APPLICATION OF SENSITIVITY ANALYSIS IN LINEAR PROGRAMMING TO MAXIMISE PROFIT

(A CASE STUDY OF ABUMEC PHARMACEUTICAL COMPANY, KADUNA)

BY

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NOVEMBER, 2005

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N PARTIAL FULFILMENT OF THE REQUIREMENT FOR THE AWARD OF BACHELOR OF ENGINEERING DEGREE (B.ENG) IN CHEMICAL ENGINEERING OF THE FEDERAL UNIVERSITY OF TECHNOLOGY MINNA, NIGER STATE, NIGERIA.

NOVEMBER, *2005*

DECLARATION

I hereby declare that this project work is my original work and has never, to my edge been submitted elsewhere.

ERIC CHIDOZIE   1/12/05

9/8153EH   DATE

CERTIFICATION

his is to certify that this project was supervise, Moderated and approved by the under- listed persons on behalf of the Chemical Engineering Department, School of ing and Engineering Technology, Federal University of Technology, Minna.

1/12/05

ZEEZ O.S. — DATE

T SUPERVISOR

RUAGBA F.O. — DATE

F DEPARTMENT

AL EXAMINER — DATE

DEDICATION

This project is dedicated to God, who saw me through this programme with his mighty

To my Father, Mother, Brothers and Sister.

ACKNOWLEDGEMENT

Thanking you God, the Father, who made and created all things through his mighty hand saw to the successful completion of this programme. I appreciate you Lord!

My special thanks to all those who supported me both financially and morally while the programme lasted; my father –Chief Mike Ejike, my mother, Mrs. Patricia Ejike, my bothers – Mr. Emeka Ejike, Mr. Chidi Ejike and Okechukwu Ejike, my sister – Mrs. Uchenna Tony, Mr. Sir Tony, my Nephew – Shaun Tony, my Uncles- Engr. Godfrey Ofoezie, Dr. Emmanuel Ofoezie, my aunties-mother Roseline Uchenna Ilonuba, Mrs. Christy Okolo, Bishop J.C. Ilonuba, my grand-parents, my cousins- Justina Udeh, Ifeanyi Ilonuba, Onyinye Ilonuba, my Friends- Patricia Alegieuno, Samuel Osakpa, Ikechukwu Mordi, Lucy Inefu, Isaac Arabala, Chukwudozie Chukwudi, Thierry Henry, Emmanuel Alex, Baba- Tunde, Father Thadeaus Umaru Akeem Babalola., Haruna Shogbo, Oyewole bamidele, Victor Oyedim, Ofioguma Tonye. Josiah Sunday,Tipsy.

To my supervisor Engr. Azeez, who gave me the opportunity to discover that I could do it and whose words of enlightment were most viable. Also Professor Onifade, who taught me optimization and thus paved the way for this project.

I do appreciate all efforts of the Head of Department, Dr. F. Aberuagba, Prof. J.O Odigure, Prof. Onifade, Dr. Edoga, Dr. Duncan Aloko, Engr. Eterigho, Dr. Olutoye, Engr. Akpan, Engr. Abdumojeed Fatai, Engr. Saka, Engr. Adeniyi, Engr. Alhasan, Mr. Kovo, Mrs. Aisha, Engr. Abajelola and all staff of Chemical Engineering Department who have contributed in one way or the other to the success of this programme.

Special gratitude to my parent in person of Chief Mike Ejike and Lolo Patricia Ejike, I Love you both greatly. Thanks for being my source of inspiration.

To all, I pray for God's abundance Mercy and Grace.

To God once more, thanks for being there.

ABSTRACT

This research work deals with the optimization process of Abumec Pharmaceutical Limited, Kaduna. This project work is aimed at exemplifying how sensitivity analysis can help in the optimization process in order to enable company managers take appropriate decisions. The simplex method of Linear programming method was used in optimizing the company's operations. By extension, a sensitivity analysis was carried out to determine sensitive parameters and their range of sensitivity. From the analysis made the unit optimal profit by linear programming was Twenty one thousand, Six hundred and Sixty seven Naira (₦21,667), which is an improvement on the Nineteen thousand five hundred and sixty Naira(₦19,560) that would have been the profit ordinarily. The profit of meclyn Cough Syrup with unit margin was ₦31, the profit of methylated spirit with unit margin was ₦70 and the profit of gentian violet with unit margin was ₦83 and the total minutes available for subdivision with range of sensitivity $85.41 \leq D_2 \leq 132.97$ and the total minutes available for packaging with the range of sensitivity $411.53 \leq D_3 \leq 7729$.

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CHAPTER ONE

1.0 INTRODUCTION

1.1 BACKGROUND STUDY

COMPANY (CASE STUDY)

Abumec pharmaceutical limited is the company I am using as a case study, all the data I used was duly obtained from them. Abumec pharmaceutical limited is a registered pharmaceutical company licensed under the Nigerian pharmaceutical registrations.

The company is a manufacturer of high qualifies pharmaceuticals such as meclyn cough syrup, methylated spirit and gentian violet. It was founded 1995 and is situated in Kaduna State – Nigeria.

I was able to see during my research work with the company that all their production is carried out using the modern specifications such as performing process map, check time. This assertion made me stand to the fact that all the data used in the course of this project is to the standard level expected.

1.2 AIMS AND OBJECTIVES

1. Maximize profit of the operations of Abumec Pharmaceutical Limited, Kaduna, or providing a higher level of service without increasing cost.
2. Analyze how the solution obtained from the model would change (if at all) if the values assigned to the parameter(s) were change to other plausible values
3. Maintaining a profitable operation while meeting imposed government regulations.

1.3 SCOPE OF STUDY

This focuses on using the simplex method to determine the optimal profit and necessary combination of product (Product-mix) in the linear programming model developed for Abumec Pharmaceutical Limited, Kaduna. Sensitivity analysis shall also be carried out.

1.4 JUSTIFICATION OF THE PROJECT

Real industrial management requires the establishment of optimization team, whose main duty is to advise management on how best to maximize profit. This research work gives an insight on how to optimize a real industrial set up.

CHAPTER TWO

RELATED LITERATURE REVIEW

2.0 OPTIMIZATION

Mathematical optimization is the branch of computational science that seeks to answer the question 'what is best?' For problems in which the quality of any answer can be expressed as a numerical value. Such problems arise in all areas of business, physical, chemical and biological sciences, engineering, architecture, economics, and management. The range of techniques available to solve them is nearly as wide (Professor Hossein Arsham- Europe Mirror Site, 2001).

If the mathematical model is a valid representation of the performance of the system, as shown by applying the appropriate analytical techniques, then the solution obtained from the model should also be the solution to the system problem. The effectiveness of the results of the application of any optimization technique is largely a function of the degree to which the model represents the system studied.

To define those conditions that will lead to the solution of a system problem, the analyst must first identify a criterion by which the performance of the system may be measured. This criterion is often referred to as the measure of the system performance or the measure of effectiveness is either cost or profit.

The mathematical (i.e. analytical) model that describes the behaviour of the measure of effectiveness is called the objective function. If the objective function is to describe the behaviour of the measure of effectiveness, it must capture the relationship between that measure and those variables that cause it to change.

System variables can be categorized as decision variables and parameters. A decision is a variables is a variables that can be directly controlled by the decision –mater. These are also some parameter whose values might be uncertain for the decision –maker. This call for sensitive analysis after finding the best strategy practice, mathematical equations rarely capture the precise relationship between all system variables and the measure of effectiveness, this mathematical relationship is the objective function that is used to evaluate the performance of the system being studied.

