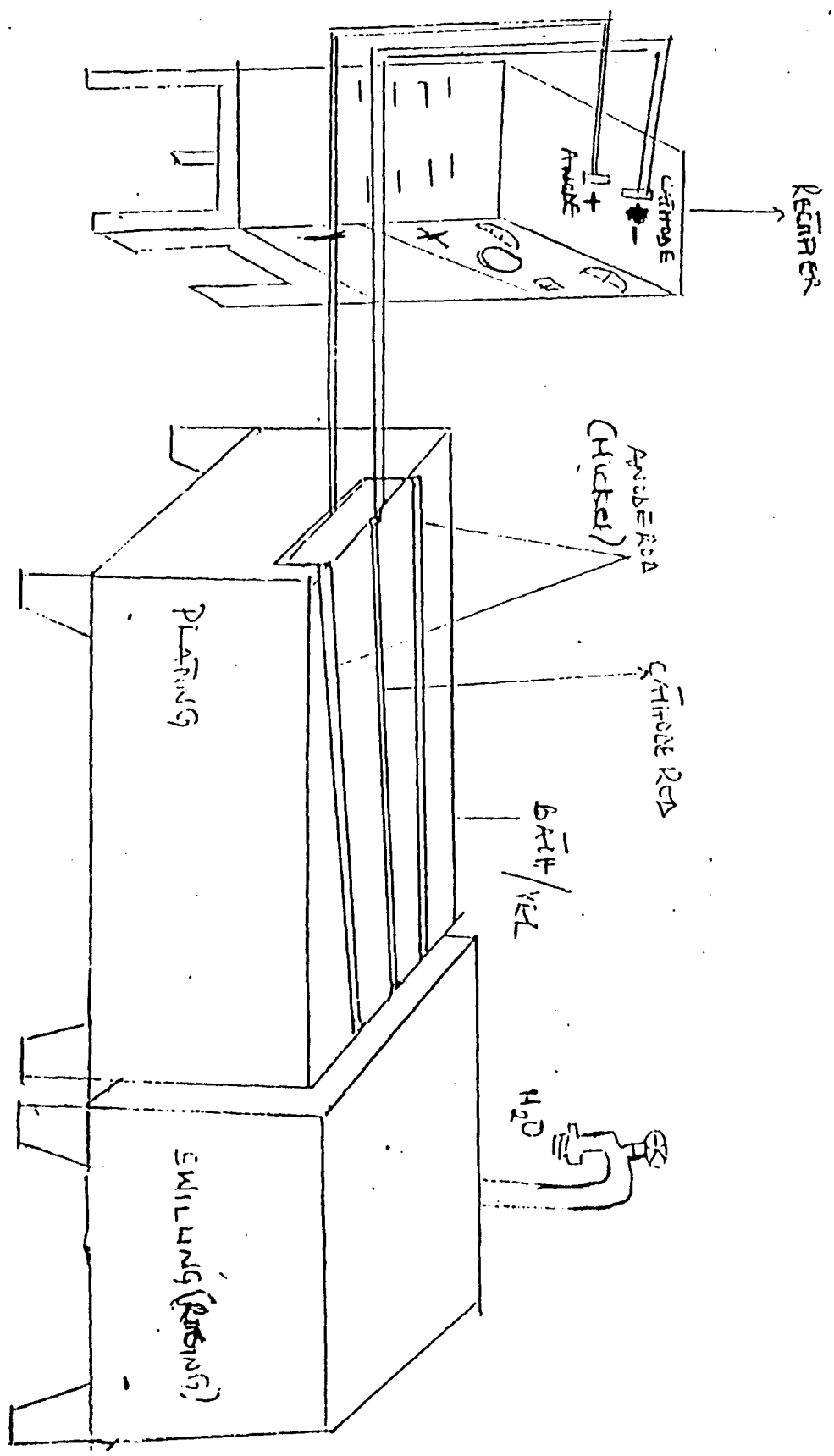
removal of all oxide coatings from the surface. They were removed and rinsed in another bath of water thoroughly.

### **3.3.3 PLATING PROCESS**

The electrolyte was heated in the plating bath and the apparatus were set up. The weighed anodes and cathodes were dipped into the solution with the aid of hooks hung on their bars respectively. The anodes and cathodes were connected to the positive and negative terminals of the D.C. source respectively. The D.C. source was switched on and adjusted and was allowed to pass through the arrangement after which the switch was turned off. The cathode was removed, rinsed thoroughly in another bath of clean distilled water and dried. The final weight of cathode was found with the aid of chemical balance so as to know the amount of nickel deposited on it. Other observations were noted. The process was repeated for eight times at different time and temperature.

