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A Transshipment Model for a State Water Board

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Abstract

The aim of this work was to model the transshipment of portable water within Minna metropolis as a transportation problem in order to determine the optimal transshipment of potable water to Minna metropolis that minimizes the transshipment and fixed costs for Niger State Water Board.The computational results provided the minimal total shipment cost that is more effective than the existing cost by the board's intuitive method.

1. Introduction

Providing sufficient water of appropriate quality and quantity has been one of the most important issues in human history. Most ancient civilizations were initiated near water sources. As populations grew, the challenge to meet user demands also increased. People began to transport water from other locations to their communities. For example, the Romans constructed aqueducts to deliver water from distant sources to their communities. Water forms the largest part of most living matter. Human beings can survive longer without food than without water (Ayoade, 1988). An average man is two-thirds water and would weigh only 13kg when completely without water (i.e., dry weight). Plants need water for photosynthesis and they take their nutrient from the soil in solution. Water is an important geomorphic agent playing a significant role in weathering the most important energy regulator in the heat budget of the earth (Ayoade, 1988).

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According to World Health Organization (WHO), 75 liters of water a day is necessary to protect against household diseases and 50 liters a day necessary for basic family sanitation. The international consumption figures released by the 4th World Water Forum (March, 2006), indicate that a person living in an urban area, uses an average of 250 liters/day; but individual consumption varies widely around the globe. At the 2002 World Summit on Sustainable Development in Johannesburg, South Africa, great concern was expressed about the 1.1 billion people in the world who do not have access to safe drinking water and the 2.4 billion who live without proper sanitation (Cech, 2005). The United Nation (UN) predicts that by 2025, two-thirds of the world population will experience water scarcities, with severe lack of water blighting the lives and livelihoods of 1.8 billion. According to the United Nation (UN) World Water Assessment Programme, by 2050, 7 billion people in 60 countries may have to cope with water scarcity (Chenoweth, 2008). People in many parts of the world today are faced with the problem of water paucity and insecurity (Udoh and Etim, 2007). The World Health Organization (WHO) carried out a survey in 1975 which revealed that only 22% of the rural population in developing countries had access to safe drinking water. The findings which were published in 1976, led to the declaration of 1981-1990 as the International Drinking Water Supply and Sanitation Decade, by the United Nations Water Conference (Dada *et al.*, 1988).
Operations Research is the application of scientific methods to the management of organized systems. From the above definitions, we can deduce that OR is the viaduct between sciences and social sciences using scientific techniques to solve managerial and human problems mathematically (Nyor *et al.*, 2014).

2. Statement of the Problem

Transshipment problem emanating from transportation is one of the most significant areas of logistic management because of its direct impact on customer service level and the firm's cost structure. Outbound transportation cost can account for as much as ten (10) to twenty (20) percent of the product price (Grant 2006) and for some production firm, the transportation cost can be as high as twenty (20) percent or more of the total production cost. Niger State water board inspite of frantic efforts being made to supply the water to its numerous customers within Minna metropolis, still face some shortage in some of their reservoirs outlets. The research work seeks to address the problem of determining the optimal transportation schedule that will minimizes the total cost of transshipment of portable water to Minna metropolis.

3. Aim of the Study

The aim of this work is to model the transshipment of portable water and its cost within Minna metropolis as a transportation problem.

4. Objectives

The study intended to:

(1) address the problem of determining the optimal transshipment of potable water to Minna metropolis;
(2) minimize the fixed cost of Niger State water board; and
(3) address the problem of shortage of water in some reservoirs within Minna metropolis.

5. Preliminary Review

5.1. **Transportation Problem.** As the name indicates, a transportation problem is one in which the objective for minimization is the cost of transporting a certain commodity from a number of origins to a number of destinations. The transportation deals with the distribution of goods from several points of supply, such as factories, often known as sources, say m sources to a number of points of demands, such as warehouses often known as destination, say n destinations. Each source is able to supply a fixed number of units of products, usually called capacity or availability and each destination has a fixed demand, usually known as requirement. Movement of goods or products are usually across a network of routes that connect each point serving as a source and another point acting as a destination thus supply routes and demand routes respectively. Each source has a given supply while each sink has a given demand and the routes connecting the two has a given transportation cost per unit of shipment. The objective is schedule shipment from source to destination so that the total transportation cost is minimized so as to maximize profit.

