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C04: Modeling the Fire Spread in a Real-Time Coupled Atmospheric-Wildland Fire

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Abstract:

This paper presents a mathematical model for predicting fire spread in a real-time coupled atmospheric-wildland fire. The equations governing the phenomenon are solved analytically using direct integration and eigenfunction expansion technique. The results obtained revealed the effect of parameters involved on the system. It is observed that the radiation parameter decrease the temperature of the medium, while it has little effect on the oxygen concentration.

Keywords and Phrases: Crown fires, forest fire spread, heat transfer, surface fire, wildland fire.

1. Introduction

The term "Wildland fire" refers to an uncontrolled or unwanted fire in an area of combustible vegetation occurring most likely in rural areas. The forest fires are a common occurrence in most parts of the world and they cause a lot of damage to biodiversity as well as to the local ecology. Wildland fires impact the lives of millions of people and cause major damage every year worldwide, yet they are a natural part of the cycle of nature. Better tools for modeling wildland fire behaviour are important for managing fire suppression, planning controlled burns to reduce the fuels, as well as to help assess fire danger. Perminov (2018) describes forest fires as a complicated phenomenon and at present, fire services can forecast the danger rating of or the specific weather elements relating to forest fire. There is need to understand and predict forest fire initiation, behaviour and spread.

Forest fire models have been developed since 1940 to the present, but a lot of chemical and thermodynamic questions related to fire behaviour are still to be resolved. Forest fires are divided into underground (peatbog) fires, surface fires, active crown fires, running crown fires (also called independent crown fires), and mass fires (Grishin, 2002).

A great deal of work has been done on the theoretical problem of how forest fire spread. Weber (1991) concentrated on physical wildland fire modelling and proposed a system by which models were described as physical, empirical or statistical, depending on whether they account for different modes of heat transfer, make no distinction between different heat transfer modes, or involve no physics at all. Pastor *et al*. (2003) proposed descriptions of theoretical, empirical and semi-empirical, again depending on whether the model was based on purely physical understanding, of a statistical nature with no physical understanding, or a combination of both.

It seems more promising to use methods of mathematical modeling that will allow taking into account the dynamics of this process in space and time. In view of this, Perminov (2018) estimated the amount of carbon dioxide and carbon monoxide emissions at crown forest fires spread using method of finite volume to obtain discrete analogies.

The present work is aimed at establish an analytical solutions capable of predicting the oxygen concentration, medium temperature and volume fractions of dry organic substance, moisture and coke in the process of fire spread in a real-time coupled atmospheric-wildland fire. This will be achieved via direct integration and eigenfunction expansion technique.

2. Model Formulations

A wildfire model is formulated based on balance equations for energy and fuel, where the fuel loss due to burning corresponds to the fuel reaction rate. The respective equations governing forest fires propagation are:

Volume fraction of dry organic substance

$$\frac{\partial \varphi_s}{\partial t} = -k_1 \varphi_s e^{-\frac{E_1}{RT}} \tag{1}$$

Volume fraction of moisture

$$\frac{\partial \varphi_m}{\partial t} = -k_2 \varphi_m T^{\frac{1}{2}} e^{-\frac{E_2}{RT}} \tag{2}$$

Volume fraction of coke

$$\rho_c \frac{\partial \varphi_c}{\partial t} = \alpha_c k_1 \rho_s \varphi_s e^{-\frac{E_1}{RT}} - \frac{M_c}{M_1} k_3 S_\sigma \rho_g \varphi_c C_{ox} e^{-\frac{E_3}{RT}} \tag{3}$$