EXPERIMENTAL DIAGRAM

NICKEL PLATING PROCESS LINE



## CHAPTER FOUR

### 4.0 RESULT

#### 4.1 EXPERIMENTAL RESULTS

Temperature °C	Voltage (Volts)	Time (mins)	Thickness (microns)	Brightness
42	2	27	$3.7 \times 10^{-6}$	Dull bright
44	2.3	25.5	$4.5 \times 10^{-6}$	Dull bright
46	2.5	24.0	$4.5 \times 10^{-6}$	Bright
48	2.85	23	$3.7 \times 10^{-6}$	Bright
50	3	20	$1.8 \times 10^{-5}$	Brighter
52	3.2	22	$1.1 \times 10^{-5}$	Brighter
54	3.4	19	$7.5 \times 10^{-6}$	Brighter
56	3.5	18	$1.9 \times 10^{-5}$	Brightest
56.	3.5	18	$1.9 \times 10^{-5}$	Brightest

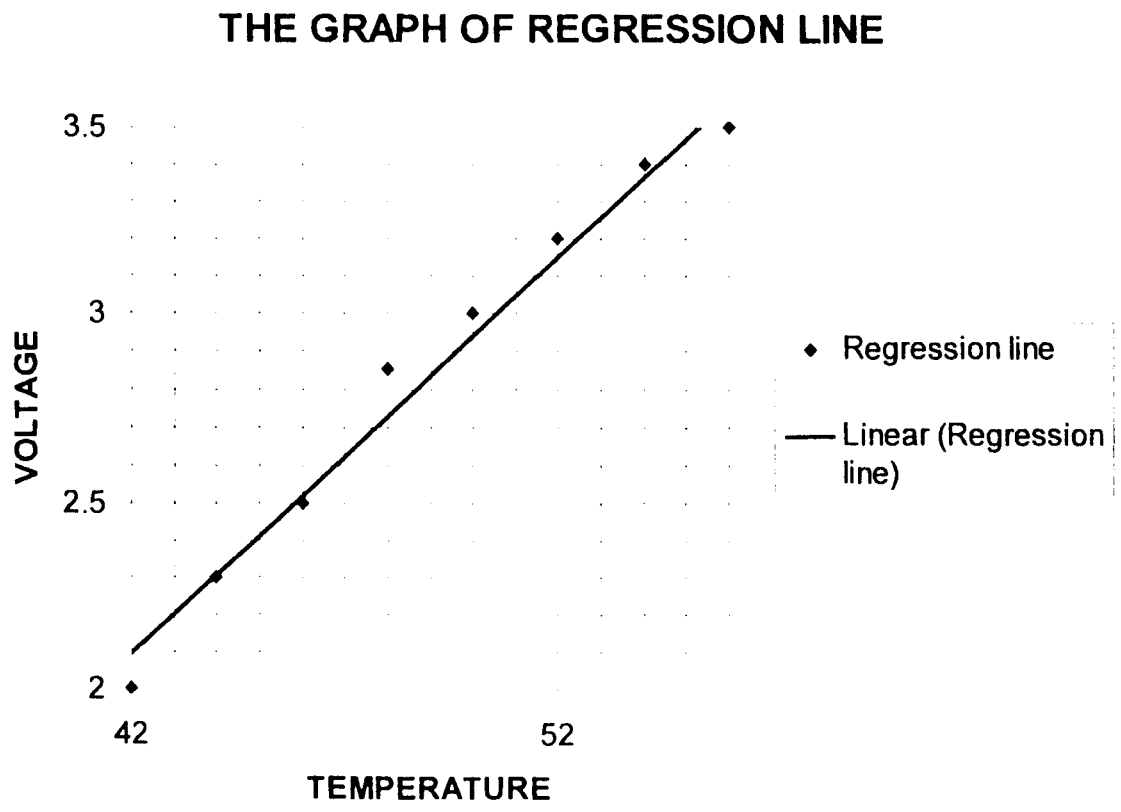
Table 1

#### 4.2: DIFFERENCE IN THICKNESS OF THE OPENER

Temp.	Voltage	Weight before plating	Weight after plating	Thickness (microns)	Difference in weight
42	2.00	22.05	22.10	$3.7 \times 10^{-6}$	0.05
44	2.30	22.04	22.10	$4.5 \times 10^{-6}$	0.06
46	2.50	21.84	21.90	$4.5 \times 10^{-6}$	0.06
48	2.85	21.75	21.80	$3.7 \times 10^{-6}$	0.05
50	3.00	22.25	22.50	$1.8 \times 10^{-5}$	0.25
52	3.20	22.25	22.40	$1.1 \times 10^{-5}$	0.15
54	3.40	22.20	22.30	$7.5 \times 10^{-6}$	0.10
56	3.50	20.84	21.10	$1.9 \times 10^{-5}$	0.26
56	3.50	20.84	21.10	$1.9 \times 10^{-5}$	0.27

Table 2

Fig 2



## CHAPTER FIVE

### 5.0 DISCUSSION OF RESULT

#### 5.1 EFFECT OF THE PARAMETERS ON STEEL

##### 5.1.1 EFFECT OF VOLTAGE

The voltage that must be applied to the bath must not be too high. Moderate voltage that is between 2V to 3.5V was applied to attain the best result. The best opener was obtained and the thickness was measured to be  $1.55 \times 10^{-6}$  and  $1.58 \times 10^{-6}$  microns but at high voltage and was very sharp.

##### 5.1.2 EFFECT OF TIME

The time used in plating is very essential in the process of plating of metals. At different plating time, different result was obtained. It was observed that at 18min, the best plating was obtained. At 27min, the surface at the metal was not attractive which indicates that the time was too long for plating. At 18min, the best plating was determined because the bright appearance of the steel was smooth and neat.

##### 5.1.3 EFFECT OF ANODE

In nickel plating solutions, nickel anodes are used to provide the necessary anode surface, to secure as uniform as possible distribution of the electrode deposit on the work and to maintain the nickel content of the plating solution. Nickel anodes were enclosed in cotton, polypropylene to prevent particles of the surface of the anodes from passing into the solution.

