Analysis of Thermo-Diffusion and Its Effects in an Inclined Hydromagnetic Boundary Layer Flow Due to Radial Stretching with Convective Boundary Conditions

Article · December 2022

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Lapai Journal of Applied and Natural Sciences

LAJANS Vol 6(1): 131 - 136

RESEARCH PAPER

Analysis of Thermo-Diffusion and Its Effects in an Inclined Hydromagnetic Boundary Layer Flow Due to Radial Stretching with Convective Boundary Conditions

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ABSTRACT

In this paper, an analysis of thermo-diffusion and diffusion-thermo effects in an inclined hydromagnetic boundary layer flow due to radial stretching with convective boundary conditions was carried out. The governing partial differential equations (PDEs) formulated were reduced with the help of similarity variables to nonlinear coupled ordinary differential equations (ODEs) which consists of velocity, temperature and concentration profiles. The implications of several parameters that occurred are presented graphically. The thermal Grashof number is found to drop the fluid velocity profile to free stream faster for higher values due to gravity.

Keywords: Radial stretching, diffusion-thermo, thermo-diffusion, hydromagnetic

INTRODUCTION

The fluid flow over a stretching sheets is important in many practical applications such as extrusion of plastic sheets, paper production, glass blowing-metal spinning, polymers in metal spinning processes, the continuous casting of metals, drawing plastic films and spinning of fibres all invoive some aspects of flow over a stretching sheet or cylindrical fibre (paullet and Weidman). The problem of radially stretching sheet has been studied by several researchers for the sole effects of rotation, velocity, MHD, chemical reaction,dufour-soret,different non-newtonian fluid,suction/injection on possible combinations of these effects. Butt *et al* (2014) studied the effects of magnetic field on enropy generation in flow and heat transfer due to radially stretching surface. It is observed that the magnetic field is a strong source of entropy production in the considered problem. Examined entropy geration analysis for viscoelastic MHD flow over a stretching sheet embedded in a porous medium. It is observed that increase in viscoelastic and magnetic parameter reduces the velocity. Increase in elastic parameter causes a greater retardation in the velocity.so also,Darcy dissipation favours higher level entropy generation in all the cases except the flow of liquid with low thermal diffusivity assuming the process to be irreversible. Wang *et al* (2007) carried out natural convection on a vertical radially stretching sheet. Aiyesimi *et al.* (2015) carried out an analytic investigation of convective boundary-layer flow of a nanofluid past a stretching sheet with radiation.

Sahoo *et al.*(2007) have investigated the MHD flow and heat transfer from a continuous surface in a uniform free stream of a non-Newtonian fluid. Mustafa *et al.* (2011) carried out in investiagation on stagnation-point flow of a nanofluid towards a stretching sheet. Wang (1990) examined Liquid film on an unsteady stretching sheet. Ariel (2001) studied axisymmetric flow of a second grade fluid past a stretching sheet.

Yusuf *et al.* (2015) carried out an analysis of unsteady hydromagnetic boundary layer flow in an inclined wavy permeable wall of a nanofluid with soret and dufour effects and heat generation with convective boundary condition. Yusuf *et al.* (2019) examined and analytical solution of unsteady boundary layer flow of a nanofluid past

Received 11 June, 2021
Accepted 22 October, 2021
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a stretching inclined sheet with effects of magnetic field. However, less attention has been paid to the boundary layer flow over a radial stretching sheet. This study is a new development in the literature in which the Analysis of thermo-diffusion and diffusion thermo effects in an inclined hydromagnetic boundary layer flow due to radial stretching with convective boundary conditions is presented.

This study will serve as a guide to Engineers as to how the physical parameters that occurred enhances the flow velocity, temperature and concentration of the fluid when the sheet is stretched radially.