5.2. **Linear Programming (LP) Formulations.** Suppose a company has m warehouses and n retail outlets. A single product is to be shipped from the warehouses to the outlets. Each warehouse has a given level of supply, and each outlet has a given level of demand. We are also given the transportation costs between every pair of warehouse and outlet, and these costs are assumed to be linear. More explicitly, the assumptions are:

(1) The total supply of the product from warehouse $i$is ai, where i= 1, 2, $\ldots$,m.
(2) The total demand for the product at outlet j is bj, where j = 1, 2, $\ldots$,n.
(3) The cost of sending one unit of the product from warehouse i to outlet j is equal to cij, where i= 1, 2, . . . ,$m$and j = 1, 2, . . . , n. The total cost of a shipment is linear in the size of the shipment.

The problem of interest is to determine an optimal transportation scheme between the warehouses and the outlets, subject to the specified supply and demand constraints. Graphically, a transportation problem is often visualized as a network with m source nodes, n sink nodes, and a set of m* n "directed arcs." This is depicted in Figure 1. We now proceed with a linear programming formulation of this problem.
The objective is to determine the amount of commodity (x_{ij}) transported from origin i to destination j such that the total transportation costs are minimized.

5.3. **The Decision Variables.** The variables in the Linear Programming (LP) model of the Transportation Problem (TP) will hold the values for the number of units shipped from one source to a destination or a transportation scheme is a complete specification of how many units of the product should be shipped from each warehouse to each factory. Therefore, the decision variables are: Xij= the size of the shipment from warehouse i to factory j, where i = 1, 2, . . .,m and j = 1, 2, . . ., n. This is a set of $m \times n$ variables.

5.4. **The Objective Function.** Consider the shipment from warehouse i to factory j. For any i and any j, the transportation cost per unit is C_{ij} and the size of the shipment is X_{ij}. Since we assume that the cost function is linear, the total cost of this shipment is given by $C_{ij}X_{ij}$. Summing over all i and all j now yields the overall transportation cost for all warehouse - factory combinations.
The objective function is:

$$\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij} \tag{5.1}$$

5.5. **The Constraints.** Consider warehouse i. The total outgoing shipment from this warehouse is the sum $x_{1j} + x_{2j} + \ldots + x_{mj}$. In summation notation, this is written as $\sum_{j=1}^{n} X_{ij}$. Since the total supply from warehouse $i$is ai, the total outgoing shipment cannot exceed a_i. That is, we must require $\sum_{i=1}^{m} X_{ij} \leq a_i$ for for $i = 1, 2, \ldots, m$.
Consider factory j. The total incoming shipment at this outlet is the sum $x_{1j} + x_{2j} + \ldots + x_{nj}$. In summation notation, this is written as $\sum_{i=1}^{m} X_{ij}$. Since the demand at outlet j is b_j, the total incoming shipment should not be less than b_j. That is, we must require $\sum_{i=1}^{m} X_{ij} \geq b_j$, for $j = 1, 2, \ldots, n$.
In summary, we arrived at the following formulation
Minimize

$$\sum_{i=1}^{m} X_{ij} \sum_{j=1}^{n} X_{ij}C_{ij}X_{ij} \tag{5.2}$$

Subject to:

(5.3) $$\sum_{i=1}^{m} X_{ij} \leq a_i \qquad \text{For } i = 1, 2, \ldots m$$

(5.4) $$\sum_{i=1}^{m} X_{ij} \geq b_j \qquad \text{For } j = 1, 2, \ldots n$$

(5.5) $$X_{ij} \geq 0 \qquad \text{For } i = 1, 2, \ldots m \text{ and } j = 1, 2, \ldots n$$

This linear programming problem (LPP) is in m x n variable and $m + n$ functional constraints. With above discussion we state that the total amount of the commodity transported from origin i to the various destinations must be equal to the amount available at origin $i(i = 1, 2, \ldots, m)$, similarly the total amount of the commodity received by destination j from all the sources must be equal to the amount required at destination $j(j = 1, 2, \ldots, n)$.
The non-negativity conditions are added since negative value for any X_{ij} has no physical meaning (that is, X_{ij} should be non-negative).

5.6. **Balancing a Transportation Problem.** It should be noted that the transportation model has feasible solution only if $\sum_{i=1}^{m} S_i = \sum_{j=1}^{n} D_j$ (that is total supply equal total demand). If this condition exits, the problem is said to be a balanced transportation problem, otherwise it is unbalanced.
If the problem has physical significance and this condition is not met, it usually means that either S_i or D_j actually represents a bound rather than an exact requirement. If this is the case, a fictitious 'source'or 'destination'(called the dummy source or the dummy destination) can be introduced to take up the slack in order to convert the inequalities into equalities and satisfy feasibility conditions.