The anode was examined from time to time to ensure that the anode bags were in good condition and when removing anode from the solution, care was taken to ensure that the material inside the bag did not return directly to the plating solution, otherwise, rough deposits may be subsequently obtained.

## **5.2 SEVERAL EFFECTS ON NICKEL DEPOSITION**

### **5.2.1 BRIGHTNESS**

A fully bright deposit must be obtained over current range and irrespective of the thickness but dark deposit must be avoided hence the depth of the column must be good and can be assessed by the human eyes; this is subjected properly and cannot be determined in a quantitative manner. The cleaner the surface of the material, the brighter is the material after plating.

### **5.2.2 ROUGHNESS AND PITTING**

Roughness of deposit is a common defect, which plagues most plating operations. From a visual standpoint, this is more serious in bright plating than when the electrodeposits were heavily buffed. From a standpoint of corrosion resistance, roughness is always bad. If the plate is buffed to eliminate roughness, particles are almost certain to be torn out, greatly reducing resistance to corrosion and often creating a pitted appearance. Roughness, like pitting may also be so fine and widely distributed that it is confused with lack of brightness.

One of the most common causes of roughness when plating on steel is faulty polishing. Insufficient cleaning may also cause roughness.

### **5.2.3 INTERNAL STRESS**

Most electrodeposits are in state of internal tensile stress. The stress can be reduced or made compressively by adding certain materials to the plating bath. The addition of saccharin to nickel-plating solution has this effect. The causes of the internal tensile stress are not well understood but there are indications that the crystallites are one important cause.



### 5.3 DISCUSSION OF THE CORRELATION AND REGRESSION OF VARIABLES

Correlation coefficient is the quantity that is independent of the units in which X and Y are measured and connected.

Correlation coefficient,  $\rho = \frac{Nu}{\sigma_x \cdot \sigma_y}$

An empirical correlation method was used to determine whether the two variables (temp and voltages) are related. The determination of the relationship between the two variables was done in the hope that any relationship to that was discovered could be used to assist in making estimate or predicting of one the variables. Correlation coefficient is not capable of solving such prediction problem. The correlation coefficient serves only as an exploratory tool for determining which variables may be worth incorporating in a regression function for prediction a given variable. The empirical problem of prediction is the problem of estimating the regression function given as:

$$\mu_{y|x} = Y + r \cdot \frac{\delta_y}{\delta_x} \cdot (x - \chi)$$

Which is the equation of a line with a

slope of

$$r \cdot \frac{\delta_y}{\delta_x}$$

That passes through the point (Y,  $\chi$ ).

The result in the predicting technique could not be used to find a relationship between X and Y, so the equation of the regression function corresponding to the predicting equation is

$$\mu_{y|x} = \alpha + \beta \cdot (x - \mu_x)$$

The empirical problem was then reduced to the problem of estimating the parameters  $\alpha$ ,  $\beta$  and  $\mu_x$  which was done by letting the values  $(x_1, y_1)$  to be plotted as a point in the plane  $x_1y_1$  plane. Then the problem of estimating a linear regression function was treated and a problem of fitting a straight line to this set of points. The fitting of a straight line to the points corresponding to the equation above was given as

$$Y^1 = a + b(x - X)$$

Where  $a = Y$  and  $b$  was given as

$$\frac{\sum [(x - X) \cdot y]}{\sum (x - X)^2}$$

The parameters  $a$  and  $b$  was determined in order to minimize the sum of the square of the errors of prediction. The values of the parameters obtained by minimization determine what is known as the best fitting curve in the sense of the least square. In the equation,  $b$  is its slope and  $a$  is the  $y$  intercept on the line  $x = X$ .

Since linear function is so simple to work with and experience shows that many sets of variables are approximately linear related, it is reasonable to attempt to predict the desired variable by means of a linear function of the remaining variables.  $y_1$ ,  $x_1$  and  $x_2$  represented the available variables. In the multiple linear regressions, the effect of  $x_1$  is more than the effect of  $x_2$ .

## CHAPTER SIX

### 6.0 CONCLUSION

#### 6.1 EXPERIMENTAL CONCLUSION

The aim of the work was to obtain a bright-coated opener using nickel to increase the strength and corrosion resistance, which can also serve as the decorative and protective functions. The openers were plated at different temperature, time and the voltage was noted. The thickness is not too much from unplated metal, hence it was concluded that thin coating with bright appearance was the best coating method for engineering material and it is of great importance in chemical industries..

In determining the flexibility of the variable used in the experiment, regression and correlation methods were used. The regression method posses the advantage of being the natural method to use in any experimental situation.

The thickness of the plated deposit is dependent upon the plating time. The estimated thickness of deposit is calculated from the surface area of the article and weight of the deposit.

#### 6.2 RECOMMENDATION

This project work can be recommended to industries to serve as their means of protection to reduce the cost of preventing corrosion.

Nickel plating can also be used as a decorative finishes. In addition to these decorative applications, electroplated and electrodes nickel are also used in industry. Application includes building up of worn or over-machined parts; the electroforming of gramophone masters, press tools and moulds.

