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8th INTERNATIONAL CONFERENCE

OF SCHOOL OF SCIENCE AND TECHNOLOGY EDUCATION (SSTE)

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CONFERENCE PROCEEDINGS

THEME:

EMERGING TRENDS IN STEM AND TVET IN THE 21ST CENTURY

DATE 6th – 8TH OCTOBER, 2021

FEDERAL UNIVERSITY OF TECHNOLOGY, MINNA

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ACKNOWLEDGEMENTS

Local Organizing Committee of the 8th International Conference of School of Science and Technology Education (SSTE), Federal University of Technology, Minna, appreciates the Management of the University for the Unflinching Support given to the team which led to the success of this Conference. We also wish to express our profound gratitude for the assistance rendered by the University Management especially in disseminating information relating to this Conference using the University website, Campus News, and FUT Search FM.

Thank the Dean, School of Science and Technology Education (SSTE) for hosting this Conference despite the economic situation and the COVID-19 Pandemic in the country. His encouragement, advice, and moral support gave the Local Organizing Committee the strength to complete this task and make sure the Conference is successful.

Sincerely thank the academic staff of the School and University Community for their essential roles played towards the success of the Conference. We thank the non-teaching staff for their roles which contributed to the success of this Conference.

Efforts of the Editorial Board are commendable for making sure that the Book of Proceedings was ready. We appreciate the efforts of the Keynote presenter, Lead Paper presenters, and others for attending this Conference despite their tight schedule.

We also commend the undergraduate and postgraduate students of the School for sparing their time for participating in all the events. Above all we thank God Almighty for the strength given to the LOC members to discharge their enormous tasks.

PREFACE

Science, Technology, Engineering and Mathematics (STEM) and Technical and Vocational Education and Training (TVET) are very relevant in the 21st-century education. 21st-century education is about giving students the skills they need to succeed in this new world and helping them develop the confidence to practice those skills. The 21st-century skills focus more on making sense of that information, sharing and using it in smart ways. A focus on STEM and TVET could help in equipping students for life and work in the 21st Century.

STEM and TVET are considered critical as they tend to removes boarder between the disciplines by making the students to comprehend the world as a whole rather than in parts. STEM is interdisciplinary approach to learning where rigorous academic concepts are coupled with real world lessons while TVET takes individuals into account by providing all kinds of skills (or jobs).

Hence, the theme of this conference is apt and provides opportunities where experts brainstorm so that international communities can benefit from one another and also respond to emerging trends in *STEM* and *TVET* in the 21st Century. It is worthy to note that, proper implementation of the 21st-century STEM and TVET issues would have a greater influence on the quality of manpower injected into the labour market which may determine the economic growth of any nation.

The theme and sub-themes of this conference, "Emerging Trends in *STEM* and *TVET* in the 21st Century" are based on the prevailing circumstance in education sectors in developing nations. I am sure this conference has provided an avenue for researchers and educators to share their ideas on the 21st-century emerging trends in STEM and TVET that can enhance quality education and self-reliance in underdeveloped and developing nations across the world. I hope the theme and sub-themes meet the needs of the stakeholders in education.

The sub-themes are:

- STEM and TVET for Economic Diversification
- STEM and TVET for Human Resource Development
- STEM and TVET for National Security
- Curricular and Assessment issues in STEM and TVET
- Entrepreneurship in STEM and TVET
- Gender Equality/ Issues STEM and TVET
- STEM and TVET for Life-Long Learning and Skills
- STEM and TVET for Special Education
- Instructional Approaches in STEM and TVET
- ICT in STEM and TVET
- Communication in STEM and TVET
- STEM and TVET for Global health Challenges
- STEM and TVET for Innovative Technology and Engineering Education
- STEM and TVET as Solution to COVID-19 Pandemic

The Local Organizing Committee is thankful to the participants of 2021 conference for their contributions. God bless you all.

Dr. I. Y. Umar
LOC Chairman

A WELCOME ADDRESS GIVEN BY PROF. AMOSA ISIAKA GAMBARI, DEAN, SCHOOL OF SCIENCE AND TECHNOLOGY EDUCATION, FEDERAL UNIVERSITY OF TECHNOLOGY, MINNA, NIGERIA, AT THE OPENING CEREMONY OF THE 8TH INTERNATIONAL CONFERENCE OF THE SCHOOL. 1ST – 5TH OCTOBER, 2021

Protocol

It is with great delight that i warmly welcome you all, on behalf of the staff and students of the school of science and technology education (SSTE), Federal university of technology, Minna, to the 8th international conference of the school. I am glad that you all found time to be part of this ceremony to usher in another moment of robust deliberations on issues bothering on the progress of our immediate society in Nigeria and elsewhere in Africa and beyond.

