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Advancements in Fixed Point Results of Generalized Metric Spaces: A Survey

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Abstract:

The increasing significance of metric spaces and their applications in sciences and engineering has manifested over the years. This has led to the emergence of fixed point theory in metric spaces which in turn, has many practical usefulness in inequalities, approximation theory, optimization theory, image restoration and filtering, to mention but a few. Following up this development, in this paper, various results on G-metric spaces (also called generalized metric spaces) introduced by Mustafa and Sims are reviewed. Extensions of fixed point theorems for Lipschitzian-type mappings on G-metric spaces are compiled and a concise report on the transition in fixed point theorems on G-metric spaces are established. The aim of this survey is therefore, to examine and provide an up-to-date analysis of the important advancements in the fixed point theory of G-metric spaces. Consequently, this note is handy for researchers in the domain of metric and pseudo-metric spaces as they can easily appreciate how new results are delineated from the subsequent ones.
keywords: Metric spaces, G-metric spaces, fixed point, contraction.

1 Introduction

The increasing importance of metric spaces in mathematics and applied sciences has manifested over the years. This has led to the development of fixed point theory in metric spaces which in turn, has many applications in inequalities, approximation theory, optimization and so on.

There have been several attempts to generalize the notion of metric spaces by G*ä*hler in 1963 and Dhage in 1992 (see [1]). However, Ha et al. (see [1]) pointed out that G*ä*hler's results are independent of the known results in metric spaces. Similarly, Mustafa and Sims [2] revealed that Dhage's presentation of his generalized metric space is flawed and most of the results obtained therein are invalid.

In a PhD thesis titled "A New Structure for Generalized Metric Spaces: With Applications to Fixed Point Theory", Mustafa [3] proposed an appropriate and rigorous notion of generalized metric space called the G-metric space. This idea was first published in 2006 [1].

Subsequently, Mustafa [3] introduced the idea of the well-known Banach contraction principle into the framework of G-metric spaces and proved some fixed point theorems for contraction mappings on complete G-metric spaces satisfying certain contractive conditions. Since then, fixed point results have been generalized and extended in several directions in G-metric spaces with many interesting theorems and applications provided by different authors.

In this survey, we focus on highlighting the distinct and remarkable fixed point extensions in G-metric spaces in effort to provide researchers in the area of fixed point theory with a glimpse into the advancements in fixed point theory in G-metric spaces.

2 Preliminaries

In this section, we will highlight some fundamental notations, notions and results that will be deployed subsequently.

Throughout this paper, every set X is considered non-empty, $\mathbb{N}$ is the set of natural numbers, $\mathbb{R}$ represents the set of real numbers and $\mathbb{R}_+$, the set of non-negative real numbers.

We begin with the definition of generalized metric space due to Mustafa and Sims [1].

Definition 1.*[1] Let X be a non-empty set and let $G : X \times X \times X \longrightarrow \mathbb{R}_+$ be a function satisfying:*

$(G_1) G(x,y,z) = 0$ *if* $x = y = z$*;*

(G_2) $0 < G(x,x,y)$ *for all* $x,y \in X$ *with* $x \neq y$;
(G_3) $G(x,x,y) \leq G(x,y,z)$, *for all* $x,y,z \in X$ *with* $z \neq y$;
(G_4) $G(x,y,z) = G(x,z,y) = G(y,x,z) = \dots$ *(symmetry in all three variables);*
(G_5) $G(x,y,z) \leq G(x,a,a) + G(a,y,z)$, *for all* $x,y,z,a \in X$ *(rectangle inequality).*

Then the function G is called a generalized metric, or more specifically, a G-metric on X, and the pair (X,G) *is called a G-metric space.*

Example 1.[4] Let (X,d) be a usual metric space, then (X,G_s) and (X,G_m) are G-metric spaces, where

$$G_s(x,y,z) = d(x,y) + d(y,z) + d(x,z) \text{ for all } x,y,z \in X. \tag{1}$$

$$G_m(x,y,z) = \max\{d(x,y), d(y,z), d(x,z)\} \text{ for all } x,y,z \in X. \tag{2}$$

Definition 2.[4] *Let* (X,G) *be a G-metric space and let* $\{x_n\}_{n\in\mathbb{N}}$ *be a sequence of points of X. We say that* $\{x_n\}_{n\in\mathbb{N}}$ *is G-convergent to x if* $\lim_{n,m\to\infty} G(x,x_n,x_m) = 0$, *that is, for any* $\varepsilon > 0$, *there exists* $n_0 \in \mathbb{N}$ *such that* $G(x,x_n,x_m) < \varepsilon$, *for all* $n,m \geq n_0$. *We refer to x as the limit of the sequence* $\{x_n\}_{n\in\mathbb{N}}$.

Proposition 1.[4] *Let* (X,G) *be a G-metric space. Then the following are equivalent:*

(i) $\{x_n\}_{n\in\mathbb{N}}$ *is G-convergent to x.*
(ii) $G(x,x_n,x_m) \longrightarrow 0$, *as* $n,m \to \infty$.
(iii) $G(x_n,x,x) \longrightarrow 0$, *as* $n \to \infty$.
(iv) $G(x_n,x_n,x) \longrightarrow 0$, *as* $n \to \infty$.

Definition 3.[4] *Let* (X,G) *be a G-metric space. A sequence* $\{x_n\}_{n\in\mathbb{N}}$ *is called G-Cauchy if given* $\varepsilon > 0$, *there exists* $n_0 \in \mathbb{N}$ *such that* $G(x_n,x_m,x_l) < \varepsilon$, *for all* $n,m,l \geq n_0$, *that is,* $G(x_n,x_m,x_l) \longrightarrow 0$, *as* $n,m,l \to \infty$.

Proposition 2.[4] *In a G-metric space* (X,G), *the following are equivalent:*

(i) *The sequence* $\{x_n\}_{n\in\mathbb{N}}$ *is G-Cauchy.*
(ii) *For every* $\varepsilon > 0$, *there exists* $n_0 \in \mathbb{N}$ *such that* $G(x_n,x_m,x_m) < \varepsilon$, *for all* $n,m \geq n_0$.

Definition 4.[4] *Let* (X,G) *and* (X',G') *be two G-metric spaces and let* $f : X \longrightarrow X'$ *be a function. Then f is said to be G-continuous at a point* $a \in X$ *if and only if given* $\varepsilon > 0$, *there exists* $\delta > 0$ *such that* $x,y \in X$ *and* $G(a,x,y) < \delta$ *implies* $G'(f(a),f(x),f(y)) < \varepsilon$. *A function f is G-continuous on X if and only if it is G-continuous at all* $a \in X$.

Proposition 3.[4] *Let* (X,G) *and* (X',G') *be two G-metric spaces. Then a function* $f : X \longrightarrow X'$ *is said to be G-continuous at a point* $x \in X$ *if and only if it is G-sequentially continuous at x, that is, whenever* $\{x_n\}_{n\in\mathbb{N}}$ *is G-convergent to x,* $\{fx_n\}_{n\in\mathbb{N}}$ *is G-convergent to fx.*

Definition 5.[4] *A G-metric space* (X,G) *is called symmetric G-metric space if*

$$G(x,x,y) = G(y,x,x) \text{ for all } x,y \in X.$$

Proposition 4.[4] *Let* (X,G) *be a G-metric space. Then the function* $G(x,y,z)$ *is jointly continuous in all three of its variables.*

Proposition 5.[4] *Every G-metric space* (X,G) *will define a metric space* (X,d_G) *by*

$$d_G(x,y) = G(x,y,y) + G(y,x,x) \text{ for all } x,y \in X. \tag{3}$$

Note that if (X,G) *is a symmetric G-metric space, then*

$$(X,d_G) = 2G(x,y,y) \text{ for all } x,y \in X. \tag{4}$$

However, if (X,G) *is not symmetric, then it holds by the G-metric properties that*

$$\frac{3}{2}G(x,y,y) \leq d_G(x,y) \leq 3G(x,y,y) \text{ for all } x,y \in X, \tag{5}$$

and that in general, these inequalities are sharp.