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## APPENDIX 1

### CORRELATION AND REGRESSION OF VARIABLES USING LINEAR AND EMPIRICAL METHOD

Runs	y	x	$y^2$	$x^2$	xy	$(y_i - \bar{Y})^2$	$(x_i - \bar{X})^2$
1	2.0	42	4	1794	84	$(-0.9)^2$	$(-7.8)^2$
2	2.3	44	5.29	1936	101.2	$(-0.6)^2$	$(-5.8)^2$
3	2.5	46	6.25	2116	115	$(-0.4)^2$	$(-3.8)^2$
4	2.85	48	8.12	2304	136.8	$(-0.05)^2$	$(-1.8)^2$
5	3.0	50	9	2500	150	$(0.1)^2$	$(0.2)^2$
6	3.2	52	10.24	2704	166.4	$(0.3)^2$	$(2.2)^2$
7	3.4	54	11.56	2916	183.6	$(0.5)^2$	$(4.2)^2$
8	3.5	56	12.25	3136	196	$(0.6)^2$	$(6.2)^2$
9	3.5	56	12.25	3136	196	$(0.6)^2$	$(6.2)^2$
Total	26.25	448	78.96	22512	1329	0.3	26

Table 3

$$\sum_{i=1}^9 y = 2.0 + 2.3 + 2.5 + 2.85 + 3.0 + 3.2 + 3.4 + 3.5 + 3.5 = 26.25$$

$$\sum_{i=1}^9 y^2 = 4 + 5.29 + 6.25 + 8.12 + 9 + 10.24 + 11.56 + 12.25 + 12.25$$

$$\sum_{i=1}^9 y^2 = 78.96$$

$$\sum_{i=1}^9 x = 44 + 42 + 46 + 48 + 50 + 52 + 54 + 56 + 56 = 448$$

$$\sum_{i=1}^9 x^2 = 1764 + 1936 + 2116 + 2304 + 2500 + 2704 + 2916 \dots$$

$$+ 3136 + 3136$$

$$\sum_{i=1}^9 x$$

The mean  $Y = \frac{\sum_{i=1}^9 x}{n}$

$$Y = \frac{26.25}{9} = 2.9$$

$$X = \frac{448}{9} = 49.8$$

$$\text{The variance } \sigma_y^2 = \frac{\sum (y_i - Y)^2}{n - 1}$$

$$\text{The variance } \sigma_x^2 = \frac{\sum (x_i - X)^2}{n - 1}$$

$$\sigma_y^2 = \frac{(0.9)^2 + (0.6)^2 + (-0.4)^2 + (-0.05)^2 + (0.1)^2 + (0.3)^2 \dots + (0.5)^2 + (0.6)^2 + (0.6)^2}{9 - 1}$$

$$\sigma_y^2 = \frac{2.4}{8} = 0.3$$

$$\sigma_y^2 = 0.3$$

$$\sigma_x^2 = \frac{(-7.8)^2 + (-5.8)^2 + (-3.8)^2 + (-1.8)^2 + (0.2)^2 + (0.2.3)^2 \dots + (4.2)^2 + (6.2)^2 + (6.2)^2}{9 - 1}$$

$$\sigma_x^2 = \frac{211.7}{8}$$

$$\sigma_x^2 = 26$$

The Standard Deviation

$$\sigma_x = \sqrt{\sigma_x^2} = \sqrt{26} = 5$$

$$\sigma_y = \sqrt{\sigma_y^2} = \sqrt{0.3} = 0.55$$

## EMPIRICAL METHODS FOR CORRELATION AND REGRESSION

There are five parameters needed to specify a normal density (current), namely,  $\mu_x$ ,  $\mu_y$ ,  $\sigma_x$ ,  $\sigma_y$  and  $\rho$ . From earlier work on the estimation of mean from variance of the single variables in the linear regression method it follows that  $X$ ,  $Y$ ,  $S_x$  and  $S_y$ . Thus all that remains will be  $\rho$ . Since the method of moments is being employed for estimation,  $\rho$  will be estimated indirectly in the manner by replacing the parameters involved in its



definition as given by the formula

$$\rho = \frac{\mu_{11}}{\delta_x \cdot \delta_y}$$

The product moment  $\mu_x$  is defined as

$$E(X - \mu_x)(Y - \mu_y)$$

If  $\mu_x$  and  $\mu_y$  are unknown and are estimated by  $X$  and  $Y$  respectively, then the corresponding moment is chosen to be