5.6.1. *If Total Supply Exceeds Total Demand:* If this situation occurs, the modification is to add an extra column (dummy destination) to the tableau with the demand for the dummy destination equal to the excess supply. In other words, we balance the transportation problem by creating a dummy demand point that has a demand equal to the amount of excess supply. Since shipments to the dummy points are not real shipments, they are assigned a cost of zero (0). Shipments to the dummy demand point indicate unused supply capacity.

5.6.2. *If Total Supply is Less than Total Demand:* This requires a dummy source with supply equal to the excess of demand over supply. In other words, we add a dummy source that will absorb the difference (excess of supply over demand).

5.7. **The Transportation Algorithm.** The transportation algorithm consists of three stages
1. Find a transportation pattern that uses all the products available and satisfies all requirements. This is called developing an initial solution.
2. Test the solution for optimality. If the solution is optimal stop but if not move to stage three.
3. Use the stepping stone method or other method to obtain an improved solution and return to stage two

5.8. **Transportation Tableau.** The Simplex tableau serves as a very compact format for representing and manipulating linear programs. The transportation tableau represents for transportation problems that are in the standard form. For a problem with m sources and n destinations, the tableau will be a table with m rows and n columns. Specifically, each source will have a corresponding row; and each destination, a corresponding column. For ease of reference, we shall refer to the cell that is located at the intersection of the ith row and the jthcolumn as "cell (i, j)". Parameters of the problem will be entered into various parts of the table in the format below.

To Destination→ ↓From Source↓	D_1	D_2	...Dj...	D_n	Source Supply
S_1	c_{11} x_{11}	c_{12} x_{12}		c_{1n} x_{1n}	a_1
S_2	c_{21} x_{21}	c_{22} x_{22}		c_{2n} x_{2n}	a_2
$\ldots S_i \ldots$			c_{ij} x_{ij}		$\ldots a_i \ldots$
S_m	c_{m1} x_{m1}	c_{m2} x_{m2}		c_{mn} x_{mn}	a_m
Destination Requirements	b_1	b_2	$\ldots b_j \ldots$	b_m	$\sum a_i$ $\sum b_j$

Figure 1: Transportation Problems Matrix

That is, each row is labeled with its corresponding source name at the left margin; each column is labeled with its corresponding sink name at the top margin; the supply from source i is listed at the right margin of the ith row; the demand at sink j is listed at the bottom margin of the jthcolumn; the transportation cost C_{ij} is listed in a sub cell located at the upper-left corner of cell (i, j); and finally, the value of X_{ij} is to be entered at the lower-right corner of cell (i, j).

(1) The sum of product of the X_{ij} and C_{ij} is the cells is the objective function $\sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij}$

(2) Sum of X_{ij} across row give source supply constraint
(3) Sum of across column gives destination constraint X_{ij}

5.9. **The Basic Steps for Solving Transportation Model are:**

i Determine a starting basic feasible solution. We use any of the methods, North West-Corner Method (NWCM), Least Cost Method (LCM), or Vogel's Approximation Method (VAM), to find initial basic feasible solution.
ii Optimality condition. If solution is optimal then stop the iterations otherwise go to step 3.
iii Improve the solution. We use any one optimal method MODI or Stepping Stone method.

5.10. **Finding the Initial Feasible Solution.** A feasible solution of a transportation problem is a set of entries, which satisfy the following conditions:

i The entries must be non-negative, since negative shipments are not acceptable
ii The entries must sum along each row to the capacity available at that factory and down each column to the requirement of that warehouse.

The total transportation cost of any solution is obtained by multiplying each unit cost and summing overall entries. The object is to find the feasible solution(s) with the lowest total cost. This is called the optimum solution
There are several initial basic feasible solution methods for solving transportation problems satisfying supply and demand.
The following methods are always used to find initial basic feasible solution for the transportation problems and this research work use only Vogel's approximation method (VAM) to find initial basic feasible solution of the transportation problem:

i North West-Corner Method (NWCM)
ii Least Cost Method (LCM)
iii Vogel's Approximation Method (VAM)

5.11. **Algorithms of Vogel's Approximation Methed (VAM).**

i Compute penalty of each row and a column. The penalty will be equal to the difference between the two smallest shipping costs in the row or column.
ii Identify the row or column with the largest penalty and assign highest possible value to the variable having smallest shipping cost in that row or column.
iii Cross out the satisfied row or column.
iv Compute new penalties with same procedure until one row or column is left out.

v **Note:** Penalty means the difference between two smallest numbers in a row or a column.