Definition 6.[4] *A G-metric space* (X,G) *is said to be G-complete (or complete G-metric) if every G-Cauchy sequence in* (X,G) *is G-convergent in* (X,G).

Proposition 6.[4] *A G-metric space* (X,G) *is G-complete if and only if* (X,d_G) *is a complete metric space.*

Definition 7.[1] *Let* (X,G) *be a G-metric space. Then for* $x_0 \in X$, $r > 0$, *the G-ball with centre* x_0 *and radius r is*

$$B_G(x_0,r) = \{y \in X : G(x_0,y,y) < r\}. \tag{6}$$

Proposition 7.[1] *Let* (X,G) *be a G-metric space. Then for any* $x_0 \in X$, $r > 0$, *we have:*

(i) *if* $G(x_0,x,y) < r$, *then* $x,y \in B_G(x_0,r)$;
(ii) *if* $y \in B_G(x_0,r)$, *then there exists* $\delta > 0$ *such that* $B_G(y,\delta) \subseteq B_G(x_0,r)$.

It follows from (ii) of the above Proposition 7 that the family of all G-balls, $B = \{B_G(x,r) : x \in X, r > 0\}$, is the base of a topology $\tau(G)$ on X, the G-metric topology.

Definition 8.[1] *Let* (X,G) *be a G-metric space and let* $\varepsilon > 0$ *be given. Then a set* $A \subseteq X$ *is called an* ε*-net of* (X,G) *if given any x in X, there is at least one point a in A such that* $x \in B_G(a,\varepsilon)$. *If the set A is finite, then A is called a finite* ε*-net of* (X,G).

Note that if A is an ε-net, then $X = \bigcup_{a\in A} B_G(a,\varepsilon)$.

Definition 9.[1] *A G-metric space* (X,G) *is called G-totally bounded if for every* $\varepsilon > 0$, *there exists a finite* ε*-net.*

Definition 10.[1] *A G-metric space* (X,G) *is said to be a compact G-metric space if it is G-complete and G-totally bounded.*

Proposition 8.[1] *For a G-metric space (X,G), the following are equivalent:*

(i)(X,G) is a compact G-metric space.
(ii)$(X,\tau(G))$ is a compact topological space.
(iii)(X,d_G) is a compact metric space.
(iv)(X,G) is G-sequentially compact, that is, if the sequence $\{x_n\}_{n\in\mathbb{N}} \subseteq X$ is such that $\sup\{G(x_n,x_m,x_l) : n,m,l \in \mathbb{N}\} < \infty$, then $\{x_n\}_{n\in\mathbb{N}}$ has a G-convergent subsequence.

Definition 11.[5] *A function $\psi : \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ satisfying*

(i)ψ is continuous;
(ii)ψ is non-decreasing;
(iii)$\psi(t) = 0$ if and only if $t = 0$;

is called an altering distance function.

Denote by Ψ, the set of all functions $\psi : \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ satisfying (i)-(iii) of Definition 11.

Consistent with [6], let Φ be the set of all non-decreasing functions $\phi : \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ such that $\lim_{n\to\infty} \phi^n(t) = 0$ for all $t \in (0,\infty)$. If $\phi \in \Phi$, then ϕ is called a Φ-map. If ϕ is a Φ-map, then it is clear that

(i)$\phi(t) < t$ for all $t \in (0,\infty)$;
(ii)$\phi(0) = 0$.

Also, denote by Υ the set of all functions $\varphi : \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ satisfying

(i)φ is lower semi-continuous;
(ii)$\varphi(t) = 0$ if and only if $t = 0$.

Throughout this paper, unless stated otherwise, $\psi \in \Psi$, $\phi \in \Phi$ and $\varphi \in \Upsilon$ satisfy the above conditions.

The first fixed point result in G-metric space was obtained in 2005 by Mustafa [3]. It was shown that a self-mapping T on a complete G-metric space (X,G) satisfying certain contractive conditions has a unique fixed point and T is G-continuous at such a point.

Theorem 1.[3] *Let (X,G) be a complete G-metric space and let $T : X \longrightarrow X$ be a mapping satisfying*

$$G(Tx,Ty,Tz) \leq \lambda G(x,y,z), \tag{7}$$

for all $x,y,z \in X$ where $\lambda \in [0,1)$. Then T has a unique fixed point (say u, i.e., $Tu = u$) and T is G-continuous at u.

Proof. We prove the above theorem using similar approach as in Mustafa et al. [4].
Let $x_0 \in X$ be an arbitrary point and define the sequence $\{x_n\}_{n\in\mathbb{N}}$ by $x_n = T^n x_0$. Then by (7), we see that

$$G(x_n,x_{n+1},x_{n+1}) \leq \lambda G(x_{n-1},x_n,x_n).$$

Continuing in the same argument, we see that

$$G(x_n,x_{n+1},x_{n+1}) \leq \lambda^n G(x_0,x_1,x_1).$$

For all $n,m \in \mathbb{N}$ with $n < m$, we have by rectangle inequality that

$$\begin{aligned} G(x_n,x_m,x_m) &\leq G(x_n,x_{n+1},x_{n+1}) + G(x_{n+1},x_{n+2},x_{n+2}) \\ &\quad + \cdots + G(x_{m-1},x_m,x_m) \qquad (8) \\ &\leq (\lambda^n + \lambda^{n+1} + \cdots + \lambda^{m-1})G(x_0,x_1,x_1) \\ &\leq \frac{\lambda^n}{1-\lambda} G(x_0,x_1,x_1). \qquad (9) \end{aligned}$$

Then, $\lim G(x_n,x_m,x_m) = 0$ as $n,m \to \infty$, implying that $\{x_n\}_{n\in\mathbb{N}}$ is a G-Cauchy sequence. Due to the completeness of (X,G), there exists $u \in X$ such that $x_n \longrightarrow u$ as $n \to \infty$. To see that u is a fixed point of T, consider

$$G(x_n,Tu,Tu) \leq \lambda G(x_{n-1},u,u).$$

Taking limit as $n \to \infty$ and using the fact that G is continuous, we see that

$$G(u,Tu,Tu) \leq \lambda G(u,u,u),$$

implying that $G(u,Tu,Tu) = 0$ and so $Tu = u$.

To see uniqueness, assume there exists $v \in X$ such that $Tv = v$ and $u \neq v$. Then

$$G(u,v,v) \leq \lambda G(u,v,v) < G(u,v,v).$$

This is a contradiction since $\lambda \in [0,1)$. Similarly, $G(v,u,u) < G(v,u,u)$ is a contradiction. Hence, $u = v$.

To show that T is G-continuous at u, let $\{y_n\}_{n\in\mathbb{N}} \subseteq X$ be a sequence such that $y_n \longrightarrow u$ as $n \to \infty$. Then

$$G(u,Ty_n,Ty_n) = G(Tu,Ty_n,Ty_n) \leq \lambda G(u,y_n,y_n).$$

Taking limit as $n \to \infty$, we see that $G(u,Ty_n,Ty_n) \longrightarrow 0$. Hence, by Proposition 3, $Ty_n \longrightarrow u$. So, T is G-continuous at u.

3 Sequent of Mustafa's Result

In this section, we highlight the important extensions of the results of Mustafa [3]. One of the earliest generalizations of Mustafa's result was given by Mustafa et al. [4]. We first consider this result.

3.1 Mustafa, Obiedat and Awawdeh (2008)

Mustafa et al. [4] proved some fixed point results of Hardy-Rogers type for mappings satisfying certain conditions on complete G-metric space. Also, the uniqueness of these results was shown.