METHODOLOGY

Considering a steady two-dimensional incompressible, irrotational flow of a Magnetohydrodynamics due to the stretching of an inclined sheet along the radial direction with velocity $U(r) = ar$, where a is a positive constant. The location of the inclined sheet is along the plane $z = 0$ and $z \succ 0$. Following the work of Sanatan *et al* (2015) with thermo-diffusion and diffusion-thermo effects with convective boundary condition. The governing equations for continuity, momentum, temperature and concentration can be written as:

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \quad (1)$$

$$\rho\left(u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z}\right) = \mu\frac{\partial^2 u}{\partial z^2} - \sigma B_0^2 u + g\beta(T - T_\infty)\cos\Theta + g\beta(C - C_\infty)\cos\Theta \quad (2)$$

$$u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z} = \frac{k}{\rho c_p}\left(\frac{\partial^2 T}{\partial z^2}\right) - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial z} + \frac{D_M K_T}{C_S c_p}\frac{\partial^2 C}{\partial y^2} \quad (3)$$

$$u\frac{\partial C}{\partial r} + w\frac{\partial C}{\partial z} = D_B\frac{\partial^2 C}{\partial z^2} + \frac{D_M K_T}{T_M}\frac{\partial^2 T}{\partial z^2} \quad (4)$$

Subject to the boundary condition:

$$\left.\begin{array}{l} u = U(r) = ar, w = 0, -k\left(\frac{\partial T}{\partial z}\right) = h_f\left(T_f - T_\infty\right), C = C_W, z = 0 \\ u \to \infty,\ T \to T_\infty, C \to C_\infty, z \to \infty \end{array}\right\} \quad (5)$$

The velocity along the r and z axes are respectively u, and w, ρ is the density, μ is the viscosity, υ is the kinematic viscousity, σ is the electrical conductivity, B_0external magnetic field, k is the thermal conductivity, c_p is the specific heat capacity at constant pressure, T is the fluid temperature, C is the concentration, T_w and $C_w$are the wall temperature and concentration respectively, T_∞ and C_∞ are temperature and concentration at larger values of z respectively, T_M is the mean fluid temperature, C_S is concentration susceptibility, D_B is the Brownian diffusion coefficient, D_T is the thermopheric diffusion coefficient, $K_T$is the thermal-diffusion ratio.

Following Roseland approximation we have $q_r = -\frac{4\sigma^*}{3\delta}\frac{\partial T^4}{\partial z}$, where σ^* and δ are the Stefan-Boltzmann constant and the mean absorption coefficient respectively. The temperature differences within the fluid are assumed sufficiently small such that T^4 may be expressed as a linear function of Temperature.

The Taylor's series expansion of T about T_∞ is given as

$$T = T(T_\infty) + (T - T_\infty)T^{/}(T_\infty) + \frac{(T-T_\infty)^2}{2}T^{//}(T_\infty) + \ldots \quad (6)$$

Taking $T(T_\infty) = T_\infty^4$ and neglecting higher order terms we have:

$$T^4 \cong 4TT_\infty^3 - 3T_\infty^4 \quad (7)$$

Therefore, $\frac{\partial q_r}{\partial z} = -\frac{16\sigma^* T_\infty^3}{3\delta}\frac{\partial^2 T}{\partial z^2}$ (8)

introducing (8) into (3) we have

$$u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z} = \frac{k}{\rho c_p}\left(\frac{\partial^2 T}{\partial z^2}\right) + \frac{1}{\rho c_p}\frac{16\sigma^* T_\infty^3}{3\delta}\frac{\partial^2 T}{\partial z^2} + \frac{D_M K_T}{C_S C_p}\frac{\partial^2 C}{\partial y^2} \quad (9)$$

Introducing the following similarity variables by Butt and Ali (2014):

$$\eta = z\sqrt{\frac{a}{\upsilon}}, u = arf^{/}(\eta), w = -2\sqrt{a\upsilon}, \theta = \frac{T-T_\infty}{T_w-T_\infty}, \ \phi = \frac{C-C_\infty}{C_w-C_\infty} \quad (10)$$

Into equations (1), (2), (4) , (5) and (9). We have:

$$f^{///} + 2ff^{//} - f^{/2} - Mf^{/} + Gr_T\theta \cos\Theta + Gr_C\phi \cos\Theta = 0$$
$$(1+R)\theta^{//} + 2\,Pr\,f\,\theta^{/} + DU\,Pr\,\phi^{//} = 0$$
$$\chi^{//} + 2Lef\chi^{/} + LeS_r\theta^{//} = 0 \qquad (11)$$