5.12. **Finding the Optimum Solution.** There is the need to check to see if the **initial feasible solution** is the optimum cost. This is done by calculating what are known as "shadow cost"and comparing these with the actual costs to see whether a change of allocation is desirable. We start by calculating "dispatch"and "reception"costs for each used cell. It is assumed that the transportation cost in each cell can be split into two costs: "dispatch "and "reception". The dispatch costs are denoted by U_j and the reception costs are denoted by V_j.
Thus a feasible solution is optimal if and only if $(C_{ij}-U_i-V_j)$ for every (i,j) such that X_{ij} in the unused cells. The only work required by the optimality test is the derivation of the values of U_i and V_j for the used cells and then the calculation of $(C_{ij} - U_i - V_j)$ for the unused cells.
Since $(C_{ij} - U_i - V_j)$ is required to be zero if X_ij is in a used cell, the d_j and Si satisfy the set of equation $C_{ij} = s_i + d_j$ for each (i,j) such that X_{ij} is in a used cell. If $(C_{ij} - U_i - V_j)$, then optimality is reached. Otherwise, we conclude that the current feasible solution is not optimal. Therefore the transportation simplex method must go to the iteration step to find a better feasible solution.

6. Methodology of Transshipment Model

Transshipment problem is a transportation problem in which each origin and destination can act as an intermediate point through which goods can be temporarily received and then transshipped to other points or to the final destination. (Gass, 1969).
A transshipment model is a multi-phase transportation problem in which the flow of goods (such as raw materials) and services between the source and the origin is interrupted in at least one point. The product is not sent directly from the supplier (origin) to the point of demand; rather, it is first transported to a transshipment point, and from there to the point of demand (destination). (Barkovi; 2002).
In this model two questions must be answered with a view of minimizing the costs:

(1) How to transport the goods from the origin to the transshipment point;
(2) How to transport the goods from the transshipment point to the destination

6.1. **Transshipment Model Equations.** Minimize

$$\sum_{i=1}^{m}\sum_{k=1}^{r} C_{ik}X_{ik} + \sum_{k=1}^{r}\sum_{j=1}^{n} C_{jk}X_{jk} \tag{5.6}$$

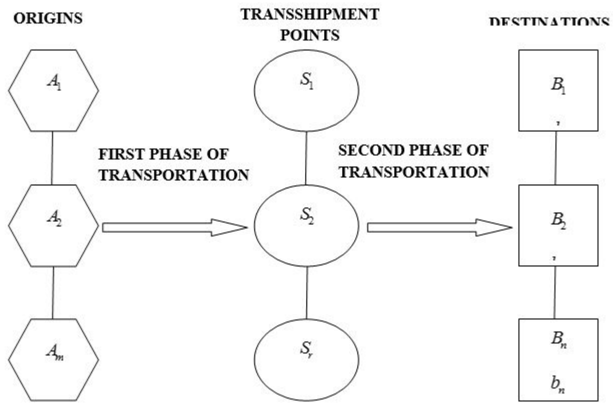

Figure 1. Product flow from origin to destination

Subject to:

$$\sum_{k=1}^{r} X_{jk}, \qquad j = 1, 2, \ldots, n \tag{5.7}$$

$$\sum_{i=1}^{m} X_{ik} = \sum_{j=1}^{n} X_{jk} \tag{5.8}$$

$$\sum_{k=1}^{r} X_{ik} \leq a_i \tag{5.9}$$

$$X_{ik} \geq 0, \quad i = 1, 2, \ldots, m,\ k = 1, 2, \ldots, r \tag{5.10}$$

$$X_{jk} \geq 0, \quad k = 1, 2, \ldots, r,\ j = 1, 2, \ldots, n \tag{5.11}$$

In the mathematical formulation (5.6) – (5.11) above, the following symbols are used: i - the symbol for origins A_i with available quantities on offer $a_i(i = 1, 2, \ldots, m)$; k - the symbol for transshipment point S_k with quantities $S_k(k = 1, 2, \ldots, r)$; j - the symbol for destinations B_j with demands $b_j(j = 1, 2, \ldots, n)$; X_{ik} - the quantity being transported from the origin A_i to the transshipment point S_k; X_{ij} - the quantity being transported from the transshipment point S_k to the destination B_j; C_{ik}- transportation costs per unit of goods from the origin A_i to the transshipment point S_k; C_{kj}- transportation costs per unit from the transshipment point S_k to the destination B_j.