Theorem 2.[4] *Let (X,G) be a complete G-metric space and let $T : X \longrightarrow X$ be a mapping satisfying one of the following conditions:*

$$G(Tx,Ty,Tz) \leq \{aG(x,y,z) + bG(x,Tx,Tx) + cG(y,Ty,Ty) + dG(z,Tz,Tz)\}$$

or

$$G(Tx,Ty,Tz) \leq \{aG(x,y,z) + bG(x,x,Tx) + cG(y,y,Ty) + dG(z,z,Tz)\}$$

for all $x,y,z \in X$ where $0 \leq a+b+c+d < 1$. Then T has a unique fixed point (say u) and T is G-continuous at u.

Corollary 1.[4] *Let (X,G) be a complete G-metric space and let $T : X \longrightarrow X$ be a mapping satisfying one of the following conditions for some $m \in \mathbb{N}$:*

$$G(T^m x,T^m y,T^m z) \leq \{aG(x,y,y) + bG(x,T^m x,T^m x) + cG(y,T^m y,T^m y) + dG(z,T^m z,T^m z)\}$$

or

$$G(T^m x,T^m y,T^m z) \leq \{aG(x,y,y) + bG(x,x,T^m x) + cG(y,y,T^m y) + dG(z,z,T^m z)\}$$

for all $x,y,z \in X$ where $0 \leq a+b+c+d < 1$. Then T has a unique fixed point (say u) and T^m is G-continuous at u.

Several other fixed point results have been stated in this manner, satisfying various contractive conditions (see e.g., [4,7,8,9,10,11]). Throughout, they adopted the method of proof used in Theorem 1.

3.2 Mustafa, Awawdeh and Shatanawi (2010)

Mustafa et al. [12] defined expansive mapping in the setting of G-metric space and proved some fixed point results.

Definition 12.[12] *Let (X,G) be a G-metric space and T be a self-mapping on X. Then T is called expansive mapping if there exists a constant $\lambda > 1$ such that for all $x,y,z \in X$, we have*

$$G(Tx,Ty,Tz) \geq \lambda G(x,y,z).$$

Their main result is the following.

Theorem 3.[12] *Let (X,G) be a complete G-metric space. If there exists a constant $\lambda > 1$ and a surjective self-mapping T on X such that for all $x,y,z \in X$,*

$$G(Tx,Ty,Tz) \geq \lambda G(x,y,z),$$

then T has a unique fixed point.

Theorem 4.[12] *Let (X,G) be a complete G-metric space. If there exists a constant $\lambda > 1$ and a surjective self-mapping T on X such that for all $x,y \in X$,*

$$G(Tx,Ty,Ty) \geq \lambda G(x,y,y),$$

then T has a unique fixed point.

3.3 Shatanawi (2010)

The main contribution of the work of Shatanawi [6] is considering the fixed points for contractive mappings satisfying the Φ-map conditions in the setting of G-metric spaces.

The main result of Shatanawi [6] is the following.

Theorem 5.[6] *Let (X,G) be a complete G-metric space. Suppose the mapping $T : X \longrightarrow X$ satisfies*

$$G(Tx,Ty,Tz) \leq \phi(G(x,y,z)),$$

for all $x,y,z \in X$. Then T has a unique fixed point (say u), and T is G-continuous at u.

Theorem 6.[6] *Let (X,G) be a complete G-metric space. Suppose the mapping $T : X \longrightarrow X$ satisfies*

$$G(Tx,Ty,Tz) \leq \phi(\max\{G(x,y,z), G(x,Tx,Tx), G(y,Ty,Ty), G(Tx,y,z)\})$$

for all $x,y,z \in X$. Then T has a unique fixed point (say u), and T is G-continuous at u.

3.4 Manro, Bhatia and Kumar (2010)

Manro, Bhatia and Kumar [13] introduced some types of R-weakly commuting mappings in G-metric space and proved some related fixed point results.

Definition 13.[13] *A pair of self-mappings $\{f,g\}$ of a G-metric space (X,G) is said to be weakly commuting if*

$$G(fgx,gfx,gfx) \leq G(fx,gx,gx) \text{ for all } x \in X.$$

Definition 14.[13] *A pair of self-mappings $\{f,g\}$ of a G-metric space (X,G) is said to be R-weakly commuting if there exists some $R \in \mathbb{R}_+$ such that*

$$G(fgx,gfx,gfx) \leq RG(fx,gx,gx) \text{ for all } x \in X.$$

Remark.[13] If $R < 1$, then R-weakly commuting mappings are weakly commuting mappings.

Definition 15.[13] *A pair of self-mappings $\{f,g\}$ of a G-metric space (X,G) is said to be*

(i) R-weakly commuting mappings of type (A_f) if there exists some $R \in \mathbb{R}_+$ such that

$$G(fgx,ggx,ggx) \leq RG(fx,gx,gx) \text{ for all } x \in X.$$

(ii) R-weakly commuting mappings of type (A_g) if there exists some $R \in \mathbb{R}_+$ such that

$$G(gfx,ffx,ffx) \leq RG(fx,gx,gx) \text{ for all } x \in X.$$

(iii) R-weakly commuting mappings of type (P) if there exists some $R \in \mathbb{R}_+$ such that

$$G(ffx,ggx,ggx) \leq RG(fx,gx,gx) \text{ for all } x \in X.$$

Definition 16.[13] *Two self-mappings f and g are said to be weakly compatible if they commute at coincidence point.*

The main result of Manro et al. [13] is the following.

Theorem 7.[13] *Let f and g be weakly compatible self-mappings of a G-metric space(X,G) satisfying the following conditions:*

(i)$f(X) \subseteq g(X)$;
(ii)any one of the subspace $f(X)$ or $g(X)$ is complete;
(iii)$G(fx,fy,fz) \leq qG(gx,gy,gz)$ for all $x,y,z \in X$ and $0 \leq q < 1$.

Then f and g have a unique common fixed point in X.

Manro et al. [13] have shown that the above Theorem 7 holds if "weakly compatible property" is replaced by any one of the following:

(i)R-weakly commuting property;
(ii)R-weakly commuting property of type (A_f);
(iii)R-weakly commuting property of type (A_g);
(iv)R-weakly commuting property of type (P);
(v)Weakly commuting property.

3.5 Saadati, Vaezpour, Vetro, Rhoades (2010)

Saadati et al. [14] considered the concept of Ω-distance on a complete partially ordered G-metric spaces and proved some fixed point theorems.

Definition 17.[14] *Let (X,G) be a G-metric space. Then a function $\Omega : X \times X \times X \longrightarrow \mathbb{R}_+$ is called an Ω-distance on X if the following conditions are satisfied:*

(i)$\Omega(x,y,z) \leq \Omega(x,a,a) + \Omega(a,y,z)$ for all $x,y,z,a \in X$;
(ii)for any $x,y \in X$, $\Omega(x,y,\centerdot),\Omega(x,\centerdot,y) : X \longrightarrow \mathbb{R}_+$ are lower semi-continuous;
(iii)for each $\varepsilon > 0$, there exists $\delta > 0$ such that $\Omega(x,a,a) \leq \delta$ and $\Omega(a,y,z) \leq \delta$ implies that $G(x,y,z) \leq \varepsilon$.

Remark.[14] The set, X is said to be Ω-bounded if there exists a constant $M > 0$ such that $\Omega(x,y,z) \leq M$ for all x,y,zX.