With the following as the corresponding boundary condition:

$$f(0) = 0,\ f^{/}(0) = 1, \theta^{/}(0) = -Bt\big(1-\theta(0)\big),\ \phi(0) = 1$$
$$f^{/}(\infty) = 0,\ \theta(\infty) = 1,\ \phi(\infty) = 0 \qquad (12)$$

Where $\eta, f(\eta), \theta(\eta)$ and $\phi(\eta)$ are the non-dimensional distance, velocity, temperature and concentration.

$$M = \frac{\sigma B_0^2}{a\rho},\quad Gr = \frac{g\beta(T_w - T_\infty)}{a^2 r},\quad Gc = \frac{g\beta(C_w - C_\infty)}{a^2 r},$$
$$Ra = \frac{16\sigma^* T_\infty^4}{3k^* k},\quad Pr = \frac{\upsilon\rho c_p}{K},\quad Du = \frac{D_m K_T (C_w - C_\infty)}{C_s c_p \upsilon (T_w - T_\infty)},$$
$$Le = \frac{\upsilon}{D_B},\ Sr = \frac{D_m K_T (T_w - C_\infty)}{T_m \upsilon (C_w - C_\infty)}, \text{ and } Bt = \frac{h_f}{k}\sqrt{\frac{\upsilon}{a}}$$

are the magnetic paramter, Thermal Grashof number, concentration Grashof number, radiation parameter, Prandtl number, Dufour number, Lewis number, Soret number, and Biot number respectively.

RESULTS AND DISCUSSION

The non linear coupled ordinary differential equations in (11) with the corresponding boundary conditions in (12) has been solved using the Runge-Kutta shooting technique. And the graphical results and their implications are presented below:-

Figures 1 to 3 displays the variations of thermal Grashof number on the fluid velocity, temperature and concentration respectively. As the Grashof number increases the velocity momentum decreases to free stream for higher Grashof number as a result of buoyancy and inclination effects , while the temperature and concentration boundary thicknesses rises.

