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Some new results on a free boundary value problem related to autoignition of combustible fluid in insulation materials

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Abstract

Autoignition of combustible fluids in insulation materials is one of the major problems facing the processing industries and many developing nations because it leads to serious environmental problem. This paper presents an analytical solution to a free boundary value problem related to autoignition of combustible fluids in insulation materials. The aim is to ascertain whether such a system is safe or if it will undergo ignition for a particular set of conditions. The conditions for the existence of unique solution of the model is established by actual solution method. The properties of solution is examined. The analytical solution is obtained via polynomial approximation method, which show the influence of the parameters involved on the system. The effect of changes in parameters such as the Frank-Kamenetskii number, Lewis number, Nusselt number, condensed reactant diffusion coefficient and the endothermicity are presented graphically and discussed.

Keywords: autoignition; combustible fluids; free boundary; heat transfer; insulation materials; polynomial approximation method

1 Introduction

The fuel's autoignition is usually caused by a leaking of a combustible liquid into insulation material surrounding a hot pipe. Leakage of flammable liquids into isolation layer during a transportation or technological process can lead to spontaneous fire of the fuel. Since the insulation is a porous medium containing an oxidizer, the exothermic oxidation reaction can lead to a substantial self-heating of the fuel and its ignition and, as a result, to destruction of the pipeline [10].
Models of combustion are characterized by two phenomena; ignition and explosion [2, 5].

The investigation of the autoignition process has been carried out by many authors. McIntosh *et al.* [7] presented a mathematical model for the autoignition of combustible liquid in an inert porous material. The simple model takes a spatially uniform approach to both the energy equation and the liquid equation for the fluid and predicts a watershed temperature and thermal runaway. McIntosh and Griffiths [6] presented the mathematical foundations for a simple theory for investigating the

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phenomenon of ignition of flammable fluids in lagging material that are used for insulation of hot pipework, for transport of heat transfer fluids, or other similar situations. A theory to explain these findings was presented which shows that there is a watershed temperature beyond which substantial self-heating will take place. Although the theory does not take account of diffusion, it simulates the main physics of the phenomenon. Truscott *et al.* [12] examined an initial value problem related to autoignition of combustible fluids in insulation materials and investigated the effect of diffusion on the system. Brindley *et al.* [3] developed the theoretical background and the limiting criteria for safe operation on autoignition of combustible fluids in insulation materials and discussed the magnitudes of the parameters that may be used to compare theory with experiment.

Popoola and Ayeni [8] extended the model in [12] to a free boundary value problem by assuming that the reactant is not confined to a fixed space. Shchepakina [11] investigated the critical conditions for autoignition of combustible fluids in porous insulation materials. This work concentrated on the critical case which is concerned with the phenomenon of delayed loss of stability in the dynamical model. The realizability conditions for the critical regime are obtained. Shchepakina [10] investigated an autoignition of liquid fuel due to leaking into insulation porous material surrounding a pipeline. By application of the geometric theory of singular perturbations, he described all possible scenarios of the process, reveal the critical condition for the autoignition, and calculate the maximal safe temperature of fuel's heating.

In this paper, an approximate analytical solution capable of predicting the temperature distribution in a process of autoignition of combustible fluids in insulation materials is presented. For this we shall follow the method proposed in [9].

2 Model formulation

The work in [8] is extended by considering the model geometry which may be represented by a single characteristic dimension to be sphere. The model variables expressed as a function of spatial co-ordinate r, $0 \le r \le R$, and time t, $t > 0$, are the matrix temperature $T(r,t)$, the concentration of condensed reactant $C_f(r,t)$ and the concentration of oxygen $C_{ox}(r,t)$.The formulation of our model is guided by the following assumptions:

1. Initially, a known amount of fluid is soaked uniformly within the block of insulation and there is air present throughout the porous structure.
2. The initial temperature of the system is known.
3. The reactant is adsorbed in the liquid state on the pore surface within the insulation matrix. It may react in this condensed state or it may evaporate.
4. The exothermic oxidation occurs by reaction between gaseous oxygen in the pores and the condensed reactant.
5. The order of reaction with respect to the gas-phase oxygen concentration and the condensed fluid concentration is greater than one.
6. The vaporization is related to the condensed fluid concentration and its temperature dependence is expressed in an Arrhenius-like exponential form.
7. The convective heat transport of gaseous components is ignored, as are pressure gradients within the matrix.