Formulation of a meaningful objective function is usually a tedious and frustrating task. Attempts to develop the objective function may fail. Failure could result because the analyst chose the wrong set of variables for incursion in the model, because he fails to identify the proper relationship between these variables and the measure of effectiveness. Returning to the drawing board, the analyst attempts to discover additional variables that may be seen to have little or no learning. However, whether or not these factors do in fact improve the model, can only be determined after formulation may require multiple reiteration before a satisfactorily objective function is developed. The analyst hopes to achieve some improvement in the model at each iteration, although it is not usually the case. Ultimate success is more often preceded by a string of failures and small successes.

Optimization, also called mathematical programming, helps find the answer that yields the best result. The one that attains the highest profit, output, or happiness, or the one that attains the lowest cost, waste, or discomfort. Often these problems involve making the most efficient use of resources – including money, time, machinery, staff, inventory, and more. Optimization problems are often classified as linear or non linear, depending on whether the relationship in the problem is linear with respect to the variables.

Optimization problems are made up of four basic ingredients.

I. An objection function that we want to minimize or maximize that is, the quality you want to maximize or minimized is called the objective function. Most optimization problems have a single function if they do not, they can often be formulated so that they do.

II. Decision variables: - the controllable input are the set of decision variables which affect the value of the objective function. The allocation of different available resources, or the labour spent on each activity. Decision variables are essential if there are no variables, we cannot define the objective function and the problem constraint.

III. The uncontrollable inputs are called parameters the input values may be fixed numbers associated with the particular problem. We call these values parameters of the model. Often you will have several 'cases' in variables of the same problem to solve, and the parameter value will change in each problem variation.

IV. Constraints are relations between decision variable and the parameters. A set of constraints allows some of the decision variables to take on certain values, and exclude others. For the manufacturing problem, it does not make sense to spend a negative amount of time on any activity, so we constrain all the 'time' variables to be non-negative. Constraints are not always essential. in fact, the field of unconstrained optimization is a large and important one for which a lot of algorithms and software are variable in practice, answers that make good sense about the underlying physical or economic problem; cannot often be obtained without putting constraints on the decision variables.

A solution value for decision variable, where all of the constraints are satisfied is called a feasible solution. Most solutions algorithms proceed by first finding a finally changing the decision to move from feasible solution to another feasible solution. This process is repeated until the objective function has reached it's maximum or minimum -- this result is called optimal solution.

2.1 WHY IS OPTIMIZATION NECESSARY?

Why are engineers interested in optimization? Engineers work to improve the initial design of equipment, and strive for enhancements in the operation of the equipment once it is installed in order to realize the largest production, the greatest profit, at minimum cost and energy usage and so on.

In plant operations, benefits of optimization arise from improved plant performance, such as improved yields of valuable products (or reduced yields of contaminants), reduced energy consumption, higher processing rates and longer times between shutdowns. Optimization can also lead to reduced maintenance costs, less equipment wear and better staff utilization. (Edgar and Himmelblau, 1989).

What about the argument that the formal application optimization is really not warranted because of the uncertainty that exist in the mathematical representation of the process and for the data used in the model of the process? Certainly such an argument has some merits. Engineers have to use judgment in applying optimization techniques to problems that have considerable uncertainty associated with them, both from the stand point of accuracy and the fact that the plant operating parameters and environs are not always static. In some cases it may be possible to carry out an analysis to yield quantitative productions of the degree of uncertainty. "Whenever the model of a process is idealized and the input and parameter data only known approximately. the optimization results must be treated judiciously. They can provide upper limits on expectations. Another way to evaluate the influence of uncertain parameters in optimal design is to perform a sensitivity analysis.

2.2 SCOPE AND HIERARCHY OF OPTIMIZATION

Optimization can take place at many levels in a company, ranging from a complex combination of plants and distribution facilities down through individual plants, combination of units, individual pieces of equipment, subsystems in a piece of equipment or even smaller entities (Beveridge and Schechter, 1970). Optimization problems can be found at all levels. Thus the scope of an optimization problem can be the entire company, a plant, a process, a single unit operation, a single piece of operation in that operation or in any intermediate system between these. The complexity of analysis may involve only gross features, or may examine minute detail, depending upon the use to which the results will be put. The availability of accurate data and the time available in which to carry out the optimization. In a typical industrial company there are three levels (area) in which optimization are used:

I. Management

II. Process design and equipment specification and

III. Plant operation (Edgar and Himmelblau, 1989).

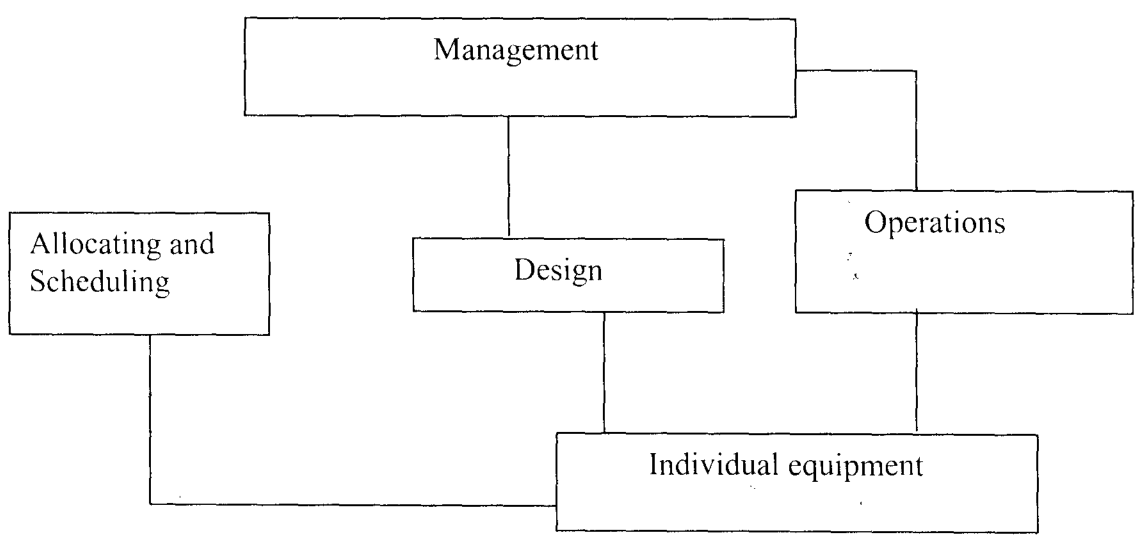

Fig.1: Hierarchy or level of optimization.

Management makes decision concerning project evaluation, product selection, corporate budget, investment in sales versus research and development, new plant construction, i.e. when and where should new plants be constructed and so forth. At this level much of the information that is available is at best qualitative or has a high degree of uncertainty, in general the magnitude of the objective function as measured in dollars is much at the management level than at the other two levels.

Process design and equipment specification specialists are concerned with the choice of a process and nominal operating conditions. They answer questions such as: Do we design a batch or a continuous process? What should the configurations of the plant be and how do we arrange the processes so that the operating efficiency of the plant is at a maximum? What is the optimum size of a unit or combination of units? Such questions can be resolved with the aid of so called process design simulators or flow sheeting programs. These are large computer program that carry out the material and energy balances for individual pieces of equipment and combine them into an overall production unit. Iterative use of such a simulator is often required in order to arrive at a desirable process flow sheet. Often decision such as actual choice of equipment and the material of construction of various process units are made at the design and equipment specification level.