The function of goal includes transportation costs from the origin to the transshipment point, transportation costs from the transshipment point to the destination and according to (5.6) it has to be minimized. The demand of all destinations

Figure 2. TABLE 1: Distance (INKILOMETER)

FROM	CHAN T	BOS T	INEC	TOP	SHIR	BIW	DUTS	IBB	PAID	BAH	BSS	POG	PAIK
BOSS	36.688	6.125	11.327	17.452	19.348	19.437	10.059	12.938	11.07	8.987	8.356	42.446	47.757
TAG	5.6255	36.688	27.876	22.001	27.313	25.188	34.564	34.864	28.801	38.499	39.599	40.196	43.876
CHAN	0	35.688	0.017374	0.00425	0.014874	0.010624	0.029376	0.029976	0.03385	0.037124	0.039446	0.04264	0.05
BOS T	35.688	0	0.022654	0.03262	0.023446	0.026624	0.007868	0.013626	0.009964	0.005724	0.004462	0.078642	0.087264

will be satisfied thanks to the restriction (5.7). Restriction (5.8) means that the quantity of goods delivered to each transshipment point is equal to the quantity of goods transported from that transshipment point to the destination. Restriction (5.9) means that the quantity of goods transported from each origin to all the transshipment points cannot exceed that origin's capacity. Restrictions (5.10) and (5.11) require non-negativity of decision-making variables.
In a transshipment model it is possible to introduce another limitation which ensures that the quantity of goods delivered to each transshipment point does not exceed the capacity of a particular transshipment point:

$$\sum_{i=1}^{m} X_{ik} \leq S_k,\ \ k = 1, 2, \ldots, r \tag{5.12}$$

7. Data Collection

The data was obtained from Niger State Water Board. The Board has two production points or sources in the state capital, one in Chanchaga (Tagwai dam) and the other in Bosso (Bosso dam). The data was obtained from state water board office.
The products are shipped from sources to transshipment points (treatment plants) before they are transported to the final destination.
The data is a quantitative data which is made up the distance from sources to transshipment point and from transshipment point to the destinations. The table 1 is a display name of places acting as sources, intermediate point (transshipment point) and the destinations.
Note:
The pumping machines to be used can pump 1000000 liters of water to 20km using a drum of diesel. N 40,000:00 k per drum of diesel is used in this research. An average fuel cost of N2000 is incurred in transporting products peer kilometer. The ratio of this amount to shipped 1000000 liters was found to be 0.002. This amount was used to multiply all the distances in table 2 to obtain the unit cost in

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FIGURE 3. Table 2: Unit Cost of Transporting Products

FROM	CHAN T	BOS T	INEC	TOP	SHIR	BIW	DUTS	IBB	PAID	BAH	BSS	POG	PAIK	SUPPLY
TAG	0.011252	0.14547	0.075752	0.064002	0.074626	0.070376	0.089128	0.089602	0.092114	0.096998	0.099198	0.13562	0.16382	95000000
BOSS	0.073376	0.01225	0.034904	0.04287	0.035696	0.038874	0.020118	0.025876	0.022214	0.017974	0.016712	0.08489	0.095514	7000000
CHAN T	0	0.071376	0.017374	0.00425	0.014874	0.010624	0.029376	0.02996	0.03385	0.037124	0.039446	0.04264	0.05	95000000
BOS T	0.071376	0	0.022654	0.03262	0.023446	0.026624	0.007868	0.013626	0.009964	0.005724	0.004462	0.07864	0.087264	7000000
DEMAND	95000000	7000000	21000000	5000000	6000000	13500000	30000000	12000000	3000000	5000000	1500000	250000	3750000	

transporting products from sources to intermediate point and from intermediate point to the various destinations.