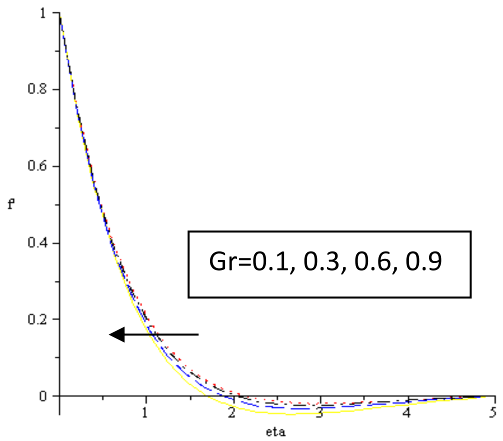

Figure 1: Variation of thermal Grashof number on velocity

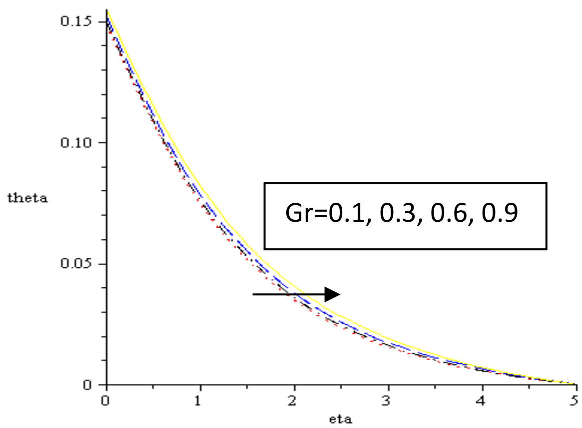

Figure 2: Variation of thermal Grashof number on temperature

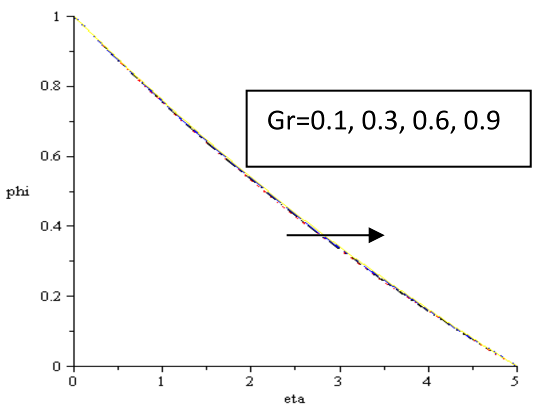

Figure 3: Variation of thermal Grashof number on concentration

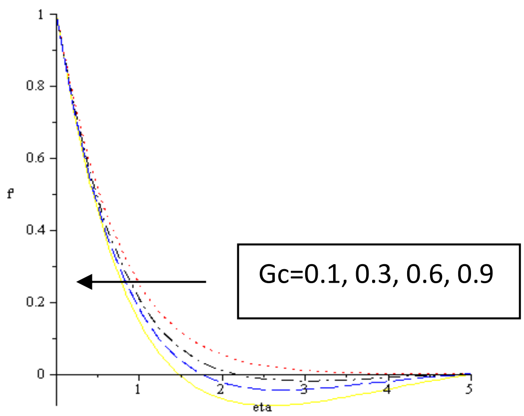

Figure 4: Variation of concentration Grashof number on velocity

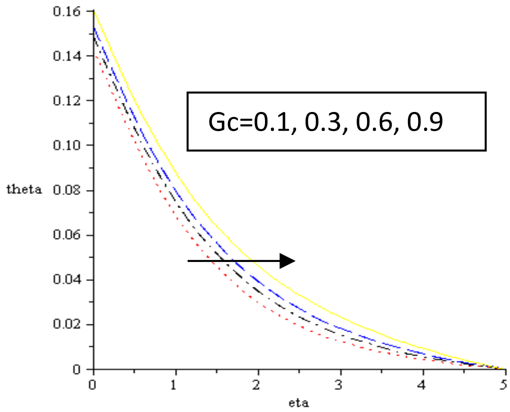

Figure 5: Variation of concentration Grashof number on temperature

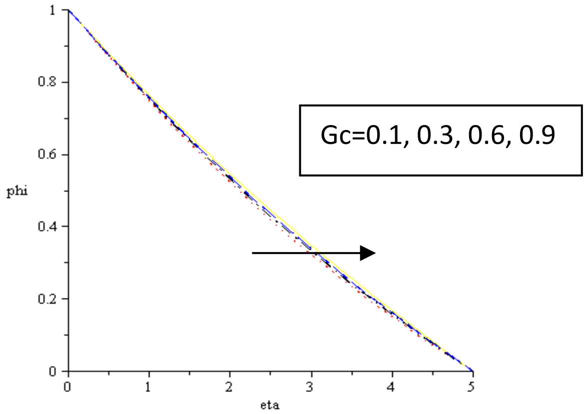

Figure 6: Variation of concentration Grashof number on concentration

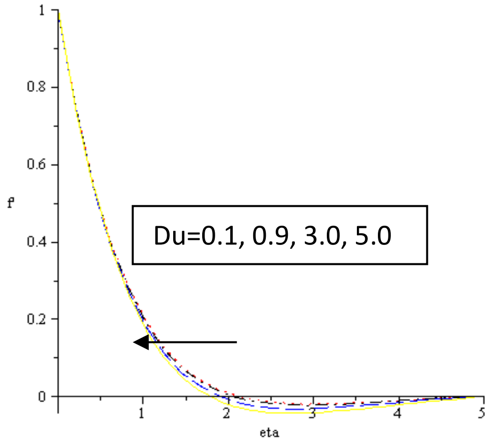

Figure 7: Variation of Dufour number on velocity

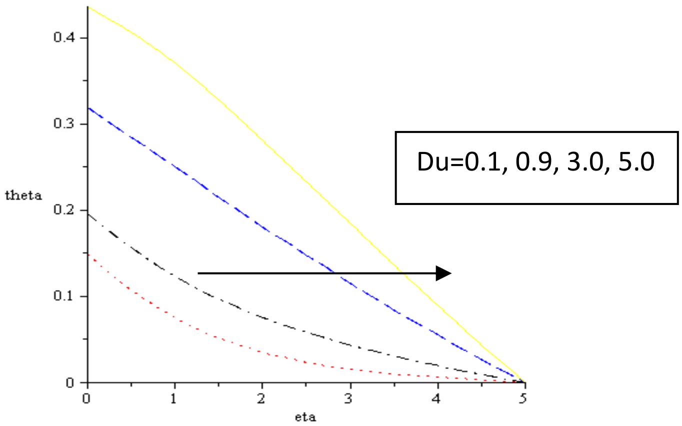

Figure 8: Variation of Dufour number on temperature

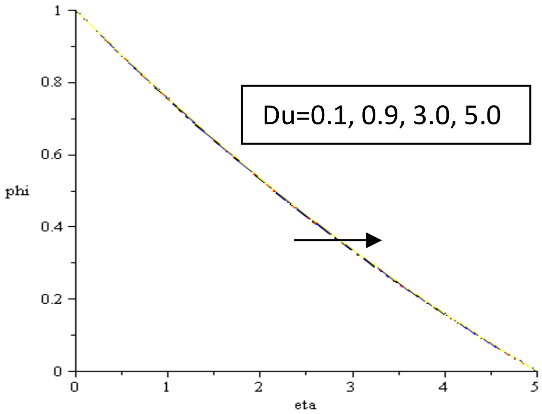

Figure 9: Variation of Dufour number on concentration

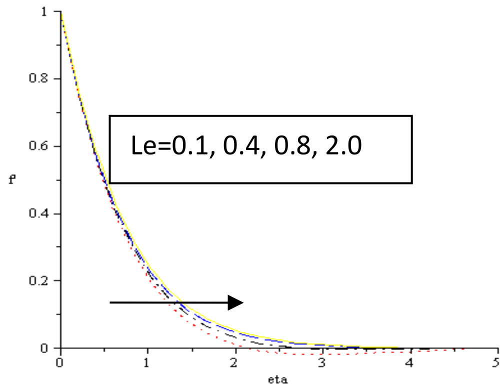

Figure 10: Variation of Lewis number on velocity

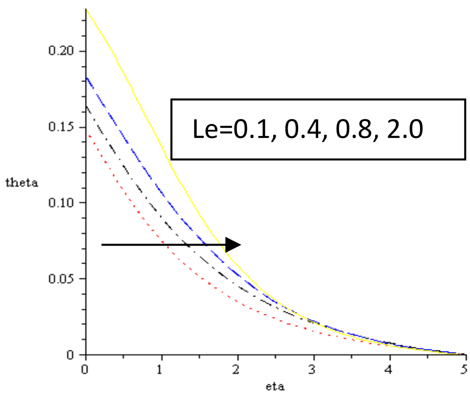

Figure 11: Variation of Lewis number on temperature

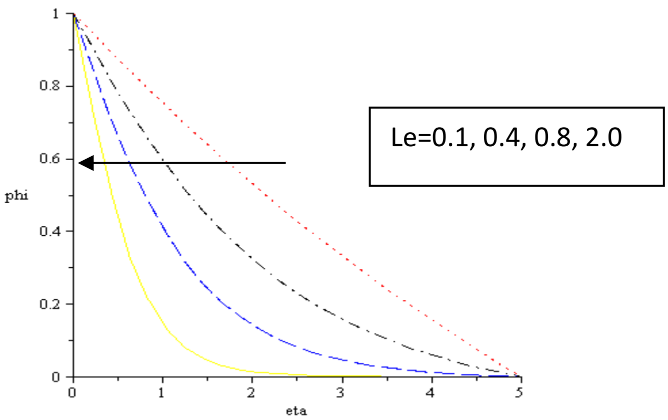

Figure 12: Variation of Lewis number on concentration

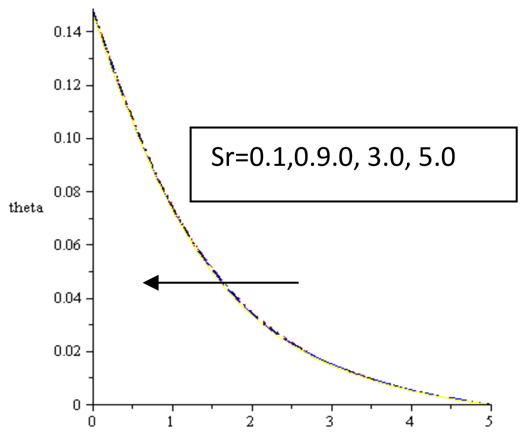

Figure 13: Variation of Soret number on temperature

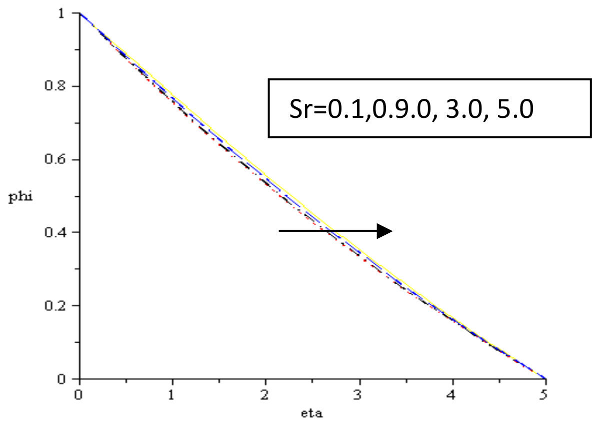

Figure 14: Variation of Soret number on concentration