Plant operational are concerned with operating control for a given unit at certain temperature, pressures, flow rates, etc, that are the best in some sense. For example the selection of the percentage of access air in a process heater is quite critical and involves a balance on the fuel- air ratio to assure complete combustion and at the same time make the maximum use of the heating potential of the fuel. Plant operation deals with allocation of raw materials on daily or weekly bases and is also concerned with the overall picture of shipping, transportation and distribution of products to engender minimal costs.

2.3 GENERAL PROCEDURE FOR SOLVING OPTIMIZATION PROBLEMS

By its very nature, optimization requires considerable ingenuity and innovation so it is impossible to write down any standard procedure that should always be followed by optimization teams (Hillier and Lieberman, 2001). Hence no single method or algorithm exists that can be applied efficiently to all problems. The method chosen for any particular case will depend primarily on:

I. The character of the objective function and whether it is known explicitly.

II. The nature of the constraints.

III. The number if independent and dependent variables.

Below is the list of the six general steps for the analysis and solution of optimization problems. You not have to follow the cited order exactly, but you should cover all the steps eventually. Shortcuts in the procedure are allowable and the easy steps can be performed first (Edger and Himmelblau, 1989).

Remember, the general objective in optimization is to choose a set of values of the variables subject to the various constraints that will produce the desired optimum response for the chosen objective function (Edgar and Himmelblau, 1989).

2.3.1 THE SIX STEPS USED TO SOLVE OPTIMIZATION PROBLEMS

1. Analyze the process itself so that the process variables and specific characteristic of interest are defined i.e. make a list of list of all the variables.

2. Determine the criterion for optimization and specify the objective function in terms of the above variables together with coefficients. This steps provides the performance model (sometimes called the economic model when appropriate).

3. Develop via mathematical expressions a valid process or equipment model that relates the inputs-output variables of the process and associated co-efficient. Include both equality an inequality constraints. Use well known physical principles (mass balances, energy balances) empirical relations implicit concepts and external restrictions. Identify the independent and dependent variables to get the number of degrees of freedom.

4. If the problem formulation is too large in scope

  a. Break it up into manageable parts and /or

  b. Simplify the objective function and model

5. Apply a suitable optimization technique to the mathematical statement of the problem.

6. Check the answers and examine the **SENSITIVITY** of the result to change in the problem and the assumptions.

Steps 1, 2, and 3 deal with mathematical definition of the problem i.e. Identification of variables, specification of the objective function and statement of constraints.

Step 4 suggests that the mathematical statement of the problem be simplified as much as possible without losing the essence of the problem. You might decide to neglect those variables which have an insignificant effect on the objective function.

A step 5 involves the computer to obtained numerical answers.

Sept 6 involves checking the candidate solution to determine that it is indeed optimal. It also involves the determination of how sensitive is the optimum to changes in parameters in the problem statement.

2.4 OBSTACLES TO OPTIMIZATION

If the objective function and constraints in an optimization problem are "nicely behave" optimization presents no great difficulty. In particular, if the objective function and constraints are all linear there is a powerful method known as linear programming for solving the optimization problem. However, most optimization problems in their natural formulation are not linear. (Edgar and Himmelblau, 1989) to make it possible to work with the relative simplicity of a linear problem, we often modify the mathematical description of the physical process so that it fits the available method of solution.

Often optimization problems exhibit one or more of the following characteristics causing difficulty and / or failure to calculate the desired optimal solution.

1. The objective function and/ or the constraints functions may have finite discontinuities in the continuous parameter values. For example, the price of a compressor or reactor may not change as a function of variables such as site, pressure, temperature and so on. Consequently increasing the level of a parameter in some ranges has no effect on cost, where as in other ranges a jump in cost occurs.
2. The objective function and /or the constraint functions may be nonlinear functions of the variables. When one considers real process equipment, the existence of truly linear behaviour and system behaviour is somewhat of a rarity. This does not preclude the use of linear approximations, but one must interpret the results of such approximations with considerable care.

3. The objective function and /or the constraint functions may be defined in terms of complicated interactions of the variable. A familiar case of interaction is the temperature and pressure dependence in the design of pressure vessels. For example, if the objective function is given as $F = 15.5 x_1 x_2^{1/2}$, the interaction between x_1 and x_2 precludes the determination of unique values of x_1 and x_2 the interaction prevents calculations of unique values of the variables at the optimum.

4. The objective function and / or the constraint functions may exhibit nearly 'flat' behaviour for some ranges of variables or exponential behaviour for other ranges. This means that the value of the objective function or constraint is not sensitive, or is very sensitive respectively, to changes in the value of the variables.

5. The objective function may exhibit many local optima whereas the global optimum is sought. A solution to the optimization problem may be obtained that is less satisfactory than another solution elsewhere in the region. The better solution may be reached only by initiating the search for the optimum from a different starting point.

CHAPTER THREE

3.0 LINEAR PROGRAMMING

Mathematical programming may be divided into linear and non – linear programming. It can thus be used to solve problems, which conform to the following:-

- The problem must be capable of being stated in numeric terms.
- All factors involved in the problem must.
- The problem must permit a choice or choices between alternative courses of restriction on the factors involved.

Linear programming is defined as the maximization of a linear objective function whose variables satisfy a system of linear inequalities (Professor Hossein Arsham - Europe mirror site).

Linear programming has proven to be an extremely powerful tool for selecting alternatives in a decision, problem and, consequently has been applied in a variety of problem settings, is often a favourite topic for both processors and student.

The widespread of availability of linear programming software package, and the wide application make linear programming accessible even to students with extremely weak mathematical background.

Linear programming deal with a class of programming problems where both the objective function to be optimized is linear and the relations among the variables corresponding to resources are linear. This problem was first formulated and solved in the late 1940's.

3.1 SIMPLEX METHOD

A step – by – step method of solving linear problems whereby one moves progressively from a position of zero production and therefore zeros contribution, until no further contribution can be made. Such step produces feasible solution as each step produces an answer better than the one before; greater contribution in maximizing problems. The mathematics behind the simplex methods is complex.

Simple method of solving linear problems may be the algebraic method, graphical method or by the simplex tableaux method. Slack variables represent any unused capacity in the constraints and can take any value from zero production unused capacity to zero unused capacity.

For any linear programming problem in our standard form (including functional constraints in less than or equals to form). The appearance of the functional constraints after slack variables are introduction is as follows:

1. $a_{11}X_1 + a_{12}X_2 + \ldots\ldots\ldots + a_{1n}X_n + X_{n+1} = b_1$
2. $a_{21}X_1 + a_{22}X_2 + \ldots\ldots\ldots + a_{2n}X_n + X_{n+2} = b_2$

....

M $a_{m1}X_1 + a_{m2}X_2 + \ldots\ldots\ldots + a_{mn}X_n + X_{n+m} = b_m$

Where X_{n+1}, X_{n+2} .....X_{n+m} are slack variables for other linear programming problems proper form can be obtained by introducing artificial variables. e.t.c. thus, the original solutions $(X_1, X_2 ......X_n)$ now are augmented by the corresponding values of the slack or artificial variables $(X_{n+1}, X_{n+2} ......X_{n+m})$ and perhaps some surplus variables as well.

Although the simplex method is an algebraic procedure, it is based on some fairly simple geometric concepts. These concepts enable one to use the algorithm to examine only a relatively small number of basic feasible solutions before reaching and identifying an optimal solution.

3.1.1 OBJECTIVE FUNCTION

The first step in linear programming is to decide what result is required, that is the objective. This is to maximize profit; in this project the profit of each product will be maximized. Once the objective has been defined, maximization of the products profit, it is now stated mathematically with the elements involved.

3.1.2 LIMITATIONS OR CONSTRAINTS

These are factors, which exist and govern the achievement of the objective. The limitations in this project are clearly identified, quantified and expressed mathematically, they are also linear.