I extend my warm welcome to our distinguished Keynote Speaker, Professor Joseph De-Beer of North West University, South Africa and our lead paper presenters, Professor Aloysius Uzoagulu of Enugu State University of Science and Technology, Dr. Islam Alhaji Abdou of Science, STEM and Bioethics Education, Egypt, and Professor Binyao Zheng of Kennesaw State University, USA. These carefully sourced and selected scholars are true representatives of 21st century resource persons in science and technology education. They are chosen because of their passion and dedication to the development of the individual through science and technology education and of the society at large. I appreciate all participants who are connected to us from different parts of the world and we believe that together, we shall make the conference a success.

STEM and TVET are two very significant areas in science and technology education which deal directly with human capacity development for the advancement of the larger society. These are areas that have what it takes to build up or destroy the future of an individual or a society.

But, education is all about the development of the individual to the benefit of the society. Science and Technology Education is factor to the technological growth and stability of every economy irrespective of its size or status. It is, therefore, of great importance to expose our generations to what it takes to grow and develop in this 21st century. Thus, this international conference is organised to bring together world experts to deliberate on emerging trends in STEM and TVET in the 21st century world of technology which are considered driving factors of economic growth and development.

The sub-themes of this conference were carefully extracted from the main theme ***Emerging Trends in STEM and TVET in the 21st Century*** to cover the major areas of impact of science and technology education globally including Covid-19 pandemic. The conference has great interest in capacity building and innovation as modern trends emerge in STEM and TVET.

This conference also covers current and fundamental issues relating to research and innovations in education which seeks to unravel the currency of the trends in science and technology to align education seekers to the realities of the time. This conference is, therefore, an avenue to bring together academic researchers and teachers of no mean reputation to deliberate on the new trends and the future of science and technology, globally. Global knowledge and experience in top level research that can engender scientific knowledge will be shared, presented and discussed internationally focusing attention on recent outstanding academic breakthroughs in science and technology education.

The urgent needs of education in the present, are necessary skills designed to move learners to the top of the ladder of scientific breakthrough in education and technology. Hence, the various sub-themes cover such interests from both local and international perspectives to equip both teachers, learners, education providers and policy makers to revolutionalise and renew initiatives.

Nigeria is struggling to make in-road into the league of technologically developed Nations, the impact is still far from being felt. The slow motion being experienced in the field of education can be attributed to archaic curriculum being circulated and used in schools. This type of curriculum that has little to show in the current dispensation of technological initiatives in science and technology cannot translate any learner to a scientist. Such obsolete learning can bedevil the good and spirited proposals of scientific development and equally weaken economic viability.

No society can rise beyond the level of her educational system, meaning that, every country is confined to the extent of her educational provisions and aspirations. A disadvantaged curriculum produces a disadvantaged populace in a disadvantaged economy.

Nigeria is still waiting for a curriculum where new things in science and technology will be enshrined, where provisions for laboratory furniture or technology study materials are provided, where students have trending tables and chairs for reading and writing, where teaching materials are provided and where technological development will be ensured.

These challenges are issues that this conference will address and proffer solutions to. With full knowledge of emerging trends in STEM and TVET in the 21st Century, lasting solutions are expected to emerge from this conference, where economic security strategies will be unveiled to enable science and technology education run on smooth wheels.

Ladies and gentlemen, you are welcome once again to this conference. I express my heart felt gratitude to the Vice Chancellor of this great and dynamic University and his management team for standing by us in every of our conferences. I am grateful to my colleagues in the School of Science and Technology Education and to our dear students for their support at every stage of the preparation for this conference. To all our participants, I wish you successful sessions and a fruitful outcome.

Thank you very much for your attention.