FIGURE 4. TABLE 3: Vogel's Approximation Methods Manual Works

FROM	CHAN T	BOS T	INEC	TOP	SHIR	BIW	DUTS	IBB	PAID	BAH	BOSSO SEC. SCH.	POG	PAIK	SUPPLY
TAG	0.011252	0.14547	0.075752	0.064002	0.074626	0.070376	0.089128	0.089602	0.092114	0.096998	0.099198	0.13562	0.16382	95000000
BOSS	0.073376	0.01225	0.034904	0.04287	0.035696	0.038874	0.020118	0.025876	0.022214	0.017974	0.016712	0.08489	0.095514	7000000
CHAN T	0	0.071376	0.017374	0.00425	0.014874	0.010624	0.029376	0.02996	0.03385	0.037124	0.039446	0.04264	0.05	102000000
BOS T	0.071376	0	0.022654	0.03262	0.023446	0.026624	0.007868	0.013626	0.009964	0.005724	0.004462	0.07864	0.087264	102000000
DEMAND	102000000	102000000	21000000	5000000	6000000	13500000	30000000	12000000	3000000	5000000	1500000	250000	3750000	

7.1. **Data Analysis.** Note:
The iteration continued in a similar fashion until the 15^{th} iteration which is our final Distribution Table in table 6.
At Fifteenth Iteration table above, the optimum solution were obtained, since the solution satisfied $m + n - 1$ condition
$Z = 0.11252*95000000+0.01225*7000000+0*95000000+0*7000000+0.017374*21000000+0.00425*5000000+0.014874*6000000+0.010624*13500000+0.029376*30000000+0.029976*12000000+0.03385*3000000+0.03724*500000+0.04264*250000+0.05*3750000+0.004462*1500000+0.005724*5500000 = N3370709$

Excel Solver Excel Solver is a Microsoft Excel add-in program you can use to find an optimal (maximum or minimum) value for a formula in one cell —

FIGURE 5. Table 4: First Iteration

FROM	CHAN T	BOS T	INEC	TOP	SHIR	BIW	DUTS	IBB	PAID	BAH	BOSSO SEC. SCH.	POG	PAIK	SUPPLY	ROW PENALTY
TAG	0.011252	0.14547	0.075752	0.064002	0.074626	0.070376	0.089128	0.089602	0.092114	0.096998	0.099198	0.13562	0.16382	95000000	0.00425
BOSS	0.073376	0.01225	0.034904	0.04287	0.035696	0.038874	0.020118	0.025876	0.022214	0.017974	0.016712	0.08489	0.095514	7000000	0.00446
CHAN T	0	0.071376	0.017374	0.00425	0.014874	0.010624	0.029376	0.02996	0.03385	0.037124	0.039446	0.04264	0.05	95000000	0.00425
BOS T	0.071376	0	0.022654	0.03262	0.023446	0.026624	0.007868	0.013626	0.009964	0.005724	0.004462	0.07864	0.087264	7000000	0.004462
DEMAND	95000000	7000000	21000000	5000000	6000000	13500000	30000000	12000000	3000000	5000000	1500000	250000	3750000		
COLUMN PENALTY	0.01125	0.01225	0.00528	0.02837	0.012822	0.016	0.01225	0.01225	0.01225	0.01225	0.01225	0.03600	**0.037264**		

FIGURE 6. Table 5: Second Iteration

FROM	CHAN T	BOS T	INEC	TOP	SHIR	BIW	DUTS	IBB	PAID	BAH	BSS	POG	PAIK	SUPPLY	ROW PENALTY
TAG	0.011252	0.14547	0.075752	0.064002	0.074626	0.070376	0.089128	0.089602	0.092114	0.096998	0.099198	0.13562	0.16382	95000000	0.00425
BOSS	0.073376	0.01225	0.034904	0.04287	0.035696	0.038874	0.020118	0.025876	0.022214	0.017974	0.016712	0.08489	0.095514	7000000	0.00446
CHAN T	0	0.071376	0.017374	0.00425	0.014874	0.010624	0.029376	0.02996	0.03385	0.037124	0.039446	0.04264	0.05 3750000	91250000	0.00425
BOS T	0.071376	0	0.022654	0.03262	0.023446	0.026624	0.007868	0.013626	0.009964	0.005724	0.004462	0.07864	0.087264	7000000	0.004462
DEMAND	95000000	7000000	21000000	5000000	6000000	13500000	30000000	12000000	3000000	6000000	1500000	250000	---		
COLUMN PENALTY	0.01125	0.01225	0.00528	0.02837	0.012822	0.016	0.01225	0.01225	0.01225	0.01225	0.01225	**0.03600**	---		