3.1.3 SUMMARY OF STEPS IN THE SIMPLEX METHOD

1. Formulate problem in terms of an objective function and a set of constraints.

2. Convert the functional inequality constraints to equivalent equality constraint by introducing slack variables for less – than – or – equal – to - inequality constraint or surplus variables for greater – than – or – equal to – inequality constraints.

3. Set up the initial tableau.

4. Determine the entering basic variable by selecting the non-basic variable with the positive coefficient having the largest absolute value (i.e. the most positive coefficient)

5. Determine the leaving basic variable by applying the maximum ratio test.

Maximum ratio test:

I. Pick out each coefficient in the pivot column that is strictly positive (>0)

II. Divide each of these coefficients into the right side entry for the same row.

III. Identify the row that has the smallest of these ratios.

IV. The basic variable for that row is the leaving basic variable, so replace that variable by the entering basic variable in the basic variable column of the next tableau.

6. Solve for the new basic feasible solution by using elementary row operations.

The specific elementary row operations performed are:

I. Divide the pivot row by the pivot number

II. For each other row (including row Z) that has a positive coefficient in the pivot column, add to this row the product of the absolute values of this coefficient and the new pivot row.

III. For each other row that has a negative coefficient in the pivot column, subtract from this row the product of this coefficient and the new pivot row.

7. Return to optimality test by checking if row (Z) still has a positive coefficient. It has, the solution is not yet optimal. The highest absolute value is chosen again and the iteration processes of 1-6 done again till there is no positive coefficient on row (Z), then the solution becomes optimal and the iteration stopped. (Hillier and Lieberman, 2001).

3.2 DUALITY THEORY AND SENSITIVITY ANALYSIS

One of the most important discoveries in the early development of linear

Programming was the concept of duality and its many important ramifications. This discovery Revealed that every linear programming problem has associated with it, another linear Programming problem called the dual.

The relationship between the dual problem and the original problem (called the primal) proves to be extremely useful in variety of ways. One of the key uses of duality theory is in the interpretation and implementation of sensitivity analysis.

3.2.1 THE ESSENCE OF DUALITY THEORY

Given our standard form for the primal problem at the left (perhaps after conversion from another form), its dual problem has the form shown to the right.

PRIMAL PROBLEM

Maximize $Z = \sum_{j=1}^{n} C_j X_j$

Subject to

$\sum_{j=1}^{n} a_{ij} X_j \leq b_i$ for i = 1,2....m

and $X_j = 0$ for j = 1,2....n

DUAL PROBLEM

Minimize $W = \sum_{i=1}^{m} b_i Y_i$

Subject to

$\sum_{i=1}^{m} a_{ij} Y_i \geq C_i$ for j = 1,2...n

and $Y_i \geq 0$ for i = 1,2...m

Thus, the dual problem was exactly the same parameter as the primal problem, but in different locations.

3.2.2 DUAL PROBLEM CONSTRUCTION

- If the primal is a maximization problem, then its dual is a minimization problem (and vise versa).
- Use the variable type of one problem to find the constraint type of the other problem.
- Use the constraint type of one problem to find the variable type of the other problem.
- The RHS elements of one problem become the objective function coefficients of the other problem (and vise versa).
- The matrix coefficients of the constraints of one problem are the transpose of the matrix coefficients of the constraint for the other problem. That is, rows of the matrix become column and vise versa.

PRIMAL PROBLEM	DUAL PROBLEM
Maximize Z = CX	Minimize W = Yb
Subject to	Subject to
$AX \leq b$ and $X \geq 0$	$YA \geq C$ and $Y \geq 0$

3.2.3 APPLICATION

One important application of duality theory is that the dual problem can be solved directly by the simplex method in order to identify an optimal solution for the primal problem. The dual problem has fewer functional constraints that the primal problem, then applying the simplex method directly to the dual problem instead of the primal problem probably will achieve a substantial reduction in computational effort.

The weak and strong duality property describes key relationships between the primal and dual problems. One useful application is for evaluating a proposed solution for the primal problem. For example, suppose that X is a feasible solution that has been proposed for implementation and that a feasible Y has been found by inspection for the dual problem such that Cx = yb. In this case, u must be optimal without the simplex method even being applied even if Cx< yb, then yb still provides an upper bound on the optimal value of Z so if yb – Cx is small, intangible factors favouring X may lead to its selection without further ado.

One of the key applications of the complementary solutions property is its use in the dual simplex method. This algorithm operates on the primal problem exactly as if the simplex method were being applied simultaneously to the dual problem, which can be done because of this property. Because the roles of row Z and the right side in the simple tableau have been reversed, the dual simplex method requires that row Z begin and remain non negative while the right side begins with some negative values (subsequent iterations strive to reach a non negative right side). Consequently, this algorithm occasionally is used because it is move convenient to setup the initial tableau in this form than in the form required by the simplex method.

Another important application is its use in the economic interpretation of the dual problem and the resulting insights for analyzing the primal problem.

3.2.4 TERMS DEFINATIONS

I. **WEAK DUALITY PROPERTY**

If X is a feasible solution for the primal problem and y is a feasible solution for the dual problem, then Cx < yb

II. **STRONG DUALITY PROPERTY**

If X is an optimal solution for the primal problem and y is an optimal solution for the dual problem, then Cx = yb

III. **COMPLEMENTARY SOLUTIONS PROPERTY**

At each iteration, the simplex method simultaneously identifies a corner-point feasible solution X for the primal problem and a **COMPLEMENTARY SOLUTION** y for the dual problem (found in row o, the coefficients of the slack variables), where Cx = yb if x is not optimal for the primal problem, then y is not feasible for the dual problem.

3.2.5 PRIMAL – DUAL RELATIONSHIP

Because the dual problem is a linear programming problem, it also has corner – point solutions. Furthermore, by using the augmented form of the problems, we can express these corner- point solutions as basic solution. Because the functional constraints have the > = form, this augmented form is obtained by subtracting the surplus (rather than adding the slack) from the left-hand side of each constraint $j(j = 1,2...,n)$. this surplus is

$$Z_j C_j = \sum_{i=1}^{m} a_{ij} Z_j - C_j \text{ for } j = 1,2..., n.$$

Thus $Z_j - C_j$ plays the role of the surplus variable for constraint j (or its slack variable if the constraint is multiplied through by – 1). Therefore, augmenting each corner – point solution $(y_1, y_2...y_m)$ yields a basic solution $(y_1, y_2...y_m.Z_1 - C_1, Z_2 - C_2...Z_n - C_n)$ by using this expression for $Z_j - C_j$. Since the augmented form of the dual problem has n functional constraints and n +m variables, each basic solution has n basic variables and m non basic variables.

3.3 SENSITIVITY ANALYSIS

Sensitivity analysis is a very important part of almost every linear programming study, because most of the parameter values used in the original model is just estimates future conditions, the effect on the optimal solution of other conditions prevail instead needs to be investigated. Furthermore, certain parameter values (such as resource amounts) may represent managerial decisions, in which case the choice of the parameter value be the main issue be studied, which can be done through sensitivity analysis.

3.3.1 THE ROLE OF DUALITY THEORY IN SENSITIVITY ANALYSIS

Sensitivity analysis basically involves investigating the effect in the optimal solution of making change in the values of the model parameter $a_{ij}, b_{j.}$ and C_j. However, changing parameter values in the primal problem also changes the corresponding values in the dual problem.

Therefore, you have the choice of which problem to use to investigate each change. It is easy to move back and forth between the two problems as desired; in most cases, it is more convenient to analyze the dual problem directly in order to determine the complementary effect on the primal problem.