A WELCOME ADDRESS PRESENTED BY THE VICE CHANCELLOR, PROFESSOR ABDULLAHI BALA, AT THE OPENING CEREMONY OF THE 8TH INTERNATIONAL CONFERENCE OF THE SCHOOL OF SCIENCE AND TECHNOLOGY EDUCATION, FEDERAL UNIVERSITY OF TECHNOLOGY, MINNA, ON 6TH OCTOBER, 2021

PROTOCOL

It is my pleasure to welcome every participant, on behalf of the Governing Council, Management, Staff and Students of Federal University of Technology, Minna, to the 8th edition of the annual International Conference of the School of Science and Technology Education (SSTE) of our distinguished University. I specially welcome the Keynote Speaker, Professor Josef De-Beer and the eminent lead paper presenters, Professor Aloysius Uzoagulu of Nigeria, Dr. Islam Alhaji Abdou of Egypt and Professor Binyao Zheng of United States of America.

This 8th SSTE International Conference is a unique one being the first to go virtual, a signal of greater things ahead. The conference theme **"Emerging Trends in STEM and TVET in the 21st Century"** offers another opportunity to education Technology experts to explore global emerging trends in the education industry which is regularly shaped and reshaped by Technology. It provides a platform for scholars in academia to explore greater opportunities through well researched and exhaustively articulated and documented research efforts to unveil various pedagogical challenges frustrating effective teaching and learning in today's world of science and technology. It is also an avenue for both local and international academic and research communities to share their breakthroughs in teaching and learning research as well as explore opportunities for further developments in science and technology education, not only in Nigeria but also in the world at large.

The conference presentations will provide direction to further incorporate and integrate greater and healthier methodologies into teaching and learning in STEM and TVET to reflect the 21st century skills and learning.

It is becoming evident now that the state of education in Nigeria, Africa and the world is facing pedagogical challenges that need innovative interventions to address. The current quality of education, especially in Science and Technology, has not taken Nigeria far and has no promise of doing so. This Conference is, therefore, designed to unmask and rearrange the methodological shortcomings hindering effective teaching and learning by affording brainstorming opportunities to local and international participants on the emerging global trends in Science and Technology education. The redemptive conference sub-themes have been built to address these challenges in various areas of developmental endeavour. What Nigeria needs now is the restoration of the glory of education through emerging trends in the 21st century classroom practices. This will mark the beginning of new things in STEM and TVET in a country zealously seeking for technological solutions to her numerous domestic problems.

The beginning of new things in STEM and TVET will be driven by the identification of the relevant skills needed to acquire the much desired technological innovation to upgrade learning abilities and competencies of students. Pedagogy explores the processes by which society deliberately transmits its accumulated knowledge, skills and values from one generation to another. The major methodological concern in the mind of education stakeholders is how to educate students in various institutions of learning to meet their corporate and individual needs. The ability to overcome this methodological concern will be based on willingness to overcome the teething concerns by meeting the educational goals set by the student and the teacher, and also the strategies put in place to achieve those goals. Also, such factors as the educator's own philosophical beliefs, the curriculum provision and the availability or no availability of technological devices can affect the ability to overcome those learning concerns.

The emerging trends in STEM and TVET in the 21st Century is robust technology application which drives humanity to major discoveries unlike in the old analogue world, ruled by ancient pedagogies. The world is moving on the fast lane and no learner will be left behind. I, therefore, challenge this conference to offer to our society the best and nothing but the best trends emerging now in Science and Technology education through 21st century pedagogies. The teachers of our children need to be placed in a position to produce world renown scientists and technology experts who can in turn pass the competencies to the future generations of scientific achievers. The Universities and other education providers should be willing and able to adopt emerging innovative trends and key into the accruing benefits for the survival and advancement in knowledge to the benefit of society. All hands must be on deck to ensure that our future generations align with the realities of the 21st century by ensuring genuine technological immersion.

At this juncture, let me congratulate the Dean of the School of Science and Technology Education, Professor Amosa Isiaka Gambari on his doggedness to see that this conference becomes a reality. I sincerely appreciate the efforts of the Conference Organising Committee and the entire staff of the School for their relentless efforts towards the success of this conference. Please, accept my best wishes.

Ladies and gentlemen, I welcome you once again to this International Conference, have a very fruitful and rewarding occasion. Enjoy your day.

Thank you for your attention.