Figures 4 to 6 presents the variations of concentration Grashof number on the fluid velocity, temperature and concentration respectively. As the concentration Grashof number increases the velocity profile dropped to free stream due to buoyancy effect, while the temperature and concentration boundary thicknesses rises. It is also observe that the boundary thicknesses for concentration Grashof number effect dropped to free stream faster respectively compared to thermal Grashof number.

Figures 7 to 9 showcase the effects of Dufour number on velocity, temperature and concentration profile. As Dufour number gain momentum, the velocity profile dropped while temperature and concentration boundary thicknesses thicken.

Figures 10 to 12 depicts the effects of Lewis number on the velocity, temperature and concentration profiles. Increase in Lewis number results to increase in velocity and temperature boundary thicknesses while concentration of the fluid reduces.

Figures 13 to 14 present the effect of Soret number on temperature and concentration profiles. The increase in Soret number leads to decrease in the temperature while the fluid concentration boundary thickness thickens. This phenomenon clearly shows that Dufour and Soret numbers in this study affects the temperature and concentration of the fluid in opposite manner as in Thermo-Diffusion and Diffusion-Thermo.

Figures 15 to 17 show the effects of Prandtl number, Radiation parameter and Biot number on the temperature profile respectively. Clearly, increasing prandtl number causes the fluid temperature to drop while increase in radiation and Biot number leads to increase in temperature.

Figures 18 to 20 show the effects of Magnetic parameter on the velocity, temperature and concentration profiles respectively. As expected, as the magnetic parameter increases, the velocity profile dropped due to the presence of Lorentz force while temperature and concentration profiles increases simultaneously.

Conclusion

This present work considered analysis of thermo-diffusion and diffusion thermo effects in an inclined hydromagnetic boundary layer flow due to radial stretching with convective boundary conditions by introducing the Dufour and Soret effects into the work of Sanatan et al (2015). The PDE formulated in rectangular system was reduced to ODE via some similarity variables by Butt and Ali (2014). The non linear coupled ODE depends on some physical parameters such as thermal and concentration Grashof number, Dufour number, Soret number, Prandtl number and solved using the shooting method. It is observed generally that the angle of inclination $(\Theta = 60^0)$ causes the effects of Grashof number to free stream at higher values.

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