Mass concentration of oxygen

$$\left.\begin{aligned}&\rho_g\left(\frac{\partial C_{ox}}{\partial t}+V\frac{\partial C_{ox}}{\partial x}\right)=\frac{\partial}{\partial x}\left(\rho_g D_T\frac{\partial C_{ox}}{\partial x}\right)-\frac{\alpha}{C_{pg}\Delta h}\left(C_{ox}-C_{ox_\infty}\right)-\\&\left(1-\alpha_c\right)k_1\rho_s\varphi_s C_{ox}e^{-\frac{E_1}{RT}}-k_2\rho_m T^{\frac{1}{2}}\varphi_m C_{ox}e^{\frac{-E_2}{RT}}-k_3 S_\sigma\rho_g\left(1+\frac{M_c}{M_1}C_{ox}\right)\varphi_c C_{ox}e^{\frac{-E_3}{RT}}\end{aligned}\right\} \quad (4)$$

Energy balance equation

$$\left.\begin{aligned}&\left(\phi\rho_g C_{pg}+\left(1-\phi\right)\sum_{i=1}^{s+m+c}\rho_i C_{pi}\varphi_i\right)\frac{\partial T}{\partial t}+\rho_g C_{pg}V\frac{\partial T}{\partial x}=\frac{\partial}{\partial x}\left(\lambda_T\frac{\partial T}{\partial x}\right)-\frac{\alpha}{\Delta h}\left(T-T_\infty\right)\\&-4K_R\sigma T^4-k_2\rho_m q_2 T^{\frac{1}{2}}\varphi_m e^{\frac{-E_2}{RT}}+k_3 S_\sigma\rho_g q_3\varphi_c C_{ox}e^{\frac{-E_3}{RT}}\end{aligned}\right\} \quad (5)$$

With initial and boundary conditions:

$$\left.\begin{aligned}&\varphi_s\left(x,0\right)=\varphi_{so},\ \varphi_m\left(x,0\right)=\varphi_{mo},\ \varphi_c\left(x,0\right)=\varphi_{co},\ C_{ox}\left(x,0\right)=C_{ox_o},\ C_{ox}\left(0,t\right)=C_{ox_\infty}\\&C_{ox}\left(\mathrm{L},t\right)=C_{ox_\infty},\ T\left(x,0\right)=T_o,\ T\left(0,t\right)=T_\infty,\ T\left(\mathrm{L},t\right)=T_\infty\end{aligned}\right\} \quad (6)$$

Where;

φ_s is the volume fraction of dry organic substance, φ_m is the volume fraction of moisture, φ_c is the volume fraction of coke, C_{ox} is the concentration of oxygen, T is the temperature (in Kelvin), t is the time, x is a coordinate in the system of coordinates connected with the centre of an initial fire (distance), T_∞ is the unperturbed ambient temperature, $k_j, j=1,2,3$ are the preexponential factors of chemical reactions, $E_j, j=1,2,3$ are the activation energy of chemical reactions, C is the concentration, R is the universal gas constant, S_σ is the specific surface of the condensed product of pyrolysis (coke), V is the equilibrium wind velocity vector, λ_T is the turbulent thermal conductivity, C_{ox_∞} is the unperturbed density of concentration of oxygen, $P_i, i=(s,m,c)$ is the i^{-th} phase density, that is ρ_s is the density of dry organic substance, ρ_m is the density of moisture, ρ_c is the density of coke, ρ_g is the density of gas phase (a mix of gases), Δh is the crown height, M_c is the molecular mass of carbon, M_1 is the mass of combustible forest material (CFM), C_{pg} is the thermal capacity of a gas phase, $q_j, j=2,3$ defines heat effects of processes of evaporation of burning, D_T is the diffusion coefficient, α is the coefficient of heat exchange between the atmosphere and a forest canopy, α_c is the coke number of combustible forest material (CFM), K_R is the Stefan-BoltzMann constant, $C_{p_i}, i=(s,m,c)$ is the i^{-th} phase of thermal capacity, s is the dry organic substance, m is the moisture, c is the coke, ox is the oxygen (O_2).