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Determination of Optimal Number of Servers in Banking System to Reduce Waiting Time of Customers

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Abstract
It is a common knowledge that most customers in Nigeria spent lot of their useful times in commercial banks queuing systems before being served. First City Monument Bank (FCMB) Minna branch is not left out from these times wasting. These times wasting are mostly as a result of improper allocation of banking personnel (Servers) at the different units (nodes) of the banks or redundancies in some of the units of the banks. Hence, the need to use scientific techniques to determine optimal number of banking personnel (Servers) to different units in First City Monument Bank (FCMB) Minna branch arose, in other to reduce waiting time of the customers, increase efficiency of the bank and improve its productivity. In this thesis, a network queuing model that determines optimal numbers of servers at the nodes of the Bank network queuing system to reduce waiting time of the customers is presented. The relevant data were collected for a period of four (4) weeks, through direct observations and personal interview. The number of arrivals and departures were also obtained. The total expected waiting time of customers in the current system before modification was 65 minutes with total number of 11 servers in the system while the total new expected waiting time of the customers in the system after modification was reduced to 28 minutes with optimal number of 16 servers (personnel) in all the nodes. The study has determined optimal number of servers (personnel) at the nodes of bank network system. Result from this study is an important information to the management of the First City Monument Bank, Minna branch for efficient and better service delivery.

Keywords: FCMB, Nodes, Servers, Customers, Waiting time, Network of Queue.

Introduction
A Common situation that occurs in everyday life is that of queuing or waiting in line, when the demand for a service exceeds the capacity of the service, waiting is unsurprising and inevitable (Kembe., 2012). Queues or waiting lines are usually seen at hotels, hospitals, bus stops, supermarkets, traffic, airports, gas stations, bank counters and so on. Service delay is unavoidable as a system gets blocked (Kandenmir and Cavas., 2007). When too much service is provided it's involves excessive cost and not providing enough service capacity causes the waiting line to become excessively long. The ultimate goal is to achieve an economic balance between the cost of service and the cost associated with the waiting for that service. Queuing systems theories have been used to study waiting time and predict the efficiency of services to be provided (Nityangini and Pravin., 2016). In queuing theory, there are three basic components of a queuing process which are:- Arrivals patterns, the actual waiting line and service facilities. Customers arrive to the facility from an infinite calling population, with a random arrival pattern following poison process. Once customers arrive, they are served immediately if the server(s) is empty, or otherwise the customers wait in the queue for the next empty server. Mostly, the service is on a first come first serve (FCFS) basis although other methods like service at random order (SARO) can be used. Preference service depending on the level of risk, urgency or the social, economic or political standing of the

customers and Hold on line (HL) discipline, where important arriving customer takes the lead of the queue is rampant in many facilities. Customers who may feel to have waited for long in queue can balk or renege and seek alternative equivalent services elsewhere, however, the queue length and waiting time depends on the traffic intensity, which is the ratio of arrival and service rates. The service discipline follows an exponential pattern, with individual service time variation due to different nature of the problems to be handled (Rotich., 2016).

In an open queuing network, jobs enter and depart from the network. In a closed queuing network, jobs neither enter nor depart from the network. Open queuing networks can be further divided into two categories; open feed forward queuing networks and open feedback queuing networks. In an open feed forward queuing network, a job cannot appear in the same queue for more than one time. In an open feedback queuing network, after a job is served by a queue, it may reenter the same queue (Tin yan and Veeraraghavan., 2004). In a Mixed Networks, Network has multiple job classes and is open with respect to some classes but closed with repeat to the others (Shanmugasundaram and Umarani., 2015).

Problem Formulation

The First City Monument Bank Minna branch is located adjacent to Central Bank Minna, Niger state, Nigeria. The Bank consists of five main Units, which are the Meter Greeter Unit, Customers Service Unit, Marketing Unit, Tellers Unit, and Customers Service Manager Unit. In this study, each department is regarded as node of the network system. The data used in this research were collected from the five different departments of the Bank and they were collected based on the arrival and departure rate as well as time spent at each node. The method adopted for the data collection was direct observation and personal interview. It was done for a complete one month, started from Monday to Friday. The collection of the data was for a total of six (6) hours at different time of the day, for each node. In a day, the number of arrivals and departures together with service time were taken at intervals of 5 minutes arrivals of customers into a node (□), while the departure rate was obtained also by the average number of five (5) minutes departures of customers at that particular node. However, each of the nodes was observed for a period of one (1) hour daily.

Model Assumptions

The following are the model assumptions made for Network Queuing System of the First City Monument Bank (FCMB), Minna.