$$\frac{\sum_{i=1}^n (x_i - X)(y_i - Y)}{n - 1}$$

$\rho$ , which is denoted by  $r$ , is therefore given by the formula

$$r = \frac{\sum_{i=1}^n (x_i - X)(y_i - Y)}{(n - 1) \cdot S_X \cdot Y_X}$$

Temperature = y

Voltage = x

$$X = 2.9$$

$$Y = 49.2$$

$$S_y = 0.3$$

$$S_x = 26$$

$$S_x \cdot S_y = 0.3 \cdot 26 = 7.8$$

$$r = \frac{\sum_{i=1}^n (x_i - X)(y_i - Y)}{(n - 1) \cdot S_x \cdot Y_x}$$

For  $y_1 = 2$  ,  $x_1 = 42$

$$r_1 = \frac{(2 - 2.9) \cdot (42 - 49.8)}{(9 - 1) \cdot 7.8}$$

$$r_1 = \frac{70.2}{62.4}$$

$$r_1 = 1.125$$

For  $y_2 = 2.3$  ,  $x_2 = 44$

$$r_2 = \frac{(2.3 - 2.9) \cdot (44 - 49.8)}{(9 - 1) \cdot 7.8}$$

$$r_2 = 0.055$$

For  $y_3 = 2.5$  ,  $x_3 = 46$

$$r_3 = \frac{(2.5 - 2.9) \cdot (46 - 49.8)}{(9 - 1) \cdot 7.8}$$

$$r_3 = 0.024$$

$$\text{For } y_4 = 2.85 \quad , \quad x_4 = 48$$

$$r_4 = \frac{(2.85 - 2.9) \cdot (48 - 49.8)}{(9 - 1) \cdot 7.8}$$

$$r_4 = 0.0014$$

$$\text{For } y_5 = 3 \quad , \quad x_5 = 50$$

$$r_5 = \frac{(3 - 2.9) \cdot (50 - 49.8)}{(9 - 1) \cdot 7.8}$$

$$r_5 = 0.0003$$

$$\text{For } y_6 = 3.2 \quad , \quad x_6 = 52$$

$$r_6 = \frac{(3.2 - 2.9) \cdot (52 - 49.8)}{(9 - 1) \cdot 7.8}$$

$$r_6 = 0.01$$

$$\text{For } y_7 = 3.4 \quad , \quad x_7 = 54$$

$$r_7 = \frac{(3.4 - 2.9) \cdot (54 - 49.8)}{(9 - 1) \cdot 7.8}$$

$$r_7 = 0.033$$

$$\text{For } y_8 = 3.5 \quad , \quad x_8 = 56$$

$$r_8 = \frac{(3.5 - 2.9) \cdot (56 - 49.8)}{(9 - 1) \cdot 7.8}$$

$$r_8 = 0.059$$

$$y_8 = y_9$$

and

$$x_8 = x_9$$

Therefore,

$$r_8 = r_9 = 0.059$$

$$r = \frac{\sum_{i=1}^n (x_i - X)(y_i - Y)}{(n-1) \cdot S_X \cdot Y_X}$$

$$r = 1.125 + 0.055 + 0.024 + 0.0014 + 0.0003 + 0.01 + 0.033 \dots \\ + 0.059 + 0.059$$

$$r = 1.366$$

## PREDICTION OF VARIABLE

The value of Y corresponding to X = x and it will be predicted by  $\frac{\mu_y}{x}$ , the empirical problem of prediction is the problem of estimating the regression function

$$\frac{\mu_y}{x} = \mu_y + \rho \cdot \frac{\delta_y}{\delta_x} \cdot (x - \mu_x)$$

The parameters of this function are readily estimated from the

calculations. The desired estimate of  $\frac{\mu_y}{x}$ , therefore given by

$$\frac{\mu_y}{x} = Y + r \cdot \frac{\delta_y}{\delta_x} \cdot (x - \mu_x)$$

$$x = 42$$

$$\frac{\mu_y}{x} = 2.9 + 1.366(0.011)42 - 49.8 = 2.78$$

For  $x = 44$

$$\frac{\mu_y}{x} = 2.9 + 1.366(0.011)(-5.8) = 2.81$$

For  $x = 46$

$$\frac{\mu_y}{x} = 2.9 + 1.366(0.011)(-3.8) = 2.84$$

For  $x = 48.0$

$$\frac{\mu_y}{x} = 2.9 + 1.366(0.011)(-1.8) = 2.87$$

For  $x = 50$

$$\frac{\mu_y}{x} = 2.9 + 1.366(0.011)(0.2) = 2.90$$

For  $x = 52$

$$\frac{\mu_y}{x} = 2.9 + 1.366(0.011)(2.2) = 2.93$$

For  $x = 54$

$$\frac{\mu_y}{x} = 2.9 + 1.366(0.011)(4.2) = 2.96$$

For  $x = 56$

$$\frac{\mu_y}{x} = 2.9 + 1.366(0.011)(6.2) = 2.99$$

For  $x = 56$

$$\frac{\mu_y}{x} = 2.9 + 1.366(0.011)(6.2) = 2.99$$

$$\sum \frac{\mu_y}{x} = 2.78 + 2.81 + 2.84 + 2.87 + 2.90 + 2.93 + 2.96 + 2.99 + 2.99$$

$$\sum \frac{\mu_y}{x} = 26.07$$

### LEAST SQUARE METHOD

The result in the predicting technique cannot be applied to find a relationship between  $x$  and  $y$ . so the equation of the regression function corresponding to the predicting equation is

$$\frac{\mu_y}{x} = \alpha + \beta(x - \mu_x)$$

The empirical problem of prediction is now reduced to the problem of estimating the parameters  $\alpha$ ,  $\beta$  and  $\mu_x$ . For the purpose of arriving at a method for estimating  $\alpha$  and  $\beta$ , let the value of  $(x_1, y_1), \dots, (x_n, y_n)$  be plotted as point in the  $x_1, y_1$  plane. Then the problem of estimating a linear regression function can be treated as a problem of fitting a straight line to this set of points.