3. Method of Solution

3.1 Non-dimensionalisation

Here equation (1) – (6) are non-dimensionalize using the following dimensionless variables:

$$\left.\begin{array}{l} x' = \dfrac{x}{L},\ t' = \dfrac{Ut}{L},\ v' = \dfrac{v}{U},\ \psi_1 = \dfrac{\varphi_s}{\varphi_{so}},\ \psi_2 = \dfrac{\varphi_m}{\varphi_{mo}},\ \psi_3 = \dfrac{\varphi_c}{\varphi_{co}},\ \phi = \dfrac{C_{ox} - C_{ox_\infty}}{C_{ox_o} - C_{ox_\infty}} \\ E = \dfrac{RT_0}{\in},\ \theta = \dfrac{E\left(T - T_o\right)}{RT_o^2},\ f = \dfrac{E_1}{E_3},\ r = \dfrac{E_2}{E_3} \end{array}\right\} \qquad (7)$$

and obtain

$$\left.\begin{array}{l} \dfrac{\partial \psi_1}{\partial t} = -a\psi_1 e^{\frac{f\theta}{1+\in\theta}} \\ \psi_1\left(x,0\right) = 1 \end{array}\right\} \qquad (8)$$

$$\left.\begin{array}{l} \dfrac{\partial \psi_2}{\partial t} = -b\psi_2\left(1+\in\theta\right)^{\frac{1}{2}} e^{\frac{r\theta}{1+\in\theta}} \\ \psi_2\left(x,0\right) = 1 \end{array}\right\} \qquad (9)$$

$$\left.\begin{array}{l} \dfrac{\partial \psi_3}{\partial t} = \beta\psi_1 e^{\frac{f\theta}{1+\in\theta}} - \gamma\left(\phi + q\right)\psi_3 e^{\frac{\theta}{1+\in\theta}} \\ \psi_3\left(x,0\right) = 1 \end{array}\right\} \qquad (10)$$

$$\left.\begin{aligned}
&\frac{\partial\phi}{\partial t}+V\frac{\partial\phi}{\partial x}=\frac{\partial}{\partial x}\left(D_1\frac{\partial\phi}{\partial x}\right)-\beta_1\phi-\beta_2\psi_1(\phi+q)e^{\frac{f\theta}{1+\in\theta}}\\
&-\beta_3(1+\in\theta)^{\frac{1}{2}}\psi_2(\phi+q)e^{\frac{r\theta}{1+\in\theta}}-\beta_4\psi_3(\phi+p)(\phi+q)e^{\frac{\theta}{1+\in\theta}}\\
&\phi(x,0)=1,\ \ \phi(0,t)=0,\ \ \phi(1,t)=0
\end{aligned}\right\} \tag{11}$$

$$\left.\begin{aligned}
&\frac{\partial\theta}{\partial t}+V\frac{\partial\theta}{\partial x}=\frac{\partial}{\partial x}\left(\lambda_1\frac{\partial\theta}{\partial x}\right)-\alpha_1(\theta+\gamma_1)-R_a(1+4\in\theta)-\delta\psi_2(1+\in\theta)^{\frac{1}{2}}e^{\frac{r\theta}{1+\in\theta}}\\
&+\delta_1\psi_3(\phi+q)e^{\frac{\theta}{1+\in\theta}}\\
&\theta(x,0)=0,\ \ \theta(0,t)=\sigma_1,\ \ \theta(1,t)=\sigma_1
\end{aligned}\right\} \tag{12}$$

Where;

$$a=\frac{k_1Le^{\frac{-fE_1}{RT_o}}}{U},\ \ b=\frac{k_2T_o^{\frac{1}{2}}Le^{\frac{-rE_3}{RT_0}}}{U},\ \beta=\frac{\alpha_ck_1\rho_s\varphi_{so}Le^{\frac{-fE_3}{RT_o}}}{U\rho_c\varphi_{co}},\ \gamma=\frac{M_ck_3S_\sigma\rho_gL}{M_1U\rho_c}\left(C_{ox_o}-C_{ox_\infty}\right)e^{\frac{-E_3}{RT_o}},$$

$$q=\frac{C_{ox_\infty}}{C_{ox_o}-C_{ox_\infty}},\ D_1=\frac{D_T}{LU}=\frac{1}{P_{em}},\ \beta_1=\frac{\alpha L}{C_{pg}\Delta hU},\ \beta_2=\frac{(1-\alpha_c)k_1\rho_s\varphi_{so}L}{\rho_gU}e^{\frac{-fE_3}{RT_o}},$$