FIGURE 7. Table 6: Final Distribution Table from Sources to Destinations

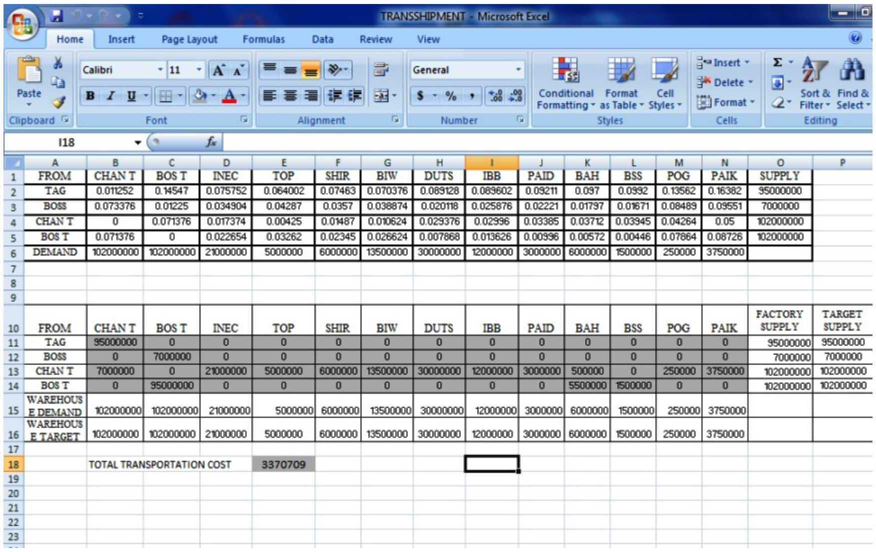
FROM	CHAN T	BOS T	INEC	TOP	SHIR	BIW	DUTS	IBB	PAID	BAH	BSS	POG	PAIK	SUPPLY
TAG	0.011252 95000000	0.14547	0.075752	0.064002	0.074626	0.070376	0.089128	0.089602	0.092114	0.096998	0.099198	0.13562	0.16382	95000000
BOSS	0.073376	0.01225 7000000	0.034904	0.04287	0.035696	0.038874	0.020118	0.025876	0.022214	0.017974	0.016712	0.08489	0.095514	7000000
CHAN T	0 7000000	0.071376	0.017374 21000000	0.00425 5000000	0.014874 6000000	0.010624 13500000	0.029376 30000000	0.02996 12000000	0.03385 3000000	0.037124 500000	0.039446	0.04264 250000	0.05 3750000	102000000
BOS T	0.071376	0 95000000	0.022654	0.03262	0.023446	0.026624	0.007868	0.013626	0.009964	0.005724 5500000	0.004462 1500000	0.07864	0.087264	102000000
DEMAND	102000000	102000000	21000000	5000000	6000000	13500000	30000000	12000000	3000000	6000000	1500000	2500000	3750000	

called the objective cell — subject to constraints, or limits, on the values of other formula cells on a worksheet. Solver works with a group of cells, called decision variables or simply variable cells that are used in computing the formulas in the objective and constraint cells. Solver adjusts the values in the decision variable cells to satisfy the limits on constraint cells and produce the result you want for the objective cell.

Applying the Excel solver to the problem, we obtained Figure 3.

SUMMARY OF THE STUDY

The following liters of water 95000000 and 7000000 were shipped from Tagwai dam and Bosso dam to Chanchaga treatment plant and Bosso water work at



FROM	CHAN T	BOS T	INEC	TOP	SHIR	BIW	DUTS	IBB	PAID	BAH	BSS	POG	PAIK	SUPPLY
TAG	0.011252	0.14547	0.075752	0.064002	0.07463	0.070376	0.089128	0.089602	0.09211	0.097	0.0992	0.13562	0.16382	95000000
BOSS	0.073376	0.01225	0.034904	0.04287	0.0357	0.038874	0.020118	0.025876	0.02221	0.01797	0.01671	0.08489	0.09551	7000000
CHAN T	0	0.071376	0.017374	0.00425	0.01487	0.010624	0.029376	0.02996	0.03385	0.03712	0.03945	0.04264	0.05	102000000
BOS T	0.071376	0	0.022654	0.03262	0.02345	0.026624	0.007868	0.013626	0.00996	0.00572	0.00446	0.07864	0.08726	102000000
DEMAND	102000000	102000000	21000000	5000000	6000000	13500000	30000000	12000000	3000000	6000000	1500000	250000	3750000	