The fitting of a straight line to a set of  $n$  points, corresponding to equation above, the equation of the line may be written in the form

$$y^l = a + b(x-X)$$

where  $b$  is its slope and  $a$  is the  $y$  intercept on the line  $x-X$ . The  $y$  intercept on  $y$ -axis is  $a - bX$ . It is convenient to express the equation of the line in this form rather than in the slope-intercept form,  $y = a + bx$ , of analytical geometry. It is now to determine the parameters  $a$  and  $b$  so that the sum of the squares of the errors of prediction will be a minimum.

Since the coordinates of the  $i$ th point are denoted by  $(x_i, y_i)$ , this sum of squares will be

$$\sum_{i=1}^n (y_i - y_i^1)^2$$

When  $y_i^1$  is replaced by its value, as given above, it becomes clear that this sum is a function of  $a$  and  $b$  only.

Let the function be denoted by  $G(a,b)$

$$G(a,b) = \sum_{i=1}^n (y_i - a - b(x_i - X))^2$$

The partial derivative of the function vanishes to have a minimum value, hence  $a$  and  $b$  must satisfy the equations

$$\frac{\partial G}{\partial a} = \sum 2 \cdot [y - a - b \cdot (x - X)] \cdot (-1) = 0$$

$$\frac{\partial G}{\partial b} = \sum 2 \cdot [y - a - b \cdot (x - X)] \cdot [-1 \cdot (x - X)] = 0$$

The subscript and range of summation have been omitted for convenience.

All the same that involve  $y$  are transpose the equation assume the form

$$a_n + b \cdot \sum (x - X) = \sum y$$

$$a \cdot \sum (x - X) + b \cdot \sum (x - X)^2 = \sum (x - X)y$$

$$\sum (x - X) = a$$

since ,the solution of these equation gives

$$a = Y \quad \text{and} \quad b = \frac{\sum [(x - X)y]}{\sum (x - X)^2}$$

The values of a and b when inserted n equation 7 yield the desired least square line

$$y^l = a + b \cdot (x - X)$$

$$y^l = Y + b \cdot (x - X)$$

$$y^l - Y = b \cdot (x - X) \longrightarrow \text{Regression\_line}$$



**USING THE LEAST SQUARE TECHNIQUE FOR ESTIMATING A LINEAR  
REGRESSION FUNCTION OF THE VARIABLE**

$$Y = 2.9$$

$$b = \frac{\sum (x_i - X) \cdot y_i}{\sum (x_i - X)^2}$$

$$(x_1 - X) \cdot y_1 = [(-7.8) - 49.8] \cdot 2.0 = -115.2$$

$$(x_2 - X) \cdot y_2 = [(-5.8) - 49.8] \cdot 2.3 = -127.88$$

$$(x_3 - X) \cdot y_3 = [(-3.8) - 49.8] \cdot 2.5 = -134$$

$$(x_4 - X) \cdot y_4 = [(-1.8) - 49.8] \cdot 2.85 = -147.06$$

$$(x_5 - X) \cdot y_5 = [(0.2) - 49.8] \cdot 3.0 = -148.8$$

$$(x_6 - X) \cdot y_6 = [(2.2) - 49.8] \cdot 3.2 = -152.32$$

$$(x_7 - X) \cdot y_7 = [(4.2) - 49.8] \cdot 3.4 = -153.68$$

$$(x_8 - X) \cdot y_8 = [(6.2) - 49.8] \cdot 3.4 = -152.6$$

$$(x_8 - X) \cdot y_8 = (x_9 - X) \cdot y_9 = -152.6$$

$$\begin{aligned} \sum (x_i - X) \cdot y_i &= -115.2 + (-127.88) + (-134) + (-147.06) \dots \\ &+ (-148.8) + (-152.32) + (-153.68) \dots \\ &+ (-152.6) + (-152.6) \end{aligned}$$

$$\sum (x_i - X) \cdot y_i = -1284.14$$

$$(x_1 - X)^2 = (-7.8 - 49.8)^2 = 3317.76$$

$$(x_2 - X)^2 = (-5.8 - 49.8)^2 = 3091.36$$

$$(x_3 - X)^2 = (-3.8 - 49.8)^2 = 2872.96$$

$$(x_4 - X)^2 = (-1.8 - 49.8)^2 = 2662.56$$

$$(x_5 - X)^2 = (0.2 - 49.8)^2 = 2460.16$$

$$(x_6 - X)^2 = (2.2 - 49.8)^2 = 2265.76$$

$$(x_7 - X)^2 = (4.2 - 49.8)^2 = 2079.36$$

$$(x_8 - X)^2 = (6.2 - 49.8)^2 = 1900.96$$

$$(x_i - X)^2 = (x_i - X)^2 = 1900.96$$

$$\begin{aligned} \sum (x_i - X)^2 &= 3317.76 + 3091.36 + 2872.96 + 2662.56 \dots \\ &+ 2460.16 + 2265.76 + 2079.36 + 1900.96 \dots \\ &+ 1900.96 \end{aligned}$$

$$\sum (x_i - X)^2 = 22551.84$$

$$b = \frac{\sum (x_i - X) \cdot y_i}{\sum (x_i - X)^2}$$

$$b = \frac{-1284.14}{22551.84}$$

$$b = -0.056$$

Hence, the equation of the regression line is given by

$$y^1 = Y + b \cdot (x - X)$$

$$y^1 = 2.9 + (-0.056) \cdot (x - 49.8)$$

$$y^1 = 2.9 - 0.056 \cdot (x - 49.8)$$

$$y^1 = 2.9 - 0.056 \cdot x + (49.8 \cdot 0.056)$$

## APPENDIX 2

### MULTIPLE LINEAR REGRESSION

Let voltage =  $y$

Temperature =  $x_1$

Time =  $x_2$

$y, x_1, x_2$  represent the available variables

Predicting the variable  $y$  by means of a linear function of the remaining variables.