1. The First City Monument Bank in the network queuing system is considered as an independent queuing system.
2. Queuing discipline is usually first come first served in the bank.
3. The external arrival of the customers in the bank followed by a Poisson arrival process.
4. Each node in the bank has one or more servers with exponential services time
5. The service rate depends on the number of customers at each node.
6. The way customers enter the bank is not restricted, but the departing rate out of the system is considered to be faster.
7. Servers represent all the banking personnel.
8. All the service providers are working in full capacity

We consider a banking network queuing system based on Jackson open network queuing model, the First City Monument Bank, Minna constitute of five units. In this study, we assumed that

customers who come in to bank for services will start by going first to the meter greeter unit and then move to the customer's service unit, then some customers proceed to tellers unit or customers service manager until all customers depart from the bank.

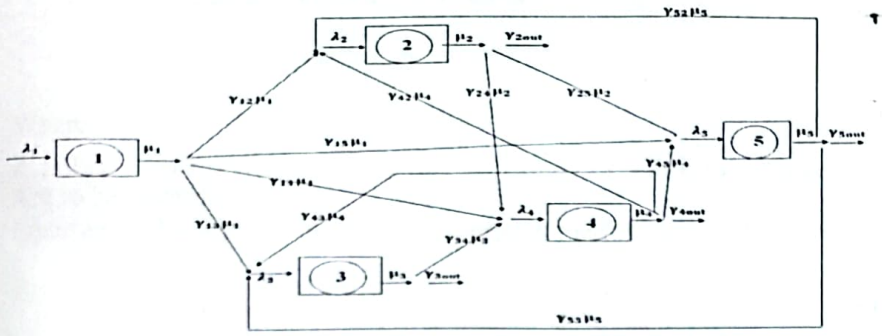

Figure 1:0 A Schematic Diagram of FCMB Queuing Network

Where: λ_i Is the arrival rate of the customer, for i = 1, 2,...5

μ_i Is the departure rate out of the system, for i = 1, 2,...5

γ_{ij} Are the weights of moving from server i to server j.

The following are the nodes in the network queuing system of the bankand Node1, Node2, Node3, Node4, Node5 are defined as follow: Meter Greeter Unit denoted by node1; Customer Service Unit denoted by node2; Marketing Unit denoted by node3; Tellers Unit denoted by node4; Customer Service Manager Unit denoted by node5.

Methodology

The following are model equations, obtained from fig 1

$$\lambda_2 = \gamma_{12}\mu_1 + \gamma_{42}\mu_4 + \gamma_{52}\mu_5 \quad (3.1)$$

$$\lambda_3 = \gamma_{13}\mu_1 + \gamma_{43}\mu_4 + \gamma_{53}\mu_5 \quad (3.2)$$

$$\lambda_4 = \gamma_{14}\mu_1 + \gamma_{24}\mu_2 + \gamma_{34}\mu_3 \quad (3.3)$$

$$\lambda_5 = \gamma_{15}\mu_1 + \gamma_{25}\mu_2 + \gamma_{45}\mu_4 \quad (3.4)$$

Also;

$$\mu_1 = \gamma_{12}\mu_1 + \gamma_{13}\mu_1 + \gamma_{14}\mu_1 + \gamma_{15}\mu_1 \quad (3.5)$$

$$\mu_2 = \gamma_{24}\mu_2 + \gamma_{25}\mu_2 + \gamma_{2out}\mu_2 \quad (3.6)$$

$$\mu_3 = \gamma_{34}\mu_3 + \gamma_{3out}\mu_3 \quad (3.7)$$

$$\mu_4 = \gamma_{42}\mu_4 + \gamma_{43}\mu_4 + \gamma_{45}\mu_4 + \gamma_{4out}\mu_4 \quad (3.8)$$

$$\mu_5 = \gamma_{52}\mu_5 + \gamma_{53}\mu_5 + \gamma_{5out}\mu_5 \quad (3.9)$$

Where:

$\gamma_{12}, \gamma_{13}, \gamma_{14}, \gamma_{15}, \gamma_{24}, \gamma_{25}, \gamma_{2out}, \gamma_{34}, \gamma_{3out}, \gamma_{42}, \gamma_{43}, \gamma_{45}, \gamma_{4out}, \gamma_{52}, \gamma_{53}, \gamma_{5out}$
Are to be determined