$$\beta_3=\frac{k_2\rho_mT_o^{\frac{1}{2}}\varphi_{mo}Le^{\frac{-rE_3}{RT_o}}}{\rho_gU},\ \beta_4=\frac{k_3S_\sigma\rho_g\dfrac{M_c}{M_1}\left[C_{ox_o}-C_{ox_\infty}\right]L\varphi_{co}e^{\frac{-E_3}{RT_o}}}{\rho_gU},\ p=\frac{\dfrac{M_1}{M_c}+C_{ox_\infty}}{C_{ox_o}-C_{ox_\infty}},$$

$$\lambda_1 = \frac{\lambda_T}{L\rho_g C_{pg} U} = \frac{1}{P_e}, \quad \alpha_1 = \frac{\alpha L}{\rho_g C_{pg} U}, \; R_a = \frac{4K_R \sigma L T_o^{\,3}}{\rho_g C_{pg} \in U}, \; \delta = \frac{k_2 \rho_m q_2 T_o^{\frac{1}{2}} L \varphi_{mo} e^{\frac{-rE_3}{RT_o}}}{\rho_g C_{pg} \in T_o U},$$

$$\delta_1 = \frac{k_3 S_\sigma \rho_g q_3 \varphi_{co} L\left(C_{ox_o} - C_{ox_\infty}\right)}{\rho_g C_{pg} \in T_o U} e^{\frac{-E_3}{RT_o}}, \; \gamma_1 = \frac{T_o - T_\infty}{\in T_o}.$$

3.2 Analytical Solution

Using direct integration and eigenfunction expansion technique, the analytical solution of equations (8)—(12) are obtained as follows:

$$\psi_1(x,t) = 1 + v\left(A_3 \sum_{n=1}^{\infty} A_2 \sin n\pi x - a_6 \left(\begin{array}{l} t + f(e-2). \\ \left(\sigma_1 t + \sum_{n=1}^{\infty} A_1 \left(t + \dfrac{e^{-c_2 t}}{c_2} \right) \sin n\pi x \right) \end{array} \right) \right) \tag{13}$$

$$\psi_2(x,t)=1+v\begin{pmatrix} a_7\sum_{n=1}^{\infty}\mathrm{A}_2\sin n\pi x\left(A_8+A_7\sum_{n=1}^{\infty}A_1\sin n\pi x\right)- \\ a_7\begin{pmatrix} t+r(e-2)\left(\sigma_1 t+\sum_{n=1}^{\infty}A_1\left(t+\dfrac{e^{-c_2 t}}{c_2}\right)\sin n\pi x\right)+ \\ \dfrac{1}{2}\in\left(\sigma_1 t+\sum_{n=1}^{\infty}A_1\left(t+\dfrac{e^{-c_2 t}}{c_2}\right)\sin n\pi x\right)+ \\ \dfrac{1}{2}\in\left(r(e-2)\right)\begin{pmatrix}\sigma^2{}_1 t+2\sigma_1\sum_{n=1}^{\infty}A_1\left(t+\dfrac{e^{-c_2 t}}{c_2}\right)\sin n\pi x \\ +\sum_{n=1}^{\infty}A_1\sum_{n=1}^{\infty}A_1\left(t+\dfrac{2}{c_2}e^{-c_2 t}-\dfrac{1}{2c_2}e^{-2c_2 t}\right)\sin^2 n\pi x\end{pmatrix}\end{pmatrix}\end{pmatrix} \qquad (14)$$