The linear predicting function may be expressed in the form

$$y^1 = b_0 + b_1 \cdot x_1 + b_2 \cdot x_2 + \dots + b_k \cdot x_k$$

Where  $b$ 's are to be determined by means of available data.

Let  $x_{1i}, \dots, x_{ki}$  and

$$i = 1 \dots n$$

Denote the values of

$$x_1 \dots x_k$$

that have been selected. Thus, the  $x$ 's are not random variables here.

Let  $y_i, i = 1 \dots, n$  denote the value of  $y$  that was obtained when an observation was taken at the point  $(x_{1i}, \dots, x_{ki})$  in the  $k$  dimensional space of the  $x$ 's.

Finding the set of  $b$ 's in equation (1) to minimize the sum

$$\sum_{i=1}^n (y_i - Y_1^1)^2$$

In terms of the notation in equation (1), this sum is written as

$$G(b_0, b_1 \dots b_k) = \sum_{i=1}^n (y_i - b_0 - b_1 \cdot x_{1i} - \dots - b_k \cdot x_{ki})^2$$

For this function to have a minimum value, its partial derivatives vanish there: hence  $b$ 's must satisfy the equation

$$\frac{\partial G}{\partial b_0} = \frac{\partial G}{\partial b_1} = \dots = \frac{\partial G}{\partial b_k} = 0$$

The differentiation of equation (2) produces the following equations, in which the summation indices have been deleted for simplicity of notation

$$\sum 2 \cdot (y - b_0 - b_1 \cdot x_1 - \dots - b_k \cdot x_k)(-1) = 0$$

$$\sum 2 \cdot (y - b_0 - b_1 \cdot x_1 - \dots - b_k \cdot x_k)(-x_1) = 0$$

$$\sum 2 \cdot (y - b_0 - b_1 \cdot x_1 - \dots - b_k \cdot x_k)(-x_k) = 0$$

If these equations are multiplied by  $\frac{1}{2}$ , the summations performed term by term, and the first sum transferred to the right side, the equation will assume the form

$$b_0 n + b_1 \cdot \sum x_1 + \dots + b_k \cdot \sum x_k = \sum y$$

$$b_0 \cdot \sum x_1 + b_1 \cdot \sum x_1^2 + \dots + b_k \cdot \sum x_1 \cdot x_k = \sum x_1 \cdot y$$

$$b_0 \cdot \sum x_k + b_1 \cdot \sum x_k \cdot x_1 + \dots + b_k \cdot \sum x_k^2 = \sum x_k \cdot y$$

$$Y = \frac{2 + 2.3 + 2.5 + 2.85 + 3 + 3.2 + 3.4 + 3.5 + 3.5}{9}$$

$$Y = 2.9$$

$$X_1 = \frac{42 + 44 + 46 + 48 + 50 + 52 + 54 + 56 + 56}{9}$$

$$X_1 = 49.7$$

$$X_2 = \frac{27 + 25.5 + 24.0 + 23 + 20 + 22 + 19 + 18 + 18}{9}$$

$$X_2 = 21.8$$

$$\frac{1}{n} \cdot \sum X_1 \cdot Y = \frac{1}{9} \cdot (2.9 \cdot 49.7) = 16.0$$

$$\frac{1}{n} \cdot \sum X_2 \cdot Y = \frac{1}{9} \cdot (2.9 \cdot 21.8) = 7.0$$

$$\frac{1}{n} \cdot \sum X_1 \cdot X_2 = \frac{1}{9} \cdot (49.7 \cdot 21.8) = 120$$