Equation (3.1 – 3.9) can also be represented in the following forms, thus:

$$\lambda_2 = \mu_1\gamma_{12} + 0\gamma_{13} + 0\gamma_{14} + 0\gamma_{15} + 0\gamma_{24} + 0\gamma_{25} + 0\gamma_{2out} + 0\gamma_{34} + 0\gamma_{3out} + \mu_4\gamma_{42} + 0\gamma_{43} + 0\gamma_{45} + 0\gamma_{4out} + \mu_5\gamma_{52} + 0\gamma_{53} + 0\gamma_{5out} \quad (3.10)$$

$$\lambda_3 = 0\gamma_{12} + \mu_1\gamma_{13} + 0\gamma_{14} + 0\gamma_{15} + 0\gamma_{24} + 0\gamma_{25} + 0\gamma_{2out} + 0\gamma_{34} + 0\gamma_{3out} + 0\gamma_{42} + \mu_4\gamma_{43} + 0\gamma_{45} + 0\gamma_{4out} + 0\gamma_{52} + \mu_5\gamma_{53} + 0\gamma_{5out} \quad (3.11)$$

$$\lambda_4 = 0\gamma_{12} + 0\gamma_{13} + \mu_1\gamma_{14} + 0\gamma_{15} + \mu_2\gamma_{24} + 0\gamma_{25} + 0\gamma_{2out} + \mu_3\gamma_{34} + 0\gamma_{3out} + 0\gamma_{42} + 0\gamma_{43} + 0\gamma_{45} + 0\gamma_{4out} + 0\gamma_{52} + 0\gamma_{53} + 0\gamma_{5out} \quad (3.12)$$

$$\lambda_5 = 0\gamma_{12} + 0\gamma_{13} + 0\gamma_{14} + \mu_1\gamma_{15} + 0\gamma_{24} + \mu_2\gamma_{25} + 0\gamma_{2out} + 0\gamma_{34} + 0\gamma_{3out} + 0\gamma_{42} + 0\gamma_{43} + \mu_4\gamma_{45} + 0\gamma_{4out} + 0\gamma_{52} + 0\gamma_{53} + 0\gamma_{5out} \quad (3.13)$$

$$\mu_1 = \mu_1\gamma_{12} + \mu_1\gamma_{13} + \mu_1\gamma_{14} + \mu_1\gamma_{15} + 0\gamma_{24} + 0\gamma_{25} + 0\gamma_{2out} + 0\gamma_{34} + 0\gamma_{3out} + 0\gamma_{42} + 0\gamma_{43} + 0\gamma_{45} + 0\gamma_{4out} + 0\gamma_{52} + 0\gamma_{53} + 0\gamma_{5out} \quad (3.14)$$

$$\mu_2 = 0\gamma_{12} + 0\gamma_{13} + 0\gamma_{14} + 0\gamma_{15} + \mu_2\gamma_{24} + \mu_2\gamma_{25} + \mu_2\gamma_{2out} + 0\gamma_{34} + 0\gamma_{3out} + 0\gamma_{42} + 0\gamma_{43} + 0\gamma_{45} + 0\gamma_{4out} + 0\gamma_{52} + 0\gamma_{53} + 0\gamma_{5out} \quad (3.15)$$

$$\mu_3 = 0\gamma_{12} + 0\gamma_{13} + 0\gamma_{14} + 0\gamma_{15} + 0\gamma_{24} + 0\gamma_{25} + 0\gamma_{2out} + \mu_3\gamma_{34} + \mu_3\gamma_{3out} + 0\gamma_{42} + 0\gamma_{43} + 0\gamma_{45} + 0\gamma_{4out} + 0\gamma_{52} + 0\gamma_{53} + 0\gamma_{5out} \quad (3.16)$$

$$\mu_4 = 0\gamma_{12} + 0\gamma_{13} + 0\gamma_{14} + 0\gamma_{15} + 0\gamma_{24} + 0\gamma_{25} + 0\gamma_{2out} + 0\gamma_{34} + 0\gamma_{3out} + \mu_4\gamma_{42} + \mu_4\gamma_{43} + \mu_4\gamma_{45} + \mu_4\gamma_{4out} + 0\gamma_{52} + 0\gamma_{53} + 0\gamma_{5out} \quad (3.17)$$

$$\mu_5 = 0\gamma_{12} + 0\gamma_{13} + 0\gamma_{14} + 0\gamma_{15} + 0\gamma_{24} + 0\gamma_{25} + 0\gamma_{2out} + 0\gamma_{34} + 0\gamma_{3out} + 0\gamma_{42} + 0\gamma_{43} + 0\gamma_{45} + 0\gamma_{4out} + \mu_5\gamma_{52} + \mu_5\gamma_{53} + \mu_5\gamma_{5out} \quad (3.18)$$