Based on the above assumptions, the respective conservation equations in spherical coordinate system are thus:

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{QA}{\rho c}C_f^{\alpha}C_{ox}^{\beta}e^{-\frac{E}{RT}} - \frac{Q_v F}{\rho c}C_f e^{-\frac{E_v}{RT}} \tag{1}$$

$$\frac{\partial C_f}{\partial t}=\frac{D_f}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial C_f}{\partial r}\right)-AC_f^{\alpha}C_{ox}^{\beta}e^{-\frac{E}{RT}}-FC_fe^{-\frac{E_v}{RT}} \tag{2}$$

$$\frac{\partial C_{ox}}{\partial t}=\frac{D_0}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial C_{ox}}{\partial r}\right)-vAC_f^{\alpha}C_{ox}^{\beta}e^{-\frac{E}{RT}}, \tag{3}$$

with the initial and boundary conditions:

$$\left.\begin{array}{lll} T(r,0)=T_0, & \left.\dfrac{\partial T}{\partial r}\right|_{r=0}=0, & k^*\left.\dfrac{\partial T}{\partial r}\right|_{r=s(t)}=h\left(T_0-T|_{r=s(t)}\right)\dot{s}(t) \\ C_f(r,0)=C_{f0}, & \left.\dfrac{\partial C_f}{\partial r}\right|_{r=0}=0, & D_f^*\left.\dfrac{\partial C_f}{\partial r}\right|_{r=s(t)}=-k_{mf}\left(C_{f0}+C_f|_{r=s(t)}\right)\dot{s}(t) \\ C_{ox}(r,0)=C_{ox0}, & \left.\dfrac{\partial C_{ox}}{\partial r}\right|_{r=0}=0, & D_0^*\left.\dfrac{\partial C_{ox}}{\partial r}\right|_{r=s(t)}=-k_{mox}\left(C_{ox0}+C_{ox}|_{r=s(t)}\right)\dot{s}(t) \\ \quad \dot{s}(t)=\dfrac{1}{t^{\gamma}}, \quad 0<r<s(t) & & \end{array}\right\}, \tag{4}$$

where C_{ox} is the concentration of Oxygen, C_f is the concentration of condensed reactant, D_0 is the Oxygen diffusion coefficient, D_f is the condensed reactant diffusion coefficient, ρ is the density of the reactant, k is the thermal conductivity of the medium, c is the heat capacity of the medium, Q is the enthalpy of oxidation (exothermicity), Q_v is the enthalpy of vaporization (endothermicity), R is the universal gas constant, T is the temperature of the medium, E is the activation energy (reaction), E_v is the activation energy (vaporization), T_0 is the initial temperature of the medium, v is the stoichiometry coefficient, C_{ox0} is the initial oxygen concentration within the insulation block, C_{f0} is the initial concentration of the uniformly distributed fluid, t is the time, A is the pre-exponent factor (reaction), F is the pre-exponent factor (vaporization), α and β are the order of reaction, r is the spatial coordinate, $s(t)$ is the moving boundary, D_0^* is the effective Oxygen diffusion coefficient, D_f^* is the effective condensed reactant diffusion coefficient, k^* is the effective thermal conductivity of the medium, k_{mf} is the condensed reactant convective mass transfer coefficient, k_{mox} is the Oxygen convective mass transfer coefficient, h is the convective heat transfer coefficient, $hT_0\dot{s}(t)$ is the heat energy released per unit time by the reaction, $hT|_{r=s(t)}\dot{s}(t)$ is the energy flux induced by the motion of the boundary preserve energy conservation, $k_{mf}C_{f0}\dot{s}(t)$ is the number of moles per unit time of condensed fluid, $k_{mf}C_f|_{r=s(t)}\dot{s}(t)$ is the mass flux of condensed fluid induced by the motion of the boundary preserve mass conservation, $k_{mox}C_{ox0}\dot{s}(t)$ is the number of moles per unit time of oxidizer that diffused into the system for the reaction, $k_{mox}C_{ox}|_{r=s(t)}\dot{s}(t)$ is the mass flux of oxidizer.