Definition 18.[14] *Suppose $(X,\preceq)$ is a partially ordered set and $T : X \longrightarrow X$ is a mapping of X into itself. Then T is said to be non-decreasing if for $x,y \in X$, $x \preceq y$ implies $Tx \preceq Ty$.*

Theorem 8.[14] *Let $(X,\preceq)$ be a partially ordered set. Suppose that there exists a G-metric on X such that (X,G) is a complete G-metric space and Ω is an Ω-distance on X and T is a non-decreasing mapping from X into itself. Let X be Ω-bounded. Suppose that there exists $\lambda \in [0,1)$ such that*

$$\Omega(Tx,T^2x,Tw) \preceq \lambda\Omega(x,Tx,w),$$

for all $x \preceq Tx$ and $w \in X$. Also, for every $x \in X$,

$$\inf\{\Omega(x,y,x) + \Omega(x,y,Tx) + \Omega(x,T^2x,y) : x \preceq Tx\} > 0$$

for every $y \in X$ with $y \neq Ty$. If there exists $x_0 \in X$ with $x_0 \preceq Tx_0$, then T has a fixed point. Moreover, if $v = Tv$, then $\Omega(v,v,v) = 0$.

Theorem 9.[14] *Let $(X,\preceq)$ be a partially ordered set. Suppose that there exists a G-metric on X such that (X,G) is a complete G-metric space and Ω is an Ω-distance on X and T is a non-decreasing mapping from X into itself. Let X be Ω-bounded. Suppose that there exists $\lambda \in [0,1)$ such that*

$$\Omega(Tx,T^2x,Tw) \preceq \lambda\Omega(x,Tx,w)$$

for all $x \preceq Tx$ and $w \in X$. Assume that either of the following holds:

(i)if $y \neq Ty$, then $\inf\{\Omega(x,y,x) + \Omega(x,y,Tx) + \Omega(x,T^2x,y) : x \preceq Tx\} > 0$ for every $x \in X$;
(ii)if $\{x_n\}_{n\in\mathbb{N}}$ and $\{Tx_n\}$ converge to y and $\Omega(v,w,\centerdot) = \Omega(w,v,\centerdot)$ for every $v,w \in X$, then $y = Ty$;
(iii)T is continuous and $\Omega(v,w,\centerdot) = \Omega(w,v,\centerdot)$ for every $v,w \in X$.

If there exists $x_0 \in X$ with $x_0 \preceq Tx_0$, then T has a fixed point. Moreover, if $v = Tv$, then $\Omega(v,v,v) = 0$.

3.6 Shatanawi (2011)

Shatanawi [15] deployed the concept of coupled coincidence point and proved some results involving coupled coincidence fixed point in the setting of G-metric space.

Definition 19.[15] *An element $(x,y) \in X \times X$ is called a coupled fixed point of a mapping $F : X \times X \longrightarrow X$ if $F(x,y) = x$ and $F(y,x) = y$.*

Definition 20.[15] *An element $(x,y) \in X \times X$ is called a coupled coincidence point of the mappings $F : X \times X \longrightarrow X$ and $g : X \longrightarrow X$ if $F(x,y) = gx$ and $F(y,x) = gy$.*

Definition 21.[15] *Let X be a nonempty set. Then the mappings $F : X \times X \longrightarrow X$ and $g : X \longrightarrow X$ are said to be commutative if $gF(x,y) = F(gx,gy)$.*

The main result of Shatanawi [15] is the following.

Theorem 10.[15] *Let (X,G) be a G-metric space. Let $F : X \times X \longrightarrow X$ and $g : X \longrightarrow X$ be two mappings such that*

$$G(F(x,y),F(u,v),F(z,w)) \leq \lambda(G(gx,gu,gz) + G(gy,gv,gw)),$$

for all $x,y,z,w,u,v \in X$ where $\lambda \in (0,\frac{1}{2})$. Assume that F and g satisfy the following conditions:

(i)$F(X \times X) \subseteq g(X)$;
(ii)$g(X)$ is G-complete;
(iii)g is G-continuous and commutes with F.

Then there is a unique x in X such that $gx = F(x,x) = x$.

3.7 Choudhury and Maity (2011)

Choudhury and Maity [17] established coupled fixed point theorems in a partially ordered G-metric space.

Definition 22.[17] *Let $(X,\preceq)$ be a partially ordered set. A mapping $F: X\times X \longrightarrow X$ is said to have mixed monotone property if $F(x,y)$ is monotone non-decreasing in x and is monotone non-increasing in y, that is, for any $x,y\in X$, $x_1,x_2\in X$, $x_1\preceq x_2$ implies $F(x_1,y)\preceq F(x_2,y)$ and $y_1,y_2\in X$, $y_1\preceq y_2$ implies $F(x,y_2)\preceq F(x,y_1)$.*

Remark.[17] Let $(X,\preceq)$ denote a partially ordered set with the partial order $\preceq$. By "$x\succeq y$ holds," we mean that "$y\preceq x$ holds" and by "$x\prec y$ holds" we mean that "$x\preceq y$ holds and $x\neq y$".

The main result of Choudhury and Maity [17] is the following.

Theorem 11.[17] *Let $(X,\preceq)$ be a partially ordered set and G be a G-metric on X such that (X,G) is a complete G-metric space. Let $F: X\times X\longrightarrow X$ be a continuous mapping having the mixed monotone property on X. Assume that there exists $\lambda\in[0,1)$ such that for $x,y,z,u,v,w\in X$, the following holds:*

$$G(F(x,y),F(u,v),F(w,z))\leq \frac{\lambda}{2}[G(x,u,w)+G(y,v,z)],$$

for all $x\succeq u\succeq w$ and $y\preceq v\preceq z$ where either $u\neq w$ or $v\neq z$. If there exist $x_0,y_0\in X$ such that $x_0\preceq F(x_0,y_0)$ and $y_0\succeq F(y_0,x_0)$, then F has a coupled fixed point in X, that is, there exist $x,y\in X$ such that $x=F(x,y)$ and $y=F(y,x)$.

Theorem 12.[17] *If in the above Theorem 11, in place of the continuity of F, we assume the following conditions:*

(i) if a non-decreasing sequence $x_n\longrightarrow x$, then $x_n\preceq x$ for all n;
(ii) if a non-increasing sequence $y_n\longrightarrow y$, then $y_n\succeq y$ for all n.

Then F has a coupled fixed point.

Theorem 13.[17] *Let $(X,\preceq)$ be a partially ordered set and G be a G-metric on X such that (X,G) is a complete G-metric space. Let $F: X\times X\longrightarrow X$ be a continuous mapping having the mixed monotone property on X and such that $F(x,y)\preceq F(y,x)$ whenever $x\preceq y$. Assume that there exists $\lambda\in[0,1)$ such that for $x,y,z,u,v,w\in X$, the following holds:*

$$G(F(x,y),F(u,v),F(w,z))\leq \frac{\lambda}{2}[G(x,u,w)+G(y,v,z)],$$

for all $x\succeq u\succeq w$, $y\preceq v\preceq z$ and $x\prec y$ where either $u\neq w$ or $v\neq z$. If there exist $x_0,y_0\in X$ such that $x_0\preceq y_0$, $x_0\preceq F(x_0,y_0)$ and $y_0\succeq F(y_0,x_0)$, then F has a coupled fixed point in X, that is, there exist $x,y\in X$ such that $x=F(x,y)$ and $y=F(y,x)$.

3.8 Aydi, Damjanović, Samet, Shatanawi (2011)

Aydi et al. [16] established coupled coincidence and coupled common fixed point results for a mixed g-monotone mapping satisfying nonlinear contractions in partially ordered G-metric spaces. Their results generalize the work of Choudhury and Maity [17].

Definition 23.[16] *Let $(X,\preceq)$ be a partially ordered set. Let us consider mappings $F: X\times X\longrightarrow X$ and $g: X\longrightarrow X$. The mapping F is said to have mixed g-monotone property if $F(x,y)$ is monotone g-non-decreasing in x and is monotone g-non-increasing in y, that is, for any $x,y\in X$, $x_1,x_2\in X$, $gx_1\preceq gx_2$ implies $F(x_1,y)\preceq F(x_2,y)$ and $y_1,y_2\in X$, $gy_1\preceq gy_2$ implies $F(x,y_2)\preceq F(x,y_1)$.*

Let $\phi\in\Phi$ such that $\lim_{r\to t^+}\phi(r)<t$ for all $t\in(0,\infty)$.