$$\psi_3(x,t)=1+v\begin{pmatrix} a_9\left(A_9 t+A_{10}\sum_{n=1}^{\infty}A_1\left(t+\dfrac{e^{-c_2 t}}{c_2}\right)\sin n\pi x\right) \\ -a_8\begin{pmatrix} -A_{11}\sum_{n=1}^{\infty}A\dfrac{e^{-c_1 t}}{c_1}\sin n\pi x+A_{12}\sum_{n=1}^{\infty}A\sum_{n=1}^{\infty}A_1\left(-\dfrac{e^{-c_1 t}}{c_1}+\dfrac{e^{-(c_1+c_2)t}}{(c_1+c_2)}\right)\sin^2 n\pi x \\ +A_{13}t+A_{14}\sum_{n=1}^{\infty}A_1\left(t+\dfrac{e^{-c_2 t}}{c_2}\right)\sin n\pi x\end{pmatrix} \\ -a_9 A_{10}\sum_{n=1}^{\infty}A_2\sin n\pi x+a_8\begin{pmatrix} A_{14}\sum_{n=1}^{\infty}A_2\sin n\pi x-A_{11}\sum_{n=1}^{\infty}\dfrac{A}{c_1}\sin n\pi x \\ -A_{12}\sum_{n=1}^{\infty}A\sum_{n=1}^{\infty}A_1 A_{15}\sin^2 n\pi x\end{pmatrix}\end{pmatrix} \qquad (15)$$

$$\phi(x,t)=\sum_{n=1}^{\infty}Ae^{-c_1t}\sin n\pi x+v\left(\sum_{n=1}^{\infty}\left(\begin{array}{l}
-2A_{35}\sum_{n=1}^{\infty}Ate^{-c_1t}-2A_{36}\sum_{n=1}^{\infty}A_1\left[\frac{1}{c_1}-\frac{e^{-c_2t}}{(c_1-c_2)}+\frac{c_2e^{-c_1t}}{c_1(c_1-c_2)}\right]- \\
2A_{37}\sum_{n=1}^{\infty}A\sum_{n=1}^{\infty}A_1A_{43}\left[te^{-c_1t}+\frac{e^{-(c_1+c_2)\tau}}{c_2}-\frac{e^{-c_1t}}{c_2}\right]- \\
2A_{34}A_{42}\left(1-e^{-c_1t}\right)+2A_{40}\sum_{n=1}^{\infty}A^2\sum_{n=1}^{\infty}A_{42}\left(e^{-2c_1t}-e^{-c_1t}\right) \\
-2A_{38}\left(\sum_{n=1}^{\infty}A\sum_{n=1}^{\infty}A^2{}_1\sum_{n=1}^{\infty}\left[te^{-c_1t}+\frac{2e^{-(c_1+c_2)t}}{c_2}-\frac{e^{-(c_1+2c_2)t}}{2c_2}-\frac{3e^{-c_1t}}{2c_2}\right]\right) \\
-2A_{39}\sum_{n=1}^{\infty}A^2{}_1\sum_{n=1}^{\infty}A_{43}\left[\begin{array}{l}\frac{1}{c_1}-\frac{2e^{-c_2t}}{(c_1-c_2)}+\frac{e^{-2c_2t}}{(c_1-2c_2)}- \\ A_{44}\left(\begin{array}{l}e^{-c_1t}(c_1-c_2)(c_1-2c_2)- \\ 2e^{-c_1t}c_1(c_1-2c_2)+e^{-c_1t}c_1(c_1-c_2)\end{array}\right)\end{array}\right] \\
-2A_{41}\sum_{n=1}^{\infty}A^2\sum_{n=1}^{\infty}A_1\sum_{n=1}^{\infty}\left[\frac{e^{-(2c_1+c_2)t}}{(c_1+c_2)}-\frac{e^{-2c_1t}}{c_1}+\frac{c_2e^{-c_1t}}{c_1(c_1+c_2)}\right]
\end{array}\right)\sin n\pi x\right) \qquad (16)$$