3 Method of solution

3.1 Dimension analysis

Dimensionless variables for space and time is been introduced as:

$$r' = \frac{2r}{\sqrt{D_f}\, s(t)}, \qquad t' = \frac{4t}{s(t)^2}. \tag{5}$$

Dimensionless variables for medium temperature, condensed fluid concentration and oxygen concentration is been introduced as follows:

$$\theta = \frac{E}{RT_0^2}\left(T - T_0\right), \qquad \psi = \frac{C_f}{C_{f0}}, \qquad \phi = \frac{C_{ox}}{C_{ox0}}, \qquad E_v = nE, \qquad \in = \frac{RT_0}{E}, \tag{6}$$

where $s(t) = (1-\gamma)^{-1} t^{1-\gamma}$ is moving boundary. Here, we choose $\gamma = \frac{1}{2}$ as in [8].

Using (5) and (6), and after dropping the prime, equations (1) - (4) become

$$\frac{\partial \theta}{\partial t} = \frac{Le}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \theta}{\partial r}\right) + \delta \psi^{\alpha} \phi^{\beta} e^{\frac{\theta}{1+\in\theta}} - \delta_1 \psi e^{\frac{n\theta}{1+\in\theta}} \tag{7}$$

$$\frac{\partial \psi}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \psi}{\partial r}\right) - \sigma_1 \psi^{\alpha} \phi^{\beta} e^{\frac{\theta}{1+\in\theta}} - \sigma_2 \psi e^{\frac{n\theta}{1+\in\theta}} \tag{8}$$

$$\frac{\partial \phi}{\partial t} = \frac{\alpha_1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \psi}{\partial r}\right) - \beta_1 \psi^{\alpha} \phi^{\beta} e^{\frac{\theta}{1+\in\theta}} \tag{9}$$

$$\left.\begin{array}{lll}
\theta(r,0) = 0, & \left.\dfrac{\partial \theta}{\partial r}\right|_{r=0} = 0, & \left.\dfrac{\partial \theta}{\partial r}\right|_{r=\frac{2}{\sqrt{D_f}}} = -Nu\left.\theta\right|_{r=\frac{2}{\sqrt{D_f}}} \\
\psi(r,0) = 1, & \left.\dfrac{\partial \psi}{\partial r}\right|_{r=0} = 0, & \left.\dfrac{\partial \psi}{\partial r}\right|_{r=\frac{2}{\sqrt{D_f}}} = -Sh_f\left(1 + \left.\psi\right|_{r=\frac{2}{\sqrt{D_f}}}\right) \\
\phi(r,0) = 1, & \left.\dfrac{\partial \phi}{\partial r}\right|_{r=0} = 0, & \left.\dfrac{\partial \phi}{\partial r}\right|_{r=\frac{2}{\sqrt{D_f}}} = -Sh_{ox}\left(1 + \left.\phi\right|_{r=\frac{2}{\sqrt{D_f}}}\right) \\
\quad 0 < r < \dfrac{2}{\sqrt{D_f}} & &
\end{array}\right\}, \tag{10}$$

where

$Le = \frac{k}{\rho c D_f}$ is the Lewis number, $\delta = \frac{QA\left(s(t)\right)^2 C_{f0}^{\alpha} C_{ox0}^{\beta} e^{-\frac{E}{RT_0}}}{4 \in T_0 \rho c}$ is the Frank-Kamenetskii number (reaction), $\delta_1 = \frac{Q_v F\left(s(t)\right)^2 C_{f0} e^{-\frac{E}{RT_0}}}{4 \in T_0 \rho c}$ is the Frank-Kamenetskii number (vaporization),

$\alpha_1 = \frac{D_0}{D_f}$, $\sigma_1 = \frac{A\left(s(t)\right)^2 C_{f0}^{\alpha-1} C_{ox0}^{\beta} e^{-\frac{E}{RT_0}}}{4}$, $\sigma_2 = \frac{F\left(s(t)\right)^2 e^{-\frac{E}{RT_0}}}{4}$,

$\beta_1 = \frac{vA\left(s(t)\right)^2 C_{f0}^{\alpha} C_{ox0}^{\beta-1} e^{-\frac{E}{RT_0}}}{4}$, $Sh_f = \frac{k_{mf}\sqrt{D_f}}{D_f^*}$ is the Sherwood number (condensed fluid),

$Sh_{ox} = \frac{k_{mox}\sqrt{D_f}}{D_0^*}$ is the Sherwood number (oxidizer), $Nu = \frac{h\sqrt{D_f}}{k^*}$ is the Nusselt number.