We now present the main result of Aydi et al. [16] as follows.

Theorem 14.[16] *Let $(X,\preceq)$ be a partially ordered set and G be a G-metric on X such that (X,G) is a complete G-metric space. Suppose that there exist $\phi\in\Phi$, $F: X\times X\longrightarrow X$ and $g: X\longrightarrow X$ such that*

$$G(F(x,y),F(u,v),F(w,z))\leq \phi\left(\frac{1}{2}[G(gx,gu,gw)+G(gy,gv,gz)]\right) \tag{10}$$

for all $x,y,u,v,w,z\in X$ with $gw\preceq gu\preceq gx$ and $gy\preceq gv\preceq gz$. Suppose also that F is continuous and has the mixed g-monotone property, $F(X\times X)\subseteq g(X)$ and g is continuous and commutes with F. If there exist $x_0,y_0\in X$ such that $gx_0\preceq F(x_0,y_0)$ and $F(y_0,x_0)\preceq gy_0$, then F and g have a coupled coincidence point. That is, there exists $(x,y)\in X\times X$ such that $gx=F(x,y)$ and $gy=F(y,x)$.

Corollary 2.[16] *Let $(X,\preceq)$ be a partially ordered set and G be a G-metric on X such that (X,G) is a complete G-metric space. Suppose that there exist $\lambda\in[0,1)$, $F: X\times X\longrightarrow X$ and $g: X\longrightarrow X$ such that*

$$G(F(x,y),F(u,v),F(w,z))\leq \frac{\lambda}{2}\left(G(gx,gu,gw)+G(gy,gv,gz)\right),$$

for all $x,y,u,v,w,z\in X$ with $gw\preceq gu\preceq gx$ and $gy\preceq gv\preceq gz$. Suppose also that F is continuous and has the mixed g-monotone property, $F(X\times X)\subseteq g(X)$ and g is continuous and commutes with F. If there exist $x_0,y_0\in X$ such that $gx_0\preceq F(x_0,y_0)$ and $F(y_0,x_0)\preceq gy_0$, then F and g have a coupled coincidence point.

Remark.[16] Taking $g=I_X$ (the identity mapping), we see that Corollary 2 coincides with Theorem 11 due to Choudhury and Maity [17].

Definition 24.[16] *Let $(X,\preceq)$ be a partially ordered set and G be a G-metric on X. Then $(X,G,\preceq)$ is said to be regular if the following conditions hold:*

(i)if a non-decreasing sequence $x_n \longrightarrow x$, then $x_n \preceq x$ for all n;
(ii)if a non-increasing sequence $y_n \longrightarrow y$, then $y \preceq y_n$ for all n.

Theorem 15.[16] *Let $(X,\preceq)$ be a partially ordered set and G be a G-metric on X such that $(X,G,\preceq)$ is regular. Suppose that there exist $\phi \in \Phi$, $F : X\times X \longrightarrow X$ and $g : X \longrightarrow X$ such that*

$$G(F(x,y),F(u,v),F(w,z)) \leq \phi\left(\frac{1}{2}[G(gx,gu,gw)+G(gy,gv,gz)]\right),$$

for all $x,y,u,v,w,z \in X$ with $gw \preceq gu \preceq gx$ and $gy \preceq gv \preceq gz$. Suppose also that $(g(X),G)$ is complete, F has the mixed g-monotone property and $F(X\times X) \subseteq g(X)$. If there exist $x_0,y_0 \in X$ such that $gx_0 \preceq F(x_0,y_0)$ and $F(y_0,x_0) \preceq gy_0$, then F and g have a coupled coincidence point.

3.9 Abbas, Nazir and Vetro (2011)

The authors proved some common fixed point theorems for three mappings defined on G-metric spaces.

Theorem 16.[18] *Let f, g and h be self-mappings on a complete G-metric space (X,G) satisfying*

$$\begin{aligned}G(fx,gy,hz) \leq & aG(x,y,z)+b[G(fx,x,x)\\ &+G(y,gy,y)+G(z,z,hz)]\\ &+c[G(fx,y,z)+G(x,gy,z)+G(x,y,hz)],\end{aligned}$$

for all $x,y,z \in X$, where $a,b,c>0$ and $a+3b+4c<1$. Then f, g and h have a unique common fixed point in X. Moreover, any fixed point of f is a fixed point of g and h and conversely.

Theorem 17.[18] *Let f, g and h be self-mappings on a complete G-metric space (X,G) satisfying*

$$\begin{aligned}G(fx,gy,hz) \leq & aG(x,y,z)+bG(x,fx,fx)\\ &+cG(y,gy,gy)+dG(z,hz,hz)\end{aligned}$$

for all $x,y,z \in X$, where $0<a+b+c+d<1$. Then f, g and h have a unique common fixed point in X. Moreover, any fixed point of f is a fixed point of g and h and conversely.

Theorem 18.[18] *Let f, g and h be self-mappings on a complete G-metric space (X,G) satisfying*

$$G(fx,gy,hz) \leq \lambda[G(x,fx,fx)+G(y,gy,gy)+G(z,hz,hz)]$$

for all $x,y,z \in X$, where $0<\lambda<\frac{1}{3}$. Then f, g and h have a unique common fixed point in X. Moreover, any fixed point of f is a fixed point of g and h and conversely.

3.10 Abbas, Khan and Nazir (2011)

Abbas et al. [19] proved the existence of unique common fixed point for four R-weakly commuting mappings in G-metric spaces. Their main result is the following.

Theorem 19.[19] *Let (X,G) be a complete G-metric space. Suppose that $\{f,S\}$ and $\{g,T\}$ are point-wise R-weakly commuting pairs of self-mappings on X satisfying*

$$G(fx,fx,gy) \leq \lambda \max\left\{\begin{array}{c}G(Sx,Sx,Ty),G(fx,fx,Sx),G(gy,gy,Ty),\\ \frac{1}{2}(G(fx,fx,Ty)+G(gy,gy,Sx))\end{array}\right\}$$

and

$$G(fx,gy,gy) \leq \lambda \max\left\{\begin{array}{c}G(Sx,Ty,Ty),G(fx,Sx,Sx),G(gy,Ty,Ty),\\ \frac{1}{2}(G(fx,Ty,Ty)+G(gy,Sx,Sx))\end{array}\right\},$$

for all $x,y \in X$, where $\lambda \in [0,1)$. Suppose that $fX \subseteq TX$, $gX \subseteq SX$, and one of the pair $\{f,S\}$ or $\{g,T\}$ is compatible. If the mappings in the compatible pair are continuous, then f, g, S and T have a unique common fixed point.

3.11 Aydi (2011)

Aydi [5] endowed the G-metric space with the notion of (ψ,φ)-weakly contractive conditions and proved some fixed point results.

Theorem 20.[5] *Let (X,G) be a complete G-metric space. Suppose the mapping $T : X \longrightarrow X$ satisfies for all $x,y,z \in X$,*

$$\psi(G(Tx,Ty,Tz)) \leq \psi(G(x,y,z))-\varphi(G(x,y,z)),$$

where ψ and φ are altering distance functions. Then T has a unique fixed point (say u) and T is G-continuous at u.

As a corollary, Aydi [5] showed that Theorem 1 can be proved using the (ψ,φ)-weakly contractive condition by setting $\psi(t)=t$ and $\varphi(t)=1-\lambda$ for $\lambda \in [0,1)$.

3.12 Aydi, Shatanawi and Vetro (2011)

Aydi, Shatanawi and Vetro [20] established some common fixed point results for two self-mappings on G-metric spaces.