CHANGE IN THE COEFFEICIENTS OF A NON BASIS VARIABLE

Suppose that the changes made in the original model occur in the coefficients of a variable that was non basis in the original optimal solution. What is the effect of these changes on these changes on this solution? Is it still feasible? Is it still optimal?

Because the variable involved is non basis (value of zero), changing its coefficients cannot affect the feasibility of the solution. Since these changes affect the dual problem by changing only one constraint, this question can be answered simply by checking whether this complementary basis solution still satisfies this revised constraint.

INTRODUCTION OF A NEW VARIABLE

Adding another activity amounts to a new variable, with the appropriate coefficients in the functional constraints and objective function, into the model. The only resulting change in the dual problem is to add a new constraint.

After these changes are made, would the original optimal solution, along with the new variable equal to zero (non basis), still be optimal for the primal problem? These questions can be answered simply by checking whether this complementary basis solution satisfies one constraint, which in this case is the new constraint for the dual problem.

3.3.2 ESSENCE OF SENSITIVITY ANALYSIS

The work of the operations research team usually is not even nearly done when the simplex method has been successfully applied to identify an optimal solution for the model. One assumption of linear programming is that all the parameters of the model $(a_{ij}, b_j \text{ and } C_j)$ are known constants. Actually, the parameter values used in the model normally are just estimates base on a prediction of future condition. The data obtained to develop these estimates often are rather crude or nonexistence.

Furthermore, the model parameters (particularly b_i) sometimes are set as a result of managerial policy decisions (e.g. the amount of certain resources to be made available to the activities), and these decisions should be reviewed after their potential consequences are recognized.

For these reasons it is important to perform sensitivity analysis to investigate the effect on the optimal solution provided by the simplex method if the parameters take on other possible values. Usually there will be some parameter that can be assigned any reasonable value without the optimality of this solution being affected. However there may also be parameters with likely alternative values that would yield a new optimal solution.

Therefore on main purpose of sensitivity analysis is to identify the sensitivity parameters (i.e. the parameter whose values cannot be changed without changing the range of values of the parameter over which the optimal solution will remain unchanged. (We call this range of values, the allowable range to stay optimal). In some cases, changing a parameter value can affect the feasibility of the optimal basis feasible solution for such parameter it is useful to determine the range of values over which the optimal basis feasible (BF) solution (with adjusted values for the basis variables) will remain feasible (we call this range of variables the allowable range to stay feasible).

3.3.3 PROCEDURE

The basis idea is that the fundamental insight immediately reveals just how any changes in the original model would change the numbers in the final simplex tableau (assuming that the same sequence of algebraic operations originally performed by the simplex method were to be duplicated. Therefore after making a few simple calculations to revise this tableau, we can check easily whether the original optimal basic feasible solution is now non optimal (or infeasible). If so, this solution would be used as the new optimal solution, if desired. If the changes in the model are not major, only a very few iterations should be required to reach the new optimal solution from this 'advanced' initial basic solution.

To describe this procedure more specifically, consider the following situation. The simplex method already has been used to obtain an optimal solution for a linear programming model with specified values for the b_i, C_j, and a_{ij} parameters. To initiate sensitivity analysis, at least one of the parameters is changed. After the changes are made, let $\overline{b}_i, \overline{C}_j$ and $\overline{a}_{ij}$ denote the values of the various parameters. Thus, in matrix notation.

$b \rightarrow \overline{b},\ C \rightarrow \overline{c},\ A \rightarrow \overline{A}.$

For the revised model.

SUMMARY OF PROCEDURE FOR SENSITIVITY ANALYSIS

1. **REVISION OF MODEL**

   Make the desired change or changes in the model to be investigated next.

2. **REVISION OF FINAL TABLEAU**

   Use the fundamental insight to determine the resulting the resulting changes in the final simplex tableau.

3. **CONVERSION TO PROPER FORM FROM GNASSIAN ELIMINATION**

   Convert this tableau to the proper form for identifying and evaluating the current basic solution by applying. (as necessary) Guassian elimination.

4. **FEASIBILITY TEST**

   Test this solution for feasibility by checking whether all its basic variable values in the right – side column of the tableau still are non negative.

5. **OPTIMALITY TEST**

Test this solution for feasibility by checking whether all its non basic variable coefficients in row Z of the tableau still are non negative.

7. REOPTIMIZATION

If this solution fails either test, the new optimal solution can be obtained (if desired) by using the current tableau as the initial simplex tableau (and making any necessary conversions) for the simplex method or dual simplex method.

CHAPTER FOUR

4.0 COLLECTION OF DATA AND ANALYSIS

To effectively demonstrate how sensitivity analysis can be applied in the optimization process, a real industrial concern – Abumec pharmaceutical Limited, Kaduna, was chosen as a case study.

The factory manager and the production manager had to go through their records to get the data used in this research. They were also assured that the data so supplied will be used for academic (research) purposes only.

4.1 PRESENTATION OF DATA

TABLE 1

PRODUCTS			
	MECLYN COUGH SYRUP	METHYLATED SPIRIT	TOTAL MINUTE AVAILABLE
UNIT COST PRICE ₦	170	345	102
UNIT SELLING PRICE ₦	200	400	180
PROFIT ₦	30	55	78

PRODUCTS				
	MECLYN COUGH SYRUP	METHYLATED SPIRIT	GENTIAN VIOLET	TOTAL MINUTE AVAILABLE
PROCESSING MINUTES	0.8	1	1.33	490
PACKAGING MINUTES	0.6	1	4	520
SUB DIVISION MINUTES	0.13	0.25	0.33	95

TABLE 2

Putting the data in linear programming form.

Maximize function objective:

$Z = 30X_1 + 55X_2 + 78X_3$

Subject to constraints equations:

$0.8X_1 + X_2 + 1.33X_3 \leq 490$

$0.6X_1 + X_2 + 4X_3 \leq 520$

$0.13X_1 + 0.25X_2 + 0.33X_3 \leq 95$

$X_1, X_2, X_3 \geq 0$

Where X_1 is the variable representing meclyn cough syrup

X_2 is the variable representing methylated spirit

X_3 is the varable representing gentian violet

Writing them in standard equality form by introduction slack variables

Maxize $Z = 30X_1 + 55X_2 + 78X_3$

Subject to:

$0.8X_1 + X_2 + 1.33_{x3} + W_1 = 490$

$0.6X_1 + X_2 + 4X_3 + W_2 = 520$

$0.13X_1 + 0.25X_2 + 0.33X_3 + W_3 = 95$

with $X_1, X_2, X_3 W_1, W_2, W_3 \geq 0$

where W_1, W_2, W_3, represent slack variables

4.2 USING THE SIMPLEX METHOD
TABLE 3

BASIS VARIABLE	X_1	X_2	X_3	W_1	W_2	W_3	VALUE	CHECK
W_1	0.8	1	1.33	1	0	0	490	494.13
W_2	0.6	1	4	0	1	0	520	526.6
W_3	0.13	0.25	0.33	0	0	1	95	96.71
Z	30	55	78	0	0	0	0	163
W_1	0.6005	0.6675	0	1	-0.3325	0	317.1	319.04
X_3	0.15	0.25	1	0	-0.25	0	130	131.65
W_3	0.0805	0.1675	0	0	-0.0825	1	52.1	53.2655
Z	18.3	35.5	0	0	-19.5	0	-10,140	-10105.7
W_1	0.2797	0	0	1	-3.76×10^{-3}	-3.9850	109.4776	106.773
X_3	0.02985	0	1	0	0.3731	-1.4925	52.2388	52.1493
X_2	0.4806	1	0	0	-0.4925	5.9701	311.0448	318.003
Z	1.2387	0	0	0	-2.016	-211.934	-21182.09	-21394.8013
X_1	1	0	0	3.5765	-0.01345	-14.2525	391.5508	381.8777
X_3	0	0	1	-0.1068	0.3735	-1.0671	40.551	40.7502
X_2	0	1	0	-1.7189	-0.4860	12.8199	122.865	134.4725
Z	0	0	0	- 4.4302	-1.9993	-194.284	-21667.103	-21561.37

$X_1 = 391.5508$
$X_2 = 122.865$
$X_3 = 40.551$

$Z = 21667.103$

4.3 APPLYING SENSITIVITY ANALYSIS

In applying sensitivity analysis, a revised final simplex tableau resulting from changes in the original model is developed, from which deductions of whether the change results in a feasible (optimal) solution or not. are made.