3.2 Existence and uniqueness of solution

Theorem 3.1: Let $Le = \alpha_1 = \sigma_1 = \sigma_2 = \beta_1 = Nu = Sh_f = Sh_{ox} = 1$ and $\delta_1 = -\frac{\delta}{2}$. Then there exists a solution of (7) – (9) which satisfies (10).

Proof: Multiply (8) and (9) by $\frac{\delta}{2}$ and adding the resulting equations, we obtain

$$\frac{\partial \varphi}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \varphi}{\partial r}\right)$$

(11)

$$\varphi(r,0) = \delta, \qquad \left.\frac{\partial \varphi}{\partial r}\right|_{r=0}, \qquad \left.\frac{\partial \varphi}{\partial r}\right|_{r=\frac{2}{\sqrt{D_f}}} = -\left(\delta + \left.\varphi\right|_{r=\frac{2}{\sqrt{D_f}}}\right), \tag{12}$$

where

$$\varphi(r,t) = \theta(r,t) + \frac{\delta}{2}\left(\psi(r,t) + \phi(r,t)\right).$$

Assuming a polynomial solution of the form (see, [9]):

$$\varphi(r,t) = c_0(t) + c_1(t)r + c_2(t)r^2, \tag{13}$$

we obtain the approximated solution of equations (11) and (12) as

$$\varphi(r,t)=\frac{\delta\sqrt{D_f}}{4}+\left(1+\frac{\sqrt{D_f}}{4}\right)\left(b+(\delta-b)e^{-at}\right)-\frac{\sqrt{D_f}}{4}\left(\delta+\left(b+(\delta-b)e^{-at}\right)\right)r^2 \qquad (14)$$

where

$$a=\frac{240D_f}{40D_f+8\sqrt{D_f}-32}, \qquad b=-\frac{12}{D_f}\delta .$$

Then, we obtain

$$\psi(r,t)=\frac{2}{\delta}\left(\begin{pmatrix}\frac{\delta\sqrt{D_f}}{4}+\left(1+\frac{\sqrt{D_f}}{4}\right)\left(b+(\delta-b)e^{-at}\right)- \\ \frac{\sqrt{D_f}}{4}\left(\delta+\left(b+(\delta-b)e^{-at}\right)\right)\end{pmatrix}-\theta(r,t)\right)-\phi(r,t) \qquad (15)$$

$$\phi(r,t)=\frac{2}{\delta}\left(\begin{pmatrix}\frac{\delta\sqrt{D_f}}{4}+\left(1+\frac{\sqrt{D_f}}{4}\right)\left(b+(\delta-b)e^{-at}\right)- \\ \frac{\sqrt{D_f}}{4}\left(\delta+\left(b+(\delta-b)e^{-at}\right)\right)\end{pmatrix}-\theta(r,t)\right)-\psi(r,t) \qquad (16)$$

$$\theta(r,t)=\begin{pmatrix}\frac{\delta\sqrt{D_f}}{4}+\left(1+\frac{\sqrt{D_f}}{4}\right)\left(b+(\delta-b)e^{-at}\right)- \\ \frac{\sqrt{D_f}}{4}\left(\delta+\left(b+(\delta-b)e^{-at}\right)\right)\end{pmatrix}-\frac{2}{\delta}\left(\psi(r,t)+\phi(r,t)\right) \qquad (17)$$

Hence, there exists a solution of problem (7) – (10). This completes the proof.

3.3 Properties of solution

Following thermal ignition theory, it is necessary to assume that there is always sufficient oxygen and fuel present in subcritical conditions so that the behaviour of the system is then governed solely by the energy conservation equation (7). That is ψ and ϕ are assumed constant.

Lemma 3.1: Let $\in\to 0$. Then the parabolic differential equation (7) with initial and boundary conditions (10) has a unique solution.

Proof: It suffices to show that the Lipschitz condition is satisfied. That is if we are able to show that $\left|f(r,t,\theta_1)-f(r,t,\theta_2)\right|\le k\left|\theta_1-\theta_2\right|$.

From (7),

$$f(r,t,\theta) = \delta\psi^{\alpha}\phi^{\beta}e^{\theta} - \delta_1\psi e^{n\theta}.$$

By mean value theorem

$$\frac{\partial f}{\partial\theta} = \delta\psi^{\alpha}\phi^{\beta}e^{\theta} - \delta_1 n\,\psi e^{n\theta}.$$

But

$\max\dfrac{\partial f}{\partial\theta}$ is achieved when $n = 0$ and $e^{\theta} = 1$ i.e. $n = 0$ and $\theta = 0$.