FROM	CHAN T	BOS T	INEC	TOP	SHIR	BIW	DUTS	IBB	PAID	BAH	BSS	POG	PAIK	FACTORY SUPPLY	TARGET SUPPLY
TAG	95000000	0	0	0	0	0	0	0	0	0	0	0	0	95000000	95000000
BOSS	0	7000000	0	0	0	0	0	0	0	0	0	0	0	7000000	7000000
CHAN T	7000000	0	21000000	5000000	6000000	13500000	30000000	12000000	3000000	500000	0	250000	3750000	102000000	102000000
BOS T	0	95000000	0	0	0	0	0	0	0	5500000	1500000	0	0	102000000	102000000
WAREHOUSE DEMAND	102000000	102000000	21000000	5000000	6000000	13500000	30000000	12000000	3000000	6000000	1500000	250000	3750000		
WAREHOUSE TARGET	102000000	102000000	21000000	5000000	6000000	13500000	30000000	12000000	3000000	6000000	1500000	250000	3750000		

TOTAL TRANSPORTATION COST 3370709

FIGURE 8. Figure 3: Excel Solver Result

FIGURE 9. Table 7: SUMMARY OF THE RESULT OF THE DATA ANALYSED.

FROM	TO	SHIPMENT	COST PER UNIT (IN NAIRA)	SHIPMENT COST
TAGWAI	CHANCHAGA	95000000	0.011252	1068940
BOSSO	BOSSO T	7000000	0.01225	85750
CHANCHAGA	INEC TANK	21000000	0.017374	1548540
CHANCHAGA	TOP MEDICAL	5000000	0.00425	21250
CHANCHAGA	SHIRORO	6000000	0.014874	89244
CHANCHAGA	BIWATER/ARMY	13500000	0.010624	143424
CHANCHAGA	DUTSEN KURA	30000000	0.029376	881280
CHANCHAGA	IBB TANK	12000000	0.02996	359520
CHANCHAGA	PAIDA	3000000	0.03385	101550
CHANCHAGA	BAHAGO TANK	500000	0.037124	18562
CHANCHAGA	POGO TANK	250000	0.04264	10660
CHANCHAGA	PAIKO TANK	3750000	0.05	187500
BOSSO T	BAHAGO	5500000	0.005724	31482
BOSSOT	BOSSO SEC.	1500000	0.004462	6693

unit cost of 0.011252 and 0.01225 respectively. A total of 21000000, 5000000, 6000000, 13500000, 30000000, 12000000, 3000000, 500000 units were shipped from Chanchaga treatment plant to INEC tank, Top Medical tank, Shiroro tank,

Biwater/Army tank, Dutesnkura tank, IBB tank, Paida tank, Bahago tank, Pogo tank and Paiko tank at a unit cost of 0.017374, 0.00425, 0.014874, 0.010624, 0.029376, 0.029976,0.03385, 0.037124, 0.04264, 0.05 respectively.
The total 5500000, 1500000 units were shipped from Bosso water work to Bahago tank and Bosso secondary tank at a unit cost of 0.005724 and 0.004462 respectively.
The total shipment cost was N3370901. The minimum shipment cost was N 6693 from Bosso water work to Bosso Secondary School tank and the maximum shipment cost was N 1068940 from Tagwai to Chanchaga treatment plant.

CONCLUSION

The transportation cost is an important element of the total cost structure of any organization.
The transportation problem was formulated as a Linear Programming and solved with the standard linear programming (LP) solvers such as the Excel solver to obtain the optimal solution.
The computational results provided the minimal total shipment cost and the values for the decision variables for optimality. Upon solving the Linear Programming (LP) problems by the computer package, the optimum solutions provided the valuable information such as sensitivity analysis for Niger State Water Board to make optimal decisions
Through the use of this mathematical model (Transportation Model) the board (Niger State Water Board) can identify easily and efficiently plan out its weekly schedule, so that it cannot only minimize the cost of shipping water but also create time utility by reaching the goods and services at the right place and right time. This research finding will assist the Niger State Water Board to minimize the weekly cost of shipping water by 14.3% of N 3.933373.396 to N 3370901. It also addresses the problem of shortage of water in some reservoirs within Minna metropolis.

RECOMMENDATION

Based on the findings of this work and the large difference that exists between the optimal solution and the weekly expenditure on distribution of products by the Niger State Water Board from the source through the transshipment points to the various destinations, it is recommended that, the Board adopt the use of the Vogel's Approximation Method so as to minimize the cost of transporting their products. In addition it is recommended that there should be further study at any of the Water Board across the nation using any of the approach to cover their entire distribution. This study employed mathematical technique to solve management problems and make timely optimal decisions. If the Niger State

Board managers are to employed the proposed transportation model it will assist them to efficiently plan out its transportation scheduled at a minimum cost.

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