Definition 25.[20] *Let (X,G) be a G-metric space and $f,g : X \longrightarrow X$ be two mappings. Then f is said to be a generalized weakly G-contraction mapping of type (A) with respect to g if for all $x,y,z \in X$, the following inequality holds:*

$$\begin{aligned}\psi(G(fx,fy,fz)) \leq & \psi\left(\frac{1}{3}[G(gx,fy,fy)+G(gy,fz,fz)+G(gz,fx,fx)]\right)\\ &-\lambda\left(G(gx,fy,fy),G(gy,fz,fz),G(gz,fx,fx)\right),\end{aligned}$$

where

(i) ψ *is an altering distance function;*
(ii) $\lambda : [0,\infty)^3 \longrightarrow [0,\infty)$ *is a continuous function with* $\lambda(t,s,u) = 0$ *if and only if* $t = s = u = 0$.

Theorem 21.[20] *Let* (X,G) *be a G-metric space and* $f,g : X \longrightarrow X$ *be two mappings such that* f *is a generalized weakly G-contraction mapping of type (A) with respect to g. Assume that* $f(X) \subseteq g(X)$, $g(X)$ *is a complete subset of* (X,G) *and the pair* $\{f,g\}$ *is weakly compatible. Then* f *and* g *have a unique common fixed point.*

Definition 26.[20] *Let* (X,G) *be a G-metric space and* $f,g : X \longrightarrow X$ *be two mappings. Then* f *is said to be a generalized weakly G-contraction mapping of type (B) with respect to* g *if for all* $x,y,z \in X$, *the following inequality holds:*

$$\begin{aligned}&\psi(G(fx,fy,fz))\\&\leq \psi\left(\frac{1}{3}[G(gx,gx,fy)+G(gy,gy,fz)+G(gz,gz,fx)]\right)\\&\quad -\lambda\left(G(gx,gx,fy),G(gy,gy,fz),G(gz,gz,fx)\right),\end{aligned}$$

where

(i) ψ *is an altering distance function;*
(ii) $\lambda : [0,\infty)^3 \longrightarrow [0,\infty)$ *is a continuous function with* $\lambda(t,s,u) = 0$ *if and only if* $t = s = u = 0$.

Theorem 22.[20] *Let* (X,G) *be a G-metric space and* $f,g : X \longrightarrow X$ *be two mappings such that* f *is a generalized weakly G-contraction mapping of type (B) with respect to g. Assume that* $f(X) \subseteq g(X)$, $g(X)$ *is a complete subset of* (X,G) *and the pair* $\{f,g\}$ *is weakly compatible. Then* f *and* g *have a unique common fixed point.*

3.13 Shatanawi (2011)

Shatanawi [21] proved some fixed point results for two weakly increasing partially ordered mappings in G-metric spaces.

Definition 27.[21] *Let* $(X,\preceq)$ *be a partially ordered set. Two mappings* $F,g : X \longrightarrow X$ *are said to be weakly increasing if* $Fx \preceq gFx$ *and* $gx \preceq Fgx$, *for all* $x \in X$.

The main result of Shatanawi [21] is the following.

Theorem 23.[21] *Let* $(X,\preceq)$ *be a partially ordered set and suppose that there exists G-metric in X such that* (X,G) *is G-complete. Let* $f,g : X \longrightarrow X$ *be two weakly increasing mappings with respect to* $\preceq$. *Suppose there exist non-negative real numbers a, b, and c with* $a+2b+2c < 1$ *such that*

$$\begin{aligned}G(fx,gy,gy) \leq& aG(x,y,y)+b[G(x,fx,fx)+G(y,gy,gy)]\\&+c[G(x,gy,gy)+G(y,fx,fx)],\\G(gx,fy,fy) \leq& aG(x,y,y)+b[G(x,gx,gx)+G(y,fy,fy)]\\&+c[G(x,fy,fy)+G(y,gx,gx)],\end{aligned}$$

for all comparative $x,y \in X$. *If* f *or* g *is continuous, then* f *and* g *have a common fixed point* $u \in X$.

Theorem 24.[21] *Let* $(X,\preceq)$ *be a partially ordered set and suppose that there exists G-metric in X such that* (X,G) *is G-complete. Let* $f,g : X \longrightarrow X$ *be two weakly increasing mappings with respect to* $\preceq$. *Suppose there exist non-negative real numbers a, b, and c with* $a+2b+2c < 1$ *such that*

$$\begin{aligned}G(fx,gy,gy) \leq& aG(x,y,y)+b[G(x,fx,fx)+G(y,gy,gy)]\\&+c[G(x,gy,gy)+G(y,fx,fx)],\\G(gx,fy,fy) \leq& aG(x,y,y)+b[G(x,gx,gx)+G(y,fy,fy)]\\&+c[G(x,fy,fy)+G(y,gx,gx)],\end{aligned}$$

for all comparative $x,y \in X$. *Assume X has the property that if* $\{x_n\}_{n\in\mathbb{N}}$ *is an increasing sequence and converges to x in X, then* $x_n \preceq x$ *for all* $n \in \mathbb{N}$. *Then f and g have a common fixed point* $u \in X$.

3.14 Rao, Sombabu and Rajendra Prasad (2011)

Rao et al. [22] obtained unique common fixed point for six expansive mappings in G-metric spaces. Their main result is the following.

Theorem 25.[22] *Let* (X,G) *be a complete G-metric space and* $S,T,R,f,g,h : X \longrightarrow X$ *be mappings such that*

$$G(Sx,Ty,Rz) \geq \lambda \max\left\{\begin{array}{c}G(fx,gy,hz),G(fx,Sx,Rz),\\G(gy,Ty,Sx),G(hz,Rz,Ty)\end{array}\right\},$$

for all $x,y,z \in X$ *and* $\lambda > 1$. *If:*

(i) $h(X) \subseteq S(X)$, $f(X) \subseteq T(X)$, $g(X) \subseteq R(X)$;
(ii)one of $f(X)$, $g(X)$ *and* $h(X)$ *is a G-complete subspace of* (X,G);
(iii)the pairs $\{f,S\}$, $\{g,T\}$ *and* $\{h,R\}$ *are weakly compatible,*

then

(i)one of the pairs $\{f,S\}$, $\{g,T\}$ *and* $\{h,R\}$ *has a coincidence point in X or;*
(ii)S, T, R, f, g and h have a unique common fixed point in X.

Theorem 26.[22] *Let* (X,G) *be a complete G-metric space and* $S,T,R,f,g,h : X \longrightarrow X$ *be mappings such that*

$$G(Sx,Ty,Rz) \geq \lambda \min\left\{\begin{array}{c}G(fx,gy,hz),G(fx,Sx,Rz),\\G(gy,Ty,Sx),G(hz,Rz,Ty)\end{array}\right\}$$

or

$$G(Sx,Ty,Rz) \geq \lambda G(fx,gy,hz),$$

for all $x,y,z \in X$ *and* $\lambda > 1$. *If:*

(i) $h(X) \subseteq S(X)$, $f(X) \subseteq T(X)$, $g(X) \subseteq R(X)$;
(ii) one of $f(X)$, $g(X)$ and $h(X)$ is a G-complete subspace of (X,G);
(iii) the pairs $\{f,S\}$, $\{g,T\}$ and $\{h,R\}$ are weakly compatible,

then

(i) one of the pairs $\{f,S\}$, $\{g,T\}$ and $\{h,R\}$ has a coincidence point in X or;
(ii) S, T, R, f, g and h have a unique common fixed point in X.

3.15 Abbas, Nazır, Shatanawi, Mustafa (2012)

Abbas et al. [23] obtained unique common fixed points of three mappings that satisfy a generalized (ψ,φ)-weak contractive condition.