From model equation (3.10 - 3.18) can be represented in the matrix form as:

$$\begin{bmatrix} \mu_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_4 & 0 & 0 & 0 & \mu_5 & 0 & 0 \\ 0 & \mu_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_4 & 0 & 0 & 0 & \mu_5 & 0 \\ 0 & 0 & \mu_1 & 0 & \mu_2 & 0 & 0 & \mu_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu_1 & 0 & \mu_2 & 0 & 0 & 0 & 0 & 0 & \mu_4 & 0 & 0 & 0 & 0 \\ \mu_1 & \mu_1 & \mu_1 & \mu_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu_2 & \mu_2 & \mu_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_3 & \mu_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_4 & \mu_4 & \mu_4 & \mu_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_5 & \mu_5 & \mu_5 \end{bmatrix} \begin{bmatrix} \gamma_{12} \\ \gamma_{13} \\ \gamma_{14} \\ \gamma_{15} \\ \gamma_{24} \\ \gamma_{25} \\ \gamma_{2out} \\ \gamma_{34} \\ \gamma_{3out} \\ \gamma_{42} \\ \gamma_{43} \\ \gamma_{45} \\ \gamma_{4out} \\ \gamma_{52} \\ \gamma_{53} \\ \gamma_{5out} \end{bmatrix} = \begin{bmatrix} \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \\ \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \\ \mu_5 \end{bmatrix} \quad (3.19)$$

Equation (3.19) can be represented in the form

$$\begin{bmatrix} \gamma_{12} \\ \gamma_{13} \\ \gamma_{14} \\ \gamma_{15} \\ \gamma_{24} \\ \gamma_{25} \\ \gamma_{2out} \\ \gamma_{34} \\ \gamma_{3out} \\ \gamma_{42} \\ \gamma_{43} \\ \gamma_{45} \\ \gamma_{4out} \\ \gamma_{52} \\ \gamma_{53} \\ \gamma_{5out} \end{bmatrix} = \begin{bmatrix} \mu_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_4 & 0 & 0 & 0 & \mu_5 & 0 & 0 \\ 0 & \mu_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_4 & 0 & 0 & 0 & \mu_5 & 0 \\ 0 & 0 & \mu_1 & 0 & \mu_2 & 0 & 0 & \mu_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu_1 & 0 & \mu_2 & 0 & 0 & 0 & 0 & 0 & \mu_4 & 0 & 0 & 0 & 0 \\ \mu_1 & \mu_1 & \mu_1 & \mu_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu_2 & \mu_2 & \mu_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_3 & \mu_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_4 & \mu_4 & \mu_4 & \mu_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_5 & \mu_5 & \mu_5 \end{bmatrix} \begin{bmatrix} \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \\ \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \\ \mu_5 \end{bmatrix} \quad (3.20)$$

Mathematical Formulation for new Departure Rate

Reducing waiting time of the customers in the banking hall and increasing the efficiency of the bank is thrust of this research, hence we formulate new departure rate of each of the nodes in our network system. This is done using equation (3.1 – 3.4), thus, we have the following equations.

$$\lambda_2 = \gamma_{12}\mu_1 + 0\mu_2 + 0\mu_3 + \gamma_{42}\mu_4 + \gamma_{52}\mu_5 \quad (3.21)$$

$$\lambda_3 = \gamma_{13}\mu_1 + 0\mu_2 + 0\mu_3 + \gamma_{43}\mu_4 + \gamma_{53}\mu_5 \quad (3.22)$$

$$\lambda_4 = \gamma_{14}\mu_1 + \gamma_{24}\mu_2 + \gamma_{34}\mu_3 + 0\mu_4 + 0\mu_5 \quad (3.23)$$

$$\lambda_5 = \gamma_{15}\mu_1 + \gamma_{25}\mu_2 + 0\mu_3 + \gamma_{45}\mu_4 + 0\mu_5 \quad (3.24)$$