$$\left.\begin{aligned}\theta(x,t)=&\left(\sigma_1+\sum_{n=1}^{\infty}A_1\left[1-e^{-c_2t}\right]\sin n\pi x\right)+\\ &v\left(\sum_{n=1}^{\infty}\left(\begin{aligned}&-2A_{57}\sum_{n=1}^{\infty}A_1\left[\frac{1}{c_2}-te^{-c_2t}-\frac{e^{-c_2t}}{c_2}\right]-\\ &2\sum_{n=1}^{\infty}A^2{}_1\sum_{n=1}^{\infty}A_{51}\left[\frac{1}{c_2}-2te^{-c_2t}-\frac{e^{-2c_2t}}{c_2}\right]\\ &+2A_{52}\sum_{n=1}^{\infty}A\left[\frac{e^{-c_1t}}{(c_2-c_1)}-\frac{e^{-c_2t}}{(c_2-c_1)}\right]-\frac{2A_{56}}{c_2}\left[1-e^{-c_2t}\right]\\ &+2\sum_{n=1}^{\infty}A\sum_{n=1}^{\infty}A_1A_{53}\left[\frac{e^{-c_1t}}{(c_2-c_1)}+\frac{e^{-(c_2+c_1)t}}{c_1}-\frac{c_2e^{-c_2t}}{c_1(c_2-c_1)}\right]\end{aligned}\right)\sin n\pi x\right)\end{aligned}\right\} \tag{17}$$

Where;

$$A=\frac{2\left[1-(-1)^n\right]}{n\pi},\ A_1=\frac{2b_2\left[(-1)^n-1\right]}{n\pi c_2},\ A_2=\frac{A_1}{c_2},\ A_3=a_6f(e-2),\ A_4=r(e-2),\ A_5=\frac{1}{2}\in,$$

$$A_6=\in\left(r(e-2)\right)\sigma_1,\ A_7=\frac{3\in\left(r(e-2)\right)}{4},\ A_8=\left(A_4+A_5+A_6\right),\ A_9=\left(1+f(e-2)\sigma_1\right),$$

$$A_{10}=f(e-2),\ A_{11}=\left(1+(e-2)\sigma_1\right),\ A_{12}=(e-2),\ A_{13}=\left(1+(e-2)\sigma_1\right)q,\ A_{14}=(e-2)q,$$

$$A_{15}=\frac{c_2}{c_1(c_1+c_2)},\ A_{16}=\left(1+f(e-2)\sigma_1\right)q,\ A_{17}=f(e-2)q,\ A_{18}=\left(1+r(e-2)\sigma_1\right),$$

$$A_{19}=r(e-2),\ A_{20}=\left(1+r(e-2)\sigma_1\right)q,\ A_{21}=r(e-2)q,\ A_{22}=\frac{1}{2}\in\left(\left(1+r(e-2)\sigma_1\right)\sigma_1\right),$$

$$A_{23}=\frac{1}{2}\in r(e-2)\sigma_1,\ A_{24}=\frac{1}{2}\in\left(\left(1+r(e-2)\sigma_1\right)\sigma_1q\right),\ A_{25}=\frac{1}{2}\in r(e-2)\sigma_1q,$$