Theorem 27. *[23] Let f, g and h be self-mappings on a complete G-metric space (X,G) satisfying*

$$\psi(G(fx,gy,hz)) \le \psi(M(x,y,z)) - \varphi(M(x,y,z)),$$

where $\psi \in \Psi$, $\varphi \in \Upsilon$ and

$$M(x,y,z) = \max\left\{\begin{array}{c} G(x,y,z), G(x,x,fx), G(y,y,gy), G(z,z,hz), \\ G(x,fx,gy), G(y,gy,hz), G(z,hz,fx) \end{array}\right\},$$

for all $x,y,z \in X$. Then f, g and h have a unique common fixed point in X. Moreover, any fixed point of f is a fixed point of g and h and conversely.

Theorem 28. *[23] Let f, g and h be self-mappings on a complete G-metric space (X,G) satisfying*

$$\psi(G(fx,gy,hz)) \le \psi(M(x,y,z)) - \varphi(M(x,y,z)),$$

where $\psi \in \Psi$, $\varphi \in \Upsilon$ and

$$M(x,y,z) = \max\{G(x,y,z), G(x,fx,fx), G(y,gy,gy), G(z,hz,hz)\},$$

for all $x,y,z \in X$. Then f, g and h have a unique common fixed point in X. Moreover, any fixed point of f is a fixed point of g and h and conversely.

3.16 Aydi, Postolache and Shatanawi (2012)

The authors [24] established coupled coincidence and common coupled fixed point theorems for (ψ,φ)-weakly contractive mappings in ordered G-metric spaces.

Theorem 29. *[24] Let $(X,\preceq)$ be partially ordered set and (X,G) be complete G-metric space. Let F and g be (ψ,φ)-weakly contractive mappings on X, with $gx \preceq gu \preceq gs$ and $gt \preceq gv \preceq gy$, or $gs \preceq gu \preceq gx$ and $gy \preceq gv \preceq gt$. Assume that F and g satisfy the following conditions:*

(i) $F(X \times X) \subseteq g(X)$;
(ii) F has the mixed g-monotone property;
(iii) F is continuous;
(iv) g is continuous and commutes with F.

If there exist $x_0, y_0 \in X$ such that $gx_0 \preceq F(x_0,y_0)$ and $F(y_0,x_0) \preceq gy_0$, then F and g have a coupled coincidence point.

Theorem 30. *[24] Let $(X,\preceq)$ be partially ordered set and G be a G-metric on X. Let F and g be (ψ,φ)-weakly contractive mappings on X, with $gx \preceq gu \preceq gs$ and $gt \preceq gv \preceq gy$, or $gs \preceq gu \preceq gx$ and $gy \preceq gv \preceq gt$. Assume that $(g(X),G)$ is complete, F has the mixed g-monotone property and $F(X \times X) \subseteq g(X)$. Also assume the following conditions:*

(i) if a non-decreasing sequence $x_n \longrightarrow x$, then $x_n \preceq x$ for all n;
(ii) if a non-increasing sequence $y_n \longrightarrow y$, then $y \preceq y_n$ for all n.

If there exist $x_0, y_0 \in X$ such that $gx_0 \preceq F(x_0,y_0)$ and $F(y_0,x_0) \preceq gy_0$, then F and g have a coupled coincidence point.

3.17 Erduran and Altun (2012)

Erduran and Altun [25] proved some fixed point theorems for φ-weakly contractive mappings on complete G-metric space. Further, they proved some homotopy result.

Theorem 31. *[25] Let (X,G) be a complete G-metric space and $T: X \longrightarrow X$ be a function such that for all $x,y,z \in X$,*

$$G(Tx,Ty,Tz) \le M(x,y,z) - \varphi(M(x,y,z)),$$

where $\varphi \in \Upsilon$ and

$$M(x,y,z) = \max\{G(x,y,z), G(x,Tx,Tx), G(y,Ty,Ty), G(z,Tz,Tz)\}.$$

Then there exists a unique point $u \in X$ such that $Tu = u$.

Theorem 32. *[25] Let (X,G) be a G-complete metric space and U be an open subset of X. Suppose that $H: \overline{U} \times [0,1] \longrightarrow X$ and:*

(i) $x \neq H(x,a)$ for every $x \in \zeta U$ and $t \in [0,1]$ (here ζU denotes the boundary of U in X);
(ii) for all $x,y,z \in U$ and $a \in [0,1]$, $\lambda \in (0,1)$ such that

$$G(H(x,a),H(y,a),H(z,a)) \le \lambda G(x,y,z);$$

(iii) there exists $M \ge 0$ such that

$$G(H(x,a),H(x,b),H(x,b)) \le M|a-b|,$$

for every $x \in \overline{U}$ and $a,b \in [0,1]$.

If $H(\cdot,0)$ has a fixed point in U, then $H(\cdot,1)$ has a fixed point in U.

3.18 Gugnani, Aggarwal and Chugh (2012)

Gugnani et al. [26] obtained a common fixed point result using EA-property for four weakly compatible mappings in the setting of G-metric spaces without exploiting the notion of continuity.

Definition 28.[26] *Let $\{T_i\}$ be a sequence of self-mappings of a G-metric space X. We say that this family has property R if $\bigcap_i F\{T_i\} = \bigcap_i F(T_i^n)$.*

Definition 29.[26] *Let (X,G) be a G-metric space and S and T be two self-mappings of X. Then S and T are said to satisfy the EA-property if there exists a sequence $\{x_n\}_{n\in\mathbb{N}}$ such that*

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t \text{ for some } t \in X.$$

The main result of Gugnani et al. [26] is the following.

Theorem 33.[26] *Let (X,G) be a G-metric space. Suppose the mappings $A,B,S,T : X \longrightarrow X$ satisfy*

$$G(Sx,Ty,Ty) \leq \phi(G(Ax,By,By)) \quad or$$
$$G(Sx,Sx,Ty) \leq \phi(G(Ax,Ax,By)),$$

for all $x,y \in X$, where $\phi \in \Phi$. If the mappings A, B, S and T satisfy the following conditions:

(i)$\overline{TX} \subseteq AX$ and $\overline{SX} \subseteq BX$;
(ii)the pair $\{A,S\}$ or $\{B,T\}$ satisfies EA-property;
(iii)the pair $\{A,S\}$ or $\{B,T\}$ are weakly compatible,

then A, B, S and T have a unique common fixed point.

Corollary 3.[26] *Let (X,G) be a G-metric space. Suppose the mappings $A,B,S,T : X \longrightarrow X$ satisfies*

$$G(Sx,Ty,Ty) \leq \lambda G(Ax,By,By) \quad or$$
$$G(Sx,Sx,Ty) \leq \lambda G(Ax,Ax,By),$$

for all $x,y \in X$, where $\lambda \in [0,1)$. If the mappings A, B, S and T satisfy the following conditions:

(i)$\overline{TX} \subseteq AX$ and $\overline{SX} \subseteq BX$;
(ii)the pair $\{A,S\}$ or $\{B,T\}$ satisfies EA-property;
(iii)the pair $\{A,S\}$ or $\{B,T\}$ are weakly compatible,

then A, B, S and T have a unique common fixed point.

3.19 Mustafa, Aydi and Karapınar (2012)

Mustafa et al. [27] introduced some new types of mappings on G-metric spaces called G-weakly commuting of type (G_f) and G-R-weakly commuting of type (G_f) and proved some related fixed point results.