Hence

$$k = \max\frac{\partial f}{\partial\theta} = \delta\psi^{\alpha}\phi^{\beta}.$$

Therefore,

$$\left|f(r,t,\theta_1) - f(r,t,\theta_2)\right| \le \delta\psi^{\alpha}\phi^{\beta}\left|\theta_1 - \theta_2\right|.$$

Since the Lipschitz condition is satisfied, equation (7) has a unique solution. This completes the proof.

For a well stirred reaction when the activation energy for reaction is the same as activation energy for vaporization, i.e. $n = 1$ and in a high activation energy situation $\in \to 0$, equation (7) reduces to

$$\frac{d\theta}{dt} = \delta\psi^{\alpha}\phi^{\beta}e^{\theta} - \delta_1\psi e^{\theta}, \qquad \theta(0) = 0 \tag{18}$$

Solving (18), we obtain

As $t = \dfrac{1}{\left(\delta\psi^{\alpha}\phi^{\beta} - \delta_1\psi\right)}$, $\theta(t) \to \infty$. (19)

Hence, $\theta(t) \to \infty$ is thermal runaway and $t = \dfrac{1}{\left(\delta\psi^{\alpha}\phi^{\beta} - \delta_1\psi\right)}$ is actually the time of thermal runaway.

3.4 Analytical solution via polynomial approximation method

In the presence of diffusion, assuming constant ψ and ϕ, equation (7) can be solve analytically in the limit $\in \to 0$. Follow the idea in [1], we assume the expression for $\exp(\theta)$, that is,

$$\exp(\theta) \approx 1 + (e - 2)\theta. \tag{20}$$

Then (7) and (10) can be written as:

$$\frac{\partial\theta}{\partial t}=\frac{Le}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\theta}{\partial r}\right)+\delta\psi^{\alpha}\phi^{\beta}\left(1+(e-2)\theta\right)-\delta_1\psi\left(1+(e-2)\theta\right)^n \tag{21}$$

$$\theta(r,0)=0, \qquad \left.\frac{\partial\theta}{\partial r}\right|_{r=0}=0, \qquad \left.\frac{\partial\theta}{\partial r}\right|_{r=\frac{2}{\sqrt{D_f}}}=-Nu\left.\theta\right|_{r=\frac{2}{\sqrt{D_f}}} \tag{22}$$

Here, we assume polynomial solution of the form (see, [9]):

$$\theta(r,t)=a_0(t)+a_1(t)r+a_2(t)r^2. \tag{23}$$

Applying the boundary conditions as given in (22), we obtain

$$a_1(t)=0, \qquad a_2(t)=-\frac{Nu\sqrt{D_f}}{4}\left.\theta\right|_{r=\frac{2}{\sqrt{D_f}}}, \qquad a_0(t)=\left(1+\frac{Nu\sqrt{D_f}}{4}\right)\left.\theta\right|_{r=\frac{2}{\sqrt{D_f}}} \tag{24}$$

Then, equation (23) becomes

$$\theta(r,t)=\left(1+\frac{Nu\sqrt{D_f}}{4}\right)\left.\theta\right|_{r=\frac{2}{\sqrt{D_f}}}-\frac{Nu\sqrt{D_f}}{4}\left.\theta\right|_{r=\frac{2}{\sqrt{D_f}}}r^2 \tag{25}$$

For long spherical shape (see, [4]), we have

$$\bar{\theta}=3\int_0^1 r^2\theta dr \tag{26}$$

where $\bar{\theta}$ is the average temperature.

Equations (26) gives the relations

$$\bar{\theta}=\left(\frac{8}{D_f\sqrt{D_f}}\left(1+\frac{Nu\sqrt{D_f}}{4}\right)-\frac{96Nu}{20D_f^2}\right)\left.\theta\right|_{r=\frac{2}{\sqrt{D_f}}} \tag{27}$$

And

$$\frac{\partial}{\partial t}\bar{\theta}=\left(\frac{8}{D_f\sqrt{D_f}}\left(1+\frac{Nu\sqrt{D_f}}{4}\right)-\frac{96Nu}{20D_f^2}\right)\left.\frac{\partial}{\partial t}\theta\right|_{r=\frac{2}{\sqrt{D_f}}}. \tag{28}$$