$$A_{26}=\frac{1}{2}\in\left(1+r\left(e-2\right)\sigma_1\right),\ A_{27}=\frac{1}{2}\in r\left(e-2\right),\ A_{28}=\frac{1}{2}\in\left(1+r\left(e-2\right)\sigma_1\right)q,$$
$$A_{29}=\frac{1}{2}\in r\left(e-2\right)q,\ A_{30}=\left(p+q\right)\left(1+\left(e-2\right)\sigma_1\right),\ A_{31}=\left(p+q\right)\left(e-2\right),$$
$$A_{32}=\left(pq\right)\left(1+\left(e-2\right)\sigma_1\right),\ A_{33}=\left(pq\right)\left(e-2\right),\ A_{34}=\left(a_1A_{16}+a_2A_{20}+a_2A_{24}+a_3A_{32}\right),$$
$$A_{35}=\frac{1}{2}\left(a_1A_9+a_2A_{18}+a_2A_{22}+a_3A_{30}\right),\ A_{36}=\frac{1}{2}\left(a_1A_{17}+a_2A_{21}+a_2A_{25}+a_2A_{28}+a_3A_{33}\right),$$
$$A_{37}=\frac{2}{3}\left(a_1A_{10}+a_2A_{19}+a_2A_{23}+a_2A_{26}+a_3A_{31}\right),\ A_{38}=\frac{3}{8}a_2A_{27},\ \ A_{39}=\frac{2}{3}a_2A_{29},$$
$$A_{40}=\frac{2}{3}a_3A_{11},\ A_{41}=\frac{3}{8}a_3A_{12},\ A_{42}=\left[\frac{1-\left(-1\right)^n}{n\pi c_1}\right],\ A_{43}=\left[\frac{1-\left(-1\right)^n}{n\pi}\right],A_{44}=\frac{1}{c_1\left(c_1-c_2\right)\left(c_1-2c_2\right)},$$
$$A_{45}=a_4A_{18}A_{43},\ A_{46}=\frac{a_4A_{19}}{2},\ A_{47}=a_4A_{22}A_{43},\ A_{48}=\frac{a_4A_{26}}{2},A_{49}=a_4A_{27}\sigma^2{}_1A_{43},\ A_{50}=a_4A_{27}\sigma_1,$$
$$A_{51}=\frac{2}{3}a_4A_{43},\ A_{52}=\frac{a_5A_{11}}{2},\ A_{53}=\frac{2a_5A_{12}A_{43}}{3},\ A_{54}=a_5A_{13}A_{43},\ A_{55}=\frac{a_5A_{14}}{2},$$
$$A_{56}=\left(A_{45}+A_{47}+A_{49}-A_{54}\right),\ A_{57}=\left(A_{46}+A_{48}+A_{50}+A_{55}\right),b_1=\left(4R_a\in+\alpha_1\right),$$
$$b_2=\left(\sigma_1\left(4R_a\in+\alpha_1\right)+\left(R_a+\alpha_1\gamma_1\right)\right),\ c_1=\left(\beta_1+D_1\left(n\pi\right)^2\right),\ c_2=\left(b_1+\lambda_1\left(n\pi\right)^2\right).$$

The computation were done using Maple 17

4. Results and discussion

To conclude this analysis we examine the effect of radiation parameter $\left(R_a\right)$ on the transient state medium temperature $\theta\left(x,t\right)$ and oxygen concentration $\phi\left(x,t\right)$. Analytical solution given by equation (13)--(17), is computed using computer symbolic algebraic package MAPLE 17. The numerical result obtain from the method are show in Figure 1 and 2.

Figure 1 depicts the graph of temperature $\theta(x,t)$ against distance x and time t for different values of Radiation parameter (R_a). It observed that the temperature decreases with time and later increases along distance but decreases as the Radiation number increases.

Figure 2 displays the graph of oxygen concentration $\phi(x,t)$ against distance x and time t for different values of Radiation parameter (R_a). It observed that the oxygen concentration does not change more with time and oscillate along the distance and does not change as Radiation number increases.