Model equation (3.21-3.24) can be transform to matrix as in equation (3.25)

$$\begin{bmatrix} \gamma_{12} & 0 & 0 & \gamma_{42} & \gamma_{52} \\ \gamma_{13} & 0 & 0 & \gamma_{43} & \gamma_{53} \\ \gamma_{14} & \gamma_{24} & \gamma_{34} & 0 & 0 \\ \gamma_{15} & \gamma_{25} & 0 & \gamma_{45} & 0 \end{bmatrix} \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \\ \mu_5 \end{bmatrix} = \begin{bmatrix} \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \end{bmatrix} \quad (3.25)$$

Where:

The arrival rate $\lambda_i = \frac{1}{mean\ number\ of\ arrival}$, for i = 1,2,...5 (3.26)

The departure rate $\mu_i = \frac{1}{mean\ number\ of\ departure}$, for i= 1,2,…5 (3.27)

$\rho = \frac{\lambda_i}{\mu_i}$, $for\ i = 1,2,\cdots 5.$ (3.28)

The expected number in the queue is given as

$l_q = \frac{\rho}{m-\rho}$, (3.29)

Where m stands for the number of servers at the node

The expected waiting time in the queue is given as:

$w_{qi} = \frac{l_q}{\lambda_i}$, (3.30)

The expected number of customers in the system is given as

$l_s = l_q + \rho$ (3.31)

Finally, the expected waiting time in the system for node 1-5 is given as

$w_i = \frac{l_s}{\lambda_i}$, For i = 1,2,…5.

Results and Discussion

The summary of the computed performance measure for determination of optimal number of servers at network queuing nodes to reduce waiting time at the First City Monument Bank, Minna **((FCMB)**, is given in the table below:

Table 1: Showing all the results obtained before modification.

Nodes i	Number of Servers (m_i)	Probabilities (α_{ij})	ρ_i	Lq	Ls	W_q	Ws
1	1	0.490525394. 0.285121877 0.02012705 0.204225679	0.90	9.0	9.9	16.7	18.5
2	2	0.204005077. 0.490675285. 0.305319638	1.00	1.0	2.0	1.00	2.0
3	3	0.460132145. 0.539867855	0.70	0.3	1.0	0.54	1.8
4	4	0.525454436. 0.275816098. 0.177498442 0.021231023	0.86	0.3	1.2	0.52	2.1
5	1	0.579712044. 0.334950037. 0.08533792	0.96	34.6	9.9	38.7	40.2
Total	**11**		**4.42**	**31.4**	**24.0**	**57.46**	**64.6**

Table 2: Showing all the results obtained after modification

Nodes i	Number of Servers (m_i)	ρ_i	Lq	Ls	W_q	Ws
1	2	1.4	2.3	3.7	4.30	6.9
2	3	1.7	1.3	3.0	1.30	3.0
[illegible]	[illegible]	0.7	[illegible]	[illegible]	[illegible]	[illegible]

4	5	0.9	0.2	1.1	0.34	11.9
5	2	1.2	1.5	2.7	2.41	4.4
Total	**16**	**5.9**	**5.5**	**11.4**	**8.71**	**27.8**

Table 3: Showing the comparison between current number of servers and optimal number of servers obtained

Nodes1	Current number of servers	Optimal number of servers obtained
1	1	2
2	2	3
3	3	4
4	4	5
5	1	2
Total	**11**	**16**

Conclusion

The FCMB network queuing system has been investigated and studied effectively. The study has determined optimal number of servers at the nodes of the FCMB network queuing system to reduce waiting time of the Customers. The result from the study is important information to the management of FCMB for proper planning and efficient service delivery. The analysis has shown that the arrival rate, departure rate as well as probabilities at each node (department) were obtained. The total expected waiting time of the customers in the system before modification is about 65 minutes. While, the total expected waiting time of the customers in the system after modification is about 28 minutes. This demonstrated that the optimal number of servers at the nodes of FCMB network queuing system is achieved.

Recommendation

To empower the FCMB consistently in order to meet up with its high standard of giving satisfactory services to her customers, the management of FCMB is advised to implement the following recommendations.

23. The number of servers at the node1 (Meter Greeter Unit) ought to be at least Two (2) servers.
24. The number of servers at the node2 (Customers Service Unit) ought to be at least Three (3) servers.

25. The number of servers at the node3 (Marketing Unit) ought to be at least Four (4) servers at each time for legitimate Proficiency.
26. The number of servers at the node4 (Tellers Unit) also, ought to be at least five (5) servers for legitimate Service delivery.
27. The number of servers at the node5 (Customers Service Manager Unit) similarly, ought to be at least Two (2) server.

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