Integrating (21) with respect to x, yield the following equation

$$\left.\frac{\partial}{\partial t}\theta\right|_{r=\frac{2}{\sqrt{D_f}}}+p\left.\theta\right|_{r=\frac{2}{\sqrt{D_f}}}=q. \tag{29}$$

Solving (29) gives

$$\theta\Big|_{r=\frac{2}{\sqrt{D_f}}}=\frac{q}{p}\left(1-e^{-pt}\right). \tag{30}$$

Substituting equation (30) into equation (25) gives

$$\theta(r,t)=\frac{q}{p}\left(1+\frac{Nu\sqrt{D_f}}{4}\right)\left(1-e^{-pt}\right)-\frac{qNu\sqrt{D_f}}{4p}\left(1-e^{-pt}\right)r^2, \tag{31}$$

where

$$p=\frac{\left(\frac{12LeNu}{D_f}+\frac{8\delta\psi^{\alpha}\phi^{\beta}(e-2)}{D_f\sqrt{D_f}}\left(\frac{3Nu}{5\sqrt{D_f}}-\left(1+\frac{Nu\sqrt{D_f}}{4}\right)\right)+\delta_1\psi n(e-2)\left(\left(1+\frac{Nu\sqrt{D_f}}{4}\right)-\frac{12Nu}{5D_f^2}\right)\right)}{\left(\frac{8}{D_f\sqrt{D_f}}\left(1+\frac{Nu\sqrt{D_f}}{4}\right)-\frac{96Nu}{20D_f^2}\right)},$$

$$q=\frac{\frac{8}{D_f\sqrt{D_f}}\left(\delta\psi^{\alpha}\phi^{\beta}-\delta_1\psi\right)}{\left(\frac{8}{D_f\sqrt{D_f}}\left(1+\frac{Nu\sqrt{D_f}}{4}\right)-\frac{96Nu}{20D_f^2}\right)}$$

The computation was done on equations (31) using computer symbolic algebraic package MAPLE 2021 version.

4 Results and discussion

Simulation was carried out to show the impact of the model parameters by employing Polynomial Approximation Method (PAM) on the equation (7). The equation was solved over spatial co-ordinate and a specific period of time using Polynomial Approximation Method. The computation equation (31) was done using MAPLE 2021 version. The parameter values used in the simulations are

$N_u = 4, 8, 12$, $D_f = 0.3, 0.5, 0.7$, $Le = 0.1, 0.5, 1.0$, $\phi = 2.75$, $\psi = 0.1$, $n = 2$, $\alpha = 2$, $\beta = 2$, $\delta_1 = 2, 4,$ · 6, $\delta = 0.4, 0.6, 0.8$

Numerical simulation of Frank-Kamenetskii number, Lewis number, Nusselt number, condensed reactant diffusion coefficient and endothermicity number was conducted to see whether or not these parameters contribute significantly to the medium temperature.

Figure 1 depicts the graph of medium temperature $\theta(r,t)$ against spatial co-ordinate r and time t for different values of Frank-Kamenetskii number δ. It is observed that the temperature of the medium increases with time and decreases along spatial co-ordinate but this medium temperature decreases as Frank-Kamenetskii number increases.

Figure 2 displays the graph of medium temperature $\theta(r,t)$ against spatial co-ordinate r and time t for different values of Lewis number Le. It is observed that the temperature of medium increases with time and decreases along spatial co-ordinate but this medium temperature increases as Lewis number increases.

Figure 3 shows the graph of medium temperature $\theta(r,t)$ against spatial co-ordinate r and time t for different values of Nusselt number Nu. It is observed that the temperature of medium increases with time and decreases along spatial co-ordinate but this medium temperature decreases as Nusselt number increases.

Figure 4 displays the graph of medium temperature $\theta(r,t)$ against spatial co-ordinate r and time t for different values of condensed reactant diffusion coefficient D_f. It is observed that the temperature of medium increases with time and decreases along spatial co-ordinate but this medium temperature increases as condensed reactant diffusion coefficient increases.

Figure 5 depicts the graph of medium temperature $\theta(r,t)$ against spatial co-ordinate r and time t for different values of endothermicity number δ_1. It is observed that the temperature of medium increases with time and decreases along spatial co-ordinate but this medium temperature increases as endothermicity number increases.