$$\frac{1}{n} \cdot \sum X_1^2 = \frac{1}{9} \cdot (49.7)^2 = 274.4$$

$$\frac{1}{n} \cdot \sum X_2^2 = \frac{1}{9} \cdot (21.8)^2 = 52.8$$

After multiplying through by n, equation (3) becomes

$$\frac{1}{n} \cdot \sum X_1^2 = \frac{1}{9} \cdot (49.7)^2 = 274.4$$

$$\frac{1}{n} \cdot \sum X_2^2 = \frac{1}{9} \cdot (21.8)^2 = 52.8$$

Solving simultaneously,

$$274.4 \cdot b_1 - 120 \cdot b_2 = 16 \cdot 120$$

$$120 \cdot b_1 - 52.6 \cdot b_2 = 7 \cdot 274.4$$

$$32928 \cdot b_1 - 14400 \cdot b_2 = 1920$$

$$32928 \cdot b_1 - 14433.44 \cdot b_2 = 1920.8$$

$$33.44 \cdot b_2 = -0.8$$

$$b_2 = \frac{-0.8}{33.44} = -0.024$$

$$120 \cdot b_1 - 52.6(-0.024) = 7$$

$$120 \cdot b_1 + 1.2624 = 7$$

$$120 \cdot b_1 = 7 - 1.2624$$

$$b_1 = \frac{5.7376}{120}$$

$$Y^1 = 0.048 \cdot X_1 + (-0.024) \cdot X_2$$

$$Y^1 = 0.048 \cdot X_1 - 0.024 \cdot X_2$$

$$Y^1 = b_0 + b_1 \cdot x_1 + b_2 \cdot x_2$$

$$Y^1 = b_0 + 0.048 \cdot X_1 - 0.024 \cdot X_2$$

$$Y^1 = b_0 + 0.048 \times 49.7 - 0.024 \times 21.8$$

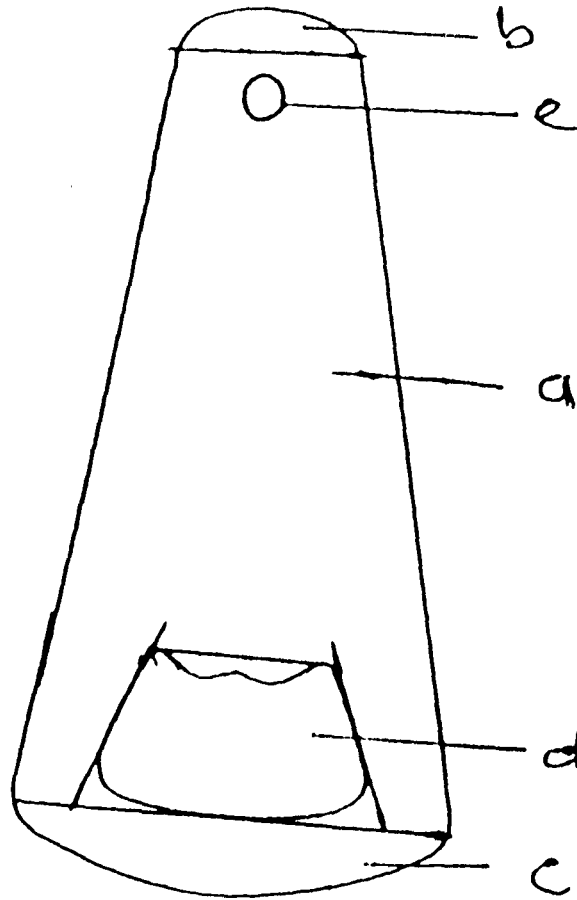
$$b_0 + 2.38 - 0.5232$$

$$b_0 + 1.85$$



### APPENDIX THREE

### DETERMINATION OF THE AREA AND THICKNESS OF THE OPENER



$$A = \frac{1}{2} \cdot (a + b)h$$

$$A = \frac{1}{2} \cdot (4 + 1.5)7.8$$

$$A = \frac{41.25}{2} = 20.625$$

AREA OF HALF SPHERE, B, (b)

$$\text{Area\_of\_half\_sphere} \quad , b = 4 \cdot \pi \cdot \frac{r^2}{2}$$

$$b = 4 \cdot 3.142 \cdot \frac{0.2^2}{2}$$

$$b = \frac{0.50272}{2} = 0.25136$$

AREA OF HALF SPHERE, C, (c)

$$\text{Area\_of\_half\_sphere} \quad , c = 4 \cdot \pi \cdot \frac{r^2}{2}$$

$$c = 4 \cdot 3.142 \cdot \frac{0.35^2}{2}$$

$$b = \frac{1.58958}{2} = 0.76979$$

AREA OF CIRCLE, e

$$e = \pi \cdot r^2$$

$$e = 3.142 \cdot 0.2^2$$

$$e = 0.12568$$

AREA OF TRAPEZIUM, D (d)

$$d = \frac{1}{2} \cdot (a + b)h$$

$$d = \frac{1}{2} \cdot (1.5 + 2.4)1.4$$

$$d = \frac{3.9 \cdot 1.4}{2} = 2.73$$

$$\text{Area\_of\_the\_opener} = (a + b + c) - d - e$$

$$= 20.625 + 0.25186 + 0.76979 - 0.12568 - 2.73 = 18.79047 \text{ m}^2$$

Thickness of the openers (microns) is given as:

	Thickness	weight_of_deposit (mg)
		$\text{Area (cm}^2) \cdot 0.71$
(1)	$\frac{0.05 \cdot 10^{-3}}{18.79047 \cdot 0.71}$	$\frac{0.05 \cdot 10^{-3}}{13.34} = 3.7 \cdot 10^{-6}$
(2)	$\frac{0.06 \cdot 10^{-3}}{13.34}$	$4.5 \cdot 10^{-6}$
(3)	$\frac{0.06 \cdot 10^{-3}}{13.34}$	$4.5 \cdot 10^{-6}$
(4)	$\frac{0.05 \cdot 10^{-3}}{13.34}$	$3.7 \cdot 10^{-6}$
(5)	$\frac{0.25 \cdot 10^{-3}}{13.34}$	$1.8 \cdot 10^{-5}$
(6)	$\frac{0.15 \cdot 10^{-3}}{13.34}$	$1.1 \cdot 10^{-5}$
(7)	$\frac{0.10 \cdot 10^{-3}}{13.34}$	$7.5 \cdot 10^{-6}$
(8)	$\frac{0.26 \cdot 10^{-3}}{13.34}$	$1.9 \cdot 10^{-6}$
(9)	$\frac{0.265 \cdot 10^{-3}}{13.34}$	$1.9 \cdot 10^{-6}$