4.3.1 Revised final simplex tableau resulting from changes in original model (Hillier and Lieberman, 2001).

TABLE FOUR

EQ	COEFFICIENT OF: Z	ORIGINAL VARIABLE	SLACK VARIABLES	RIGHT SIDE
New Initial O	1	$-\overline{C}$	0	0
Tableau 1,2..,m	0	$\overline{A}$	1	$\overline{b}$
Revised 0 final	1	$Z^* - \overline{C} = Y^*\overline{A} - \overline{C}$	Y^*	$Z^* = Y^*\overline{b}$
Tableau 1,2,…,m	0	$A^* = S^*\overline{A}$	S^*	$b^* = S^*\overline{b}$

Where $\overline{C}$ = Change in C (objective function)

$\overline{b}$ = Change in b (right side)

$\overline{A}$ = Change in the constraint values

Y^* = Shadow price

S^* = Inverse matrix

$Z^* = Y^*\overline{A}$

$Z^* = Y^*\overline{b}$

4.3.2 Change in b_i

$$b, = \begin{bmatrix} 490 \\ 520 \\ 95 \end{bmatrix} \rightarrow \overline{b} = \begin{bmatrix} 440 \\ 520 \\ 95 \end{bmatrix}$$

TABLE FIVE

E	x_1 ~~W1~~	x_2 ~~W2~~	x_3 ~~W3~~	W_1	W_2	W_3	VALUE	CHECK	RATI(
	0.8	1	1.33	1	0	0	440	444.13	333.9:
	0.6	1	4	0	1	0	520	526.6	130
	0.13	0.25	0.33	0	0	1	95	96.71	287.9
	30	55	78	0	0	0	0	163	
	0.6005	0.6675	0	1	-0.3325	0	267.1	269.0355	400.1:
	0.15	0.25	1	0	0.25	0	130	131.65	
	0.0805	0.1675	0	0	-0.0825	1	52.1	53.2655	311.0(
	18.3	35.5	0	0	-19.5	0	-10,140	-10105.7	
	0.2797	0	0	1	$-3.76X10^{-3}$	-3.9850	59.4776	56.768	
	0.02985	0	1	0	0.3731	-1.4925	52.2388	52.1493	
	0.48406	1	0	0	-0.4925	5.9701	311.0448	318.003	
	1.2387	0	0	0	-2.016	-211.934	-21182.09	-21394.8013	
	1	0	0	3.5765	-0.01345	-14.2525	212.648	202.9603	
	0	0	1	-1.1068	0.3735	-1.0671	45.8913	46.0909 b^*	
	0	1	0	-1.7189	-0.4860	12.8199	208.8462	220.4603	
	0	0	0	-4.4302	-1.9993	-194.284	21445.497	-21646.208	

Since b* are all positive then the solution is still feasible and the optimal solution is 21646.208 or since the Z row are all negative , then the solution is still feasible.

4.3.3 DETERMINATION OF RANGE OF MINUTES FOR WHICH CHANGE CAN BE EFFECTED

For any b_1. its allowable range to stay feasible is the range of values for this right hand side over which the current optimal basic feasible solution (with adjusted values for the basic variables) remains feasible assuming no change in the other right- hand sides (Hillier and Lieberman, 2001) even when the sensitivity algorithm is employed and the operation within the range shows infeasibility, the dual simplex method is applied to recover feasibility (Taha, 2002).

FOR PROCESSING MINUTES

$$\begin{bmatrix} X_1 \\ X_3 \\ X_2 \end{bmatrix} = \begin{bmatrix} 3.5765 & -0.01345 & -14.2525 \\ -0.1068 & 0.3735 & -1.0671 \\ -1.7189 & -0.4860 & 12.8199 \end{bmatrix} \begin{bmatrix} 490 + D1 \\ 520 \\ 95 \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$[X_1 \geq 0]$:

$$1752.485 + 3.5765D_1 - 6.994 - 1353.9875 \geq 0$$
$$3.576D_1 \geq 6.994 + 1353.9875 - 1752.485$$
$$3.576D_1 \geq -391.5035$$
$$D_1 \geq \frac{-391.5035}{3.576} = -109.466$$

$[X_3 \geq 0]$: X_3 Is independent of D_1

$[X_2 \geq 0]$: X_2 Is independent of D_1

Thus the current basic solution remains feasible for $D_1 \geq -109.466$
This is equivalent to varying the available hours for processing operation in the range

Processing minutes $\geq -109.466 + 490$

Processing minutes ≥ 380.534

FOR ~~PROCESSING~~ Packaging MINUTES

$$\begin{bmatrix} X_1 \\ X_3 \\ X_2 \end{bmatrix} = \begin{bmatrix} 3.5765 & -0.01345 & -14.2525 \\ -0.1068 & 0.3735 & -1.0671 \\ -1.7189 & -0.4860 & 12.8199 \end{bmatrix} \begin{bmatrix} 490 \\ 520 + D_3 \\ 95 \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$[X_1 \geq 0]$:

$$1752.485 - 6.994 - 0.01345D_3 - 1353.9875 \geq 0$$
$$-0.01345D_2 \geq -1752.485 + 6.994 + 1353.9875$$
$$-0.01345D_2 \geq -391.5035$$
$$D_3 \leq 29{,}108.07$$

$[X_3 \geq 0]$:

$$-52.332 + 194.22 + 0.3735D_3 - 101.3745 \geq 0$$
$$0.3735D_2 \geq 52.332 - 194.22 + 101.3745$$
$$0.3735D_3 \geq -40.5135$$
$$D_3 \geq \frac{-40.5134}{0.3735} \Rightarrow \geq -108.4699$$

[X

$[X_1 \geq 0]$:

$1752.485 - 6.994 - 1353.9875 - 14.2525D_2 \geq 0$

$-14.2525D_2 \geq -1752.485 + 6.994 + 1353.9875$

$-14.2525D_2 \geq -391.5035$

$D_2 \leq 27.4691$

$[X_3 \geq= 0]$:

$-52.332 + 194.22 - 101.3745 - 1.0671D_2 \geq 0$

$-1.0671D_2 \geq 52.332 - 194.22 + 101.3745$

$-1.0671D_2 \geq -40.5135$

$D_2 \leq 37.9660$

$[X_2 \geq= 0]$:

$-842.261 - 252.72 + 1217.8905 + 12.8199D_2 \geq= 0$

$12.8199D_2 \geq 842.261 + 252.72 - 1217.8905$

$12.8199D_2 \geq -122.9095$

$D_2 \geq -9.5874$

Thus the current basic solution remains feasible for

$-9.5874 \leq D_2 \leq 37.96598$

This is equivalent to varying the availability minutes of subdivision in the range

$95 - 9.5874 \leq$ Subdivision minutes $\leq 37.96598 + 95$

$85.4126 \leq$ subdivision minutes ≤ 132.96598

$[X_2 \geq 0]$:

$-842.261 - 252.72 - 0.4860D_3 + 1217.8905 \geq 0$

$-0.4860D3 \geq 842.261 + 252.72 - 1217.8905$

$-0.4860D_3 \geq -122.9095$

$D_3 \leq 252.900$

Thus the current basic solution remains feasible for

$-108.4699 \leq D_3 \leq 252.900$

This is equivalent to varying the availability minutes of packaging operation in the range

$520 - 108.4699 \leq$ packaging minutes $\leq 520 + 252.900$

$411.5301 \leq$ packaging minutes $\leq$ 77[illegible]9

FOR SUB - DIVISION MINUTES

$$\begin{bmatrix} X_1 \\ X_3 \\ X_2 \end{bmatrix} = \begin{bmatrix} 3.5765 & -0.01345 & -14.2525 \\ -0.1068 & 0.3735 & -1.0671 \\ -1.7189 & -0.4860 & 12.8199 \end{bmatrix} \begin{bmatrix} 490 \\ 520 \\ 95 + D_2 \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

4.3.4 ANALYZING SIMULTANEOUS CHANGE IN RIGHT HAND SIDES.

Since the range within the subdivision minutes covers(is above) the minutes used (i.e. 95) in arriving at the optimal solution in the simplex method analysis, as such 2.4 minutes can be added to that department. For processing department, the range of values required does not cover the addition of 2.4 minutes to the available minutes.

The right hand side (RHS) will change to

$$\begin{bmatrix} 490 + 2.4 \\ 520 \\ 95 + 2.4 \end{bmatrix} = \begin{bmatrix} 492.4 \\ 520 \\ 97.4 \end{bmatrix}$$

TABLE SIX

BASIS VARIABLES		X_1	X_2	X_3	W_1	W_2	W_3	VALUE
New initial Tableau	W_1	0.8	1	1.33	1	0	0	492.4
	W_2	0.6	1	4	0	1	0	520
	W_3	0.13	0.25	0.33	0	0	1	97.4
	Z	30	55	78	0	0	0	0
Final Simplex Tableau for Original Model	X_1	1	0	0	3.5765	-0.01345	-14.2525	365.7973
	X_3	0	0	1	-1.1068	0.3735	-1.0671	37.7377
	X_2	0	1	0	-1.7189	-0.4860	12.8199	150.5709
	Z	0	0	0	-4.4302	-1.9993	-194.284	-22,143.86
Revised Final Tableau	X_1	1	0	0	3.5765	-0.01345	-14.2525	391.5508
	X_3	0	0	1	-1.1068	0.3735	-1.0671	40.551
	X_2	0	1	0	-1.7189	-0.4860	12.8199	122.865
	Z	0	0	0	-4.4302	-1.9993	-194.284	-21667.103

All the values on the right hand side are all positive which proves the solution to be feasible or all the values on the Z row are all negative which also proves that the solution is feasible. Also the optimal solution is 22,143.86.

4.3.5 CHANGE IN THE PROFIT COEFFICIENT C_J

If C_j Represents the change in the profit coefficient; it will be necessary to determine the range at which change can be effected (to stay optimal)

For any C_j the allowable range to stay optimal is the range of values for this coefficient over which the current optimal solution remains optimal assuming no change in the other coefficients. Primal simplex method can be found after the sensitivity algorithm has been employed.

To find the range of C_1

From the optimal tableau of table 3 (simplex method)

1. Since X_1 is a basic variable, note that its coefficient in the final row (Z) is $Z^*-C_1=0$
2. Increase $C_1 = 30$ by DC_1 so $C_1= 30 + DC_1$

This changes the coefficient noted in step 1 to $Z^*- C_1 = - DC_1$, which changes row Z to

Row Z [$-DC_1$, 0, 0, +4.4302, +1.9993, +194.284, +21667.103]

3. With this coefficient row not zero, we perform elementary row operations to restore proper form from Gaussian elimination. In particular add to row Z the product of DC_1 times row X_1 to obtain row Z as shown below:

[$-DC_1$, 0, 0, 4.4302, 1.9993, 194.284, :21667.103]

+ [DC_1, 0, 0,$-3.5765DC_1$, $0.01345DC_1$, $14.2525DC_1$, :$391.5508DC_1$]

New row Z

$$\left[0,0,4.4302-3.5765DC_1,1.9993+0.01345DC_1,194.284+14.2525DC_1,.21667.103-39.5508DC_1\right]$$

4. Use this new row Z, to solve range of values of DC_1

$$4.4302 - 3.5765DC_1 \geq 0$$
$$-3/5765DC_1 \geq -4.4302$$
$$DC_1 \leq 1.2387$$

$$1.9993 + 0.01345DC_1 \geq 0$$
$$0.01345DC_1 \geq -1.9993$$
$$DC_1 \geq -148.6468$$

$$194.284 + 14.2525DC_1 \geq 0$$
$$14.2525DC_1 \geq -194.284$$
$$DC_1 \geq -13.6316$$

Thus the range of values is

$$148.6468 \leq DC_1 \leq 1.2387$$

Since $C_1 = 30 + DC_1$ add 30 to this range of values, which yields

$$-118.6468 \leq C_1 \leq 31.2387$$

For the range of C_2

1. Since X_2 is a basic variable, note that its coefficient in the final row Z is $Z^*-C_2 = 0$
2. Increase $C_2 = 55$ by DC_2, so $C_2 = 55 + DC_2$. this changes the coefficient noted in step 1 to $Z_2^*-C_2 = -DC_2$ this changes row Z to

   Row $Z[0,-DC_1,0,4.4302,1.9993,194.284,\vdots 21667.103]$

3. With this coefficient row not zero, we perform elementary row operation to restore proper form from Gaussian elimination. In particular addition the product of DC_2,

$$[0,-DC_1 0,4.4302,1.9993,194.284,\vdots 21667.103]$$
$$+[0,DC_2,0,1.7189DC_2,0.4860,DC_2 - 122.865DC_2]$$

New row Z

$$[0,0,4.4302 + 1.7189DC_2,1.9993 + 0.4860DC_2,194.284 - 12.8199DC_2 \vdots 21667.103 + 122.865DC_2$$
$$4.4302 + 1.7189DC_2 \geq 0$$
$$1.7189DC_2 \geq -4.4302$$
$$DC_2 \geq -2.5773$$

$$1.9993 + 0.4860DC_2 \geq 0$$
$$0.4860DC_2 \geq -1.9993$$
$$DC \geq -4.1138$$

$$194.284 - 12.8199DC_2 \geq 0$$
$$-12.8199DC_2 \geq -194.284$$

times row X_2 to obtain new row Z as shown below:

$DC_2 \leq 15.1549$

Thus the range of values is

$-4.1138 \leq DC_2 \leq 15.1549$

Since $C_2 = 55 + DC_2$, add 55 to this range of values, which yields

$50.8862 \leq C_2 \leq 70.1549$

for the range of C_3

1. Increase X_3 is a basic variable, note that its coefficient in the final row Z is $Z^* - C_3 = 0$
2. Increase $C_3 = 78$ by DC_3, so $C_3 = 78 + DC_2$. This changes the coefficient noted in step 1 to $Z_3^* - C_3 = - DC_3$

This changes row Z to

row Z $[0,0,-DC_3,4.4302,1.9993,194.284 \vdots 21667.103]$

3. With this coefficient row not zero, we perform elementary row operation to restore proper form from Gaussian elimination. In particular add to row Z the product of DC_3, time row X_3 to new row Z as shown below:

$[0,0,-DC_3, 4.4302, 1.9993, 194.284 \vdots 21667.103]$

$+[0,0, DC_3, 0.1068, DC_3 - 0.3735\, DC_3, 1.0671\, DC_3 \vdots - 40.551\, DC_3]$

New row Z

$[0,0, 4.4302 + 0.1068\, DC_3 1.9993 - 0.3735\, DC_3, 194.2841 - 1.0671\, DC_3 \vdots 21667.103 - 40.551 DC_3]$

4. Use this new row Z, to solve range of values of DC_3

$4.4320 + 0.1068\, DC_3 \geq 0$

$0.1068\, DC_3 \geq -4.4320$

$DC_3 \geq -41.4981$

$1.9993 - 0.3735\, DC_3 \geq 0$

$-0.3735\, DC_3 \geq -1.9993$

$DC_3 \leq 5.3529$

$194.284 + 1.0617\, DC_3 \geq 0$

$1.0671\, DC_3 \geq -194.284$

$DC_3 \geq -182.0673$

Thus the range of values is

$-182.0673 \leq DC_3 \leq 5.3529$

Since $C_3 = 78 + DC_3$ add 78 to this range of values, which yields

$-104.0673 \leq C_3 \leq 83.3529$

Change in the coefficient of the objective function

1. Changing the coefficient of methylated spirit from ₦55 to ₦60 in the objective function.

   The new objective function becomes

TABLE SEVEN

	BASIS VARIABLES	X_1	X_2	X_3	W_1	W_2	W_3	VALUE
New Initial Tableau	W_1	0.8	1	1.33	1	0	0	490
	W_2	0.6	1	4	0	1	0	520
	W_3	0.13	0.25	0.33	0	0	1	95
	Z	30	57	78	0	0	0	0
Final Tableau for Original Model	X_1	1	0	0	3.5765	-0.01345	14.2525	391.5508
	X_3	0	0	1	-0.1068	0.3735	-1.0671	40.551
	X_2	0	1	0	-1.7189	-0.4860	12.8199	122.865
	Z	0	0	0	-4.4302	-1.9993	-194.284	-21667.103
Revised Final Tableau	X_1	1	0	0	3.5765	-0.01345	-14.2525	391.409
	X_3	0	0	1	-0.1068	0.3735	-1.0671	40.555
	X_2	0	1	0	-1.7189	-0.4860	12.8199	122.935
	Z	0	0	0	-0.9925	-1.02752	-219.9240	-21912.835

Since all the values on the Z row is negative the solution is optimal, and the optimal value is 21912.835

2. Changing the coefficient of Gentian violet from ₦78 to N80. The objective function becomes

$30X_1+55X_2+80X_3$

TABLE EIGHT

	BASIS VARIABLES	X_1	X_2	X_3	W_1	W_2	W_3	VALUE
ew itial ableau	W_1	0.8	1	1.33	1	0	0	490
	W_2	0.6	1	4	0	1	0	520
	W_3	0.13	0.25	0.33	0	0	1	95
	Z	30	57	80	0	0	0	0
inal ableau or Original Model	X_1	1	0	0	3.5765	-0.01345	14.2525	391.5508
	X_3	0	0	1	-0.1068	0.3735	-1.0671	40.551
	X_2	0	1	0	-1.7189	-0.4860	12.8199	122.865
	Z	0	0	0	-4.4302	-1.9993	-194.284	-21667.103
Revised Final Tableau	X_1	1	0	0	3.5765	-0.01345	-14.2525	391.409
	X_3	0	0	1	-0.1068	0.3735	-1.0671	40.555
	X_2	0	1	0	-1.7189	-0.4860	12.8199	122.935
	Z	0	0	0	-4.2167	-2.7466	-192.1498	-21748.20

Since all the values pm the Z row is negative, the solution is optimal, and the optimal value is 21748.20

CHAPTER FIVE

5.0 DISCUSSION OF RESULTS

The essence of the research is to maximize the profit of abumec pharmaceutical Limited, Kaduna.

The optimal value profit (considering the given model) calculated was found by solving the model developed in chapter four using the simplex method and it was found to be Twenty one thousand six hundred and sixty seven Naira (₦ 21,667). This is an increase from the original profit which is Nineteen thousand five hundred and sixty Naira (₦19, 560). It was found that the value of X_1 (representing meclyn cough syrup) was 381.89 $\approx$ 382, that of X_2 (representing methylated spirit) was 122.87 $\approx$ 123 and that of X_3 (representing gentian violet) was 40.55 $\approx$ 41.

The sensitivity analysis carried out indicated that when the minutes available for processing was changed from 490 minutes to 440 minutes, it has no effect on the optimal profit, hence indicating available for subdivision was changed from as to 97.4 and the processing minutes was change from 490 to 492.4, the optimal profit changed to ₦ 22, 144, indicating that it is sensitive. So also will the optimal profit change if the minutes available for packaging is changed. So also when the profit coefficient (margin) of methylated spirit syrup is increased from ₦55 to ₦ 60, the optimal profit changes to ₦21912.84. Also when the profit coefficient of gentian violet is increase from ₦ 78 to ₦ 80, the optimal profit increase ₦21748.20. So the sensitive parameter were found to be C_1 and C_2 and C_3 representing profit coefficient (margin) of meclyn cough syrup, methylated spirit and gentian violet on one hand, and D_1, D_2 and D_3 representing minutes available for processing, subdivision and packaging on the other hand. This was so because once any of them was altered, the profitability also altered.

The feasible range of minutes for subdivision operation was $85.41 \leq D2 \leq 132.97$ whereas that of the packaging operation was $411.53 \leq D3 \leq 7729$. This implies that as the minutes available for any of these operations is increased to respective maximum, it will increase the profitability of the company. This is appealing, but its implementation will take some time.

The range of profit margin for meclyn cough syrup was found to be $-118.65 \leq C_1 \leq 31.24$ and of mythylated spirit was found to be $50.83 \leq C_2 \leq 70.15$ and that of gentian violet was found to be $-104.08 \leq C_3 \leq 83.35$ Since profit cannot be a negative figure, the profit margin of both meclyn cough syrup and gentian violet could be zero to zero or increase to

31.24 and 83.35 respectively. This implies that as the profit for any of those items is increased to maximum, the profitability of the company will be increased.

CHAPTER SIX

6.0 CONCLUSION AND RECOMMENDATION

6.1 CONCLUSION

From the analysis carried out and results obtained the following can be drawn as the conclusions of the case studied in this research project.

1. If the quantity of meclyn cough syrup is nearly thrice the quantity of methylated spirit produced, and nine times that of gentian violet produced, the profit of the company would be maximized.
2. If 133 and 7729 minutes are made available for subdivision and packaging operations respectively, the profit of the company will be maximized. However this also implies a longer time of implementation.
3. If per unit profit margin of ₦31 for meclyn cough syrup, ₦70 for methylated spirit and ₦83 for gentian violet are employed, the profit of the company will be maximized.
4. It is profitable to produce all the products.
5. The minutes available for processing must not be increased, but could be reduced to 440 minutes and, still profitability will be maintained.

6.2 RECOMMENDATIONS

The following recommendations are hereby made in order for the company to maximize its profit.

1. The quantity of meclyn cough syrup should be thrice the quantity of methylated spirit produced, and also should be nine times that of gentian violet produced.
2. 133 and 7729 minutes should be employed for subdivision and packaging operations respectively.
3. Unit profit margin of ₦ 31 for meclyn cough syrup, ₦70 for methylated spirit and ₦83 for gentian violet should be employed for maximum profitability.
4. Processing time available should be pegged at 440 minutes.

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