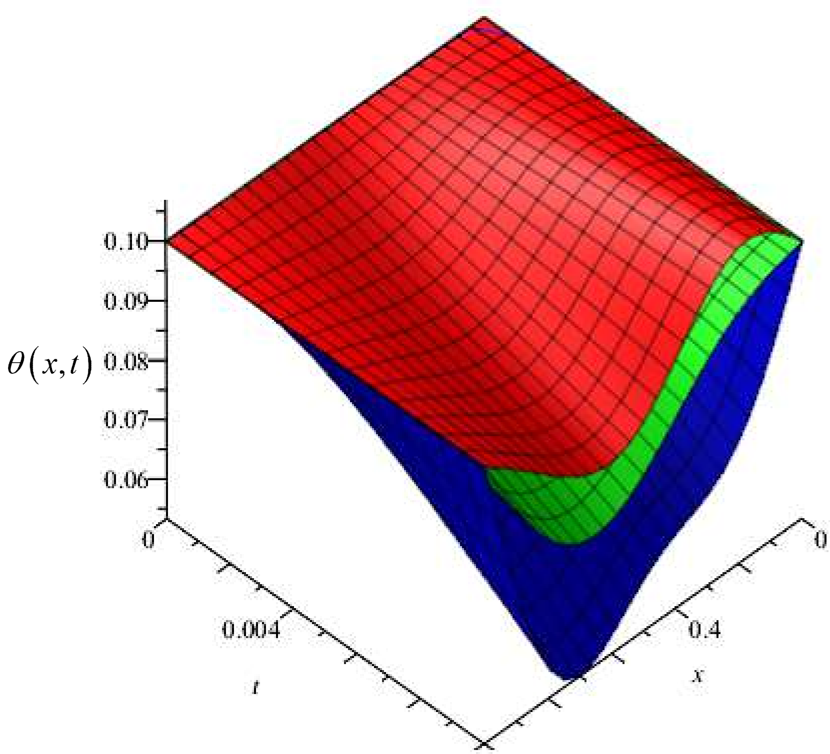

Figure 1: Graph of temperature $\theta(x,t)$ against distance x and time t for different value of Radiation parameter (R_a). $(R_a)=1$ (Read), $(R_a)=2$ (Green) and $(R_a)=4$ (Blue).

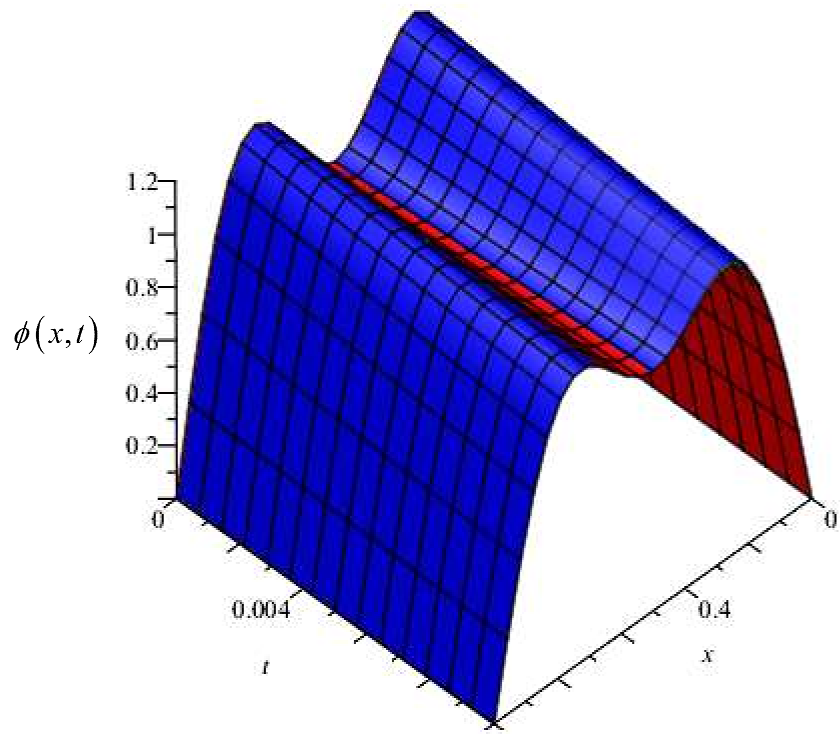

Figure 2: Graph of oxygen concentration $\phi(x,t)$ against distance x and time t for different values of Radiation Parameter (R_a). $(R_a)=1$ (Read), $(R_a)=2$ (Green) and $(R_a)=4$ (Blue).

It is worth pointing out that the effects observed in figures 1 and 2 are important for safety precaution.

5. Conclusion

For a high activation energy situation (i.e. as $\in \to 0$), we have solved the equations governing the fire spread model using direct integration and eigenfunction expansion technique. From the result obtained, we can conclude that, Radiation number reduced the medium temperature and does not have much effect on the oxygen concentration.

This results obtained are not only expected to guide fire services to forecast the danger rating of forest fire but to forecast the specific weather elements relating to forest fire.

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