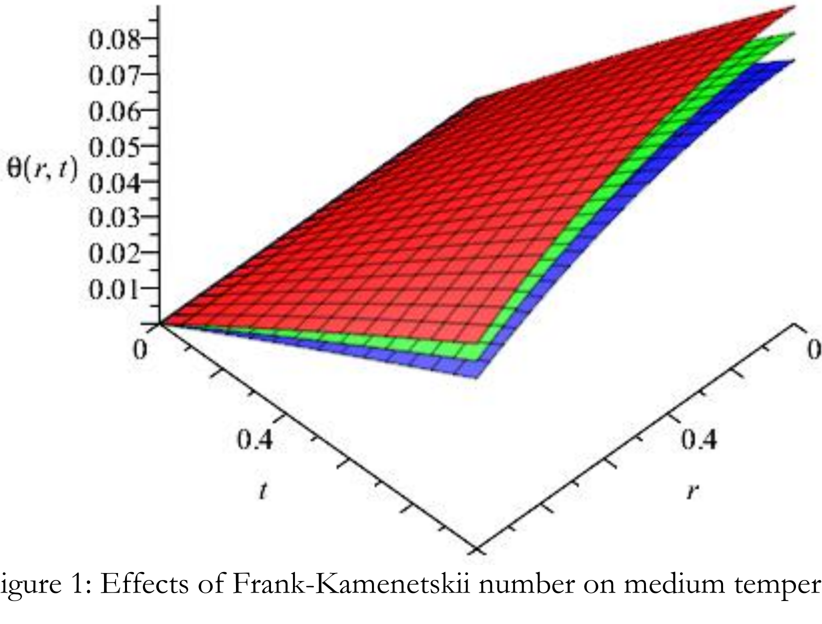

Figure 1: Effects of Frank-Kamenetskii number on medium temperature

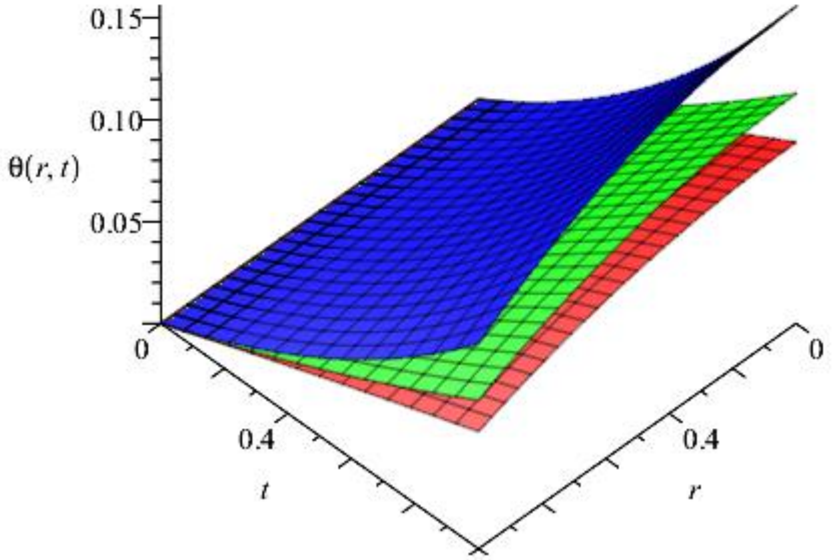

Figure 2: Effects of Lewis number on medium temperature

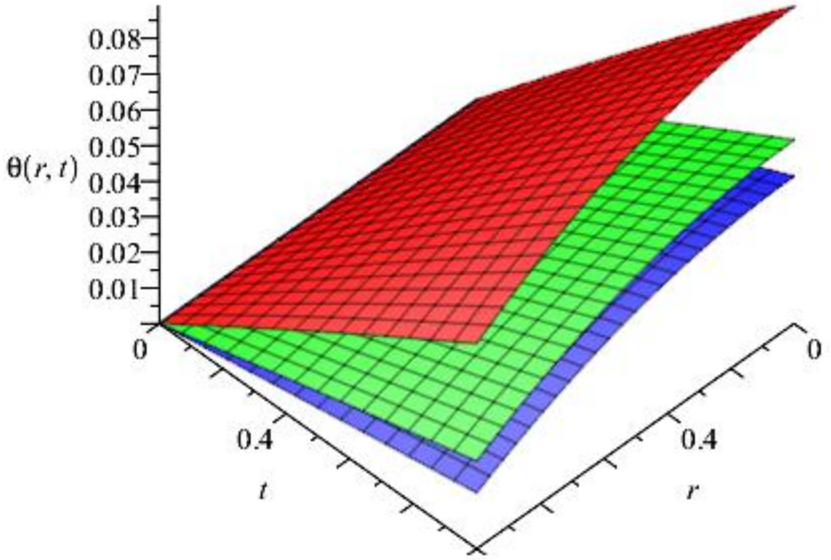

Figure 3: Effects of Nusselt number on medium temperature

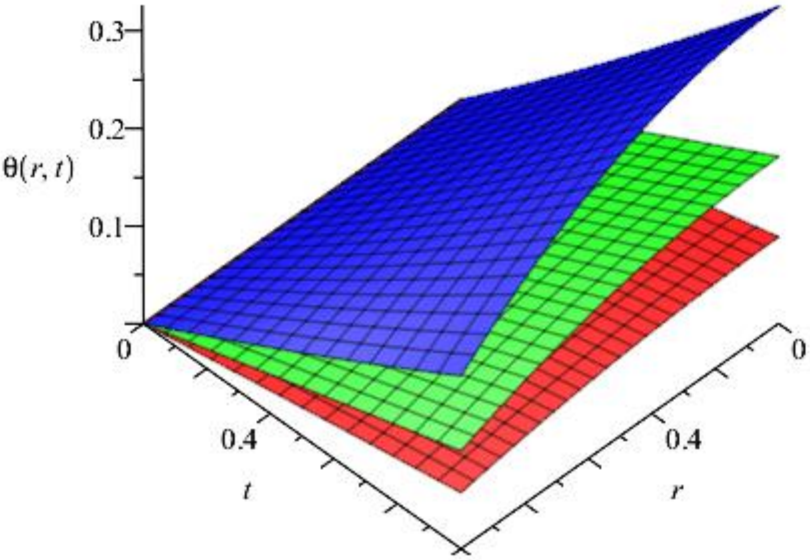

Figure 4: Effects of condensed reactant diffusion coefficient on medium temperature

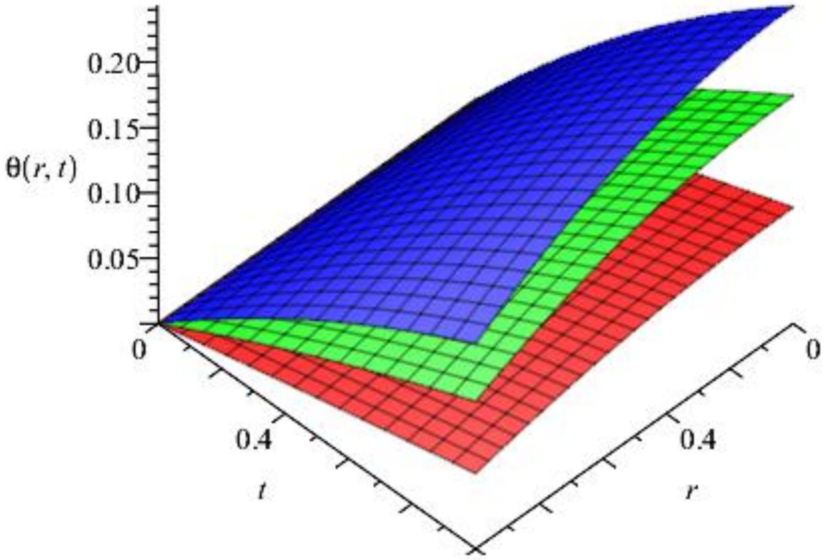

Figure 5: Effects of endothermicity number on medium temperature

It is worth pointing out that the effects observed in Figures 1 - 5, are important to guide insulation materials manufacturers so as to provide safety precautions during storage and usage.

5 Conclusion

For a well stirred reaction and in a high activation energy situation $\in \to 0$, we have found that thermal runaway could still occur when the activation energy for reaction is the same as activation energy for vaporization, i.e. $n = 1$. That is $\theta(t) \to \infty$ as $t \to \frac{1}{\left(\delta\psi^{\alpha}\phi^{\beta} - \delta_1\psi\right)}$. The existence of the unique solution of the problem implies that the problem represents a physical situation under specific conditions. In the presence of diffusion, the analytical simulations showed that increase in both Frank-Kamenetskii and Nusselt numbers reduced the medium temperature while increase in Lewis number, condensed reactant diffusion coefficient and endothermicity number enhanced the medium temperature. Therefore, the established conditions and the results obtained are not expected to guide manufacturers of insulation materials but provide safety precautions during storage and usage.

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