Definition 30.[27] *A pair of self-mappings $\{f,g\}$ of a G-metric space (X,G) is said to be G-weakly commuting of type (G_f) if*

$$G(fgx,gfx,ffx) \leq G(fx,gx,fx) \text{ for all } x \in X.$$

Definition 31.[27] *A pair of self-mappings $\{f,g\}$ of a G-metric space (X,G) is said to be G-R-weakly commuting of type (G_f) if there exists a positive real number R such that*

$$G(fgx,gfx,ffx) \leq RG(fx,gx,fx) \text{ for all } x \in X.$$

Definition 32.[27] *Let (X,G) be a G-metric space and let $T : X \longrightarrow X$ be a mapping. For $A \subset X$, let $\delta(A) = \sup\{G(a,b,c), a,b,c \in A\}$ and for all $x,y,z \in X$, define*

(i)$\mathscr{O}(x,T,n) = \{x,Tx,T^2x,\ldots,T^nx\}$;
(ii)$\mathscr{O}(x,T,\infty) = \{x,Tx,T^2x,T^3x,\ldots\}$.

Definition 33.[27] *Let $\{x_n\}_{n=0}^{\infty}$ be a sequence of elements of X. Then for i, j, let*

(i)$\mathscr{O}(x_i,j) = \{x_i,x_{i+1},x_{i+2},\ldots,x_{i+j}\}$;
(ii)$\mathscr{O}(x_i,\infty) = \{x_i,x_{i+1},x_{i+2},x_{i+3},\ldots\}$.

Theorem 34.[27] *Let (X,G) be a G-metric space. Suppose the mappings $f,g : X \longrightarrow X$ satisfy the following conditions:*

(i)f and g be G-weakly commuting of type (G_f);
(ii)$f(X) \subseteq g(X)$;
(iii)$g(X)$ is G-complete subspace of X;
(iv)$G(fx,fy,fz) \leq \phi(M(x,y,z))$, for all $x,y,z \in X$, where $\phi \in \Phi$ and

$$M(x,y,z) = \max\left\{\begin{array}{c} G(gx,gy,gz), G(gx,fy,gx), \\ G(gy,fx,gy), G(gz,fx,gz), \\ G(gz,fy,gz), G(gy,fz,gy), \\ G(gx,fz,gx) \end{array}\right\}.$$

If there exists $x_0 \in X$ such that $\delta(\mathscr{O}(x_0,f,\infty)) < \infty$, then f and g have a unique common fixed point.

Theorem 35.[27] *Let (X,G) be a G-metric space. Suppose the mappings $f,g : X \longrightarrow X$ are G-weakly commuting of type (G_f) and satisfy the following conditions:*

(i)f and g satisfy EA-property;
(ii)$g(X)$ is a closed subspace of X;
(iii)$G(fx,fy,fz) \leq \phi(M(x,y,z))$, for all $x,y,z \in X$, where $\phi \in \Phi$ and

$$M(x,y,z) = \max\left\{\begin{array}{c} G(gx,fy,fy), G(gx,fz,fz), \\ G(gy,fx,fx), G(gz,fx,fx), \\ G(gz,fy,fy), G(gy,fz,fz) \end{array}\right\}.$$

Then f and g have a unique common fixed point.

3.20 Mustafa (2012)

Mustafa [28] introduced some new types of pairs of mappings on G-metric space and obtained several common fixed point results for these mappings under certain contractive conditions.

Definition 34.[28] *A pair of self-mappings $\{f,g\}$ of a G-metric space (X,G) is said to be G-weakly commuting of type (A_f) if*

$$G(fgx,ggx,ffx) \le G(fx,gx,fx) \text{ for all } x\in X.$$

Definition 35.[28] *A pair of self-mappings $\{f,g\}$ of a G-metric space (X,G) is said to be G-R-weakly commuting of type (A_f) if there exists a positive real number R such that*

$$G(fgx,ggx,ffx) \le RG(fx,gx,fx) \text{ for all } x\in X.$$

Theorem 36.[28] *Let (X,G) be a G-metric space. Suppose the mappings $f,g : X \longrightarrow X$ satisfy the following conditions:*

(i) f and g be G-weakly commuting of type (A_f);
(ii) $f(X) \subseteq g(X)$;
(iii) $g(X)$ is G-complete subspace of X;
(iv) $G(fx,fy,fz) \le \phi(M(x,y,z))$, for all $x,y,z\in X$, where $\phi\in\Phi$ and

$$M(x,y,z) = \max\left\{\begin{array}{c} G(gx,gy,gz), G(gx,fx,fx), \frac{1}{2}G(gx,fy,fy),\\ \frac{1}{2}G(gx,fz,fz), G(gy,fy,fy), G(gy,fx,fx),\\ G(gy,fz,fz), G(gz,fz,fz), G(gz,fx,fx),\\ G(gz,fy,fy)\end{array}\right\}.$$

Then f and g have a unique common fixed point.

Theorem 37.[28] *Let (X,G) be a G-metric space. Suppose the mappings $f,g : X \longrightarrow X$ are G-weakly commuting of type (A_g) and satisfy the following conditions:*

(i) f and g satisfy EA-property;
(ii) $g(X)$ is a closed subspace of X;
(iii) $G(fx,fy,fz) \le \lambda M(x,y,z)$, for all $x,y,z\in X$, $\lambda\in[0,\frac{1}{3})$, where

$$M(x,y,z) = \max\left\{\begin{array}{c} (G(gx,fx,fx)+G(gy,fy,fy)+G(gz,fz,fz)),\\ (G(gx,fy,fy)+G(gy,fx,fx)+G(gz,fy,fy)),\\ (G(gx,fz,fz)+G(gy,fz,fz)+G(gz,fx,fx))\end{array}\right\}.$$

Then f and g have a unique common fixed point.

Theorem 38.[28] *Let (X,G) be a G-metric space and suppose the mappings $f,g : X \longrightarrow X$ are G-R-weakly commuting of type (A_f). Suppose that there exists a mapping $\mu : X \longrightarrow \mathbb{R}_+$ such that:*

(i) $f(X) \subset g(X)$;
(ii) $g(X)$ is a G-complete subspace of X;
(iii) $G(gx,fx,fx) < \mu(g(x)) - \mu(f(x))$, for all $x\in X$ and

$$G(fx,fy,fz) < \max\left\{\begin{array}{c} G(gx,gy,gz), G(gx,fx,gy),\\ G(gz,fz,fx), G(gy,fy,fz)\end{array}\right\},$$

for all $x,y,z\in X$, then f and g have a unique common fixed point.

3.21 Popa and Patriciu (2012)

Popa and Patriciu [29] proved some fixed point theorems satisfying implicit relations.

Definition 36.[29] *Let $\phi\in\Phi$ and F_ϕ be the set of all continuous functions $F(t_1,\dots,t_6) : [0,\infty)^6 \longrightarrow \mathbb{R}$ such that:*

(i) F is non-increasing in t_5;
(ii) there exists a function $\phi_1\in\Phi$ such that for all $u,v\ge 0$, $F(u,v,v,u,u+v,0)\le 0$ implies $u\le\phi_1(v)$;
(iii) there exists a function $\phi_2\in\Phi$ such that for all $t,t'>0$, $F(t,t,0,0,t,t')\le 0$ implies $t\le\phi_2(t')$.

The main result of Popa and Patriciu [29] is the following.

Theorem 39.[29] *Let (X,G) be a G-metric space. Suppose that*

$$F(G(Tx,Ty,Ty), G(x,y,y), G(x,Tx,Tx), G(y,Ty,Ty), G(x,Ty,Ty), G(y,Tx,Tx)) \le 0,$$

for all $x,y\in X$, where F satisfies condition (iii) above. Then T has at most a fixed point.

Theorem 40.[29] *Let (X,G) be a G-metric space. Suppose that*

$$F(G(Tx,Ty,Ty), G(x,y,y), G(x,Tx,Tx), G(y,Ty,Ty), G(x,Ty,Ty), G(y,Tx,Tx)) \le 0,$$

for all $x,y\in X$ and $F\in F_\phi$. Then T has at most a fixed point.

3.22 Shatanawi and Postolache (2012)

Shatanawi and Postolache [30] introduced the concepts of a G-weak contraction mapping of types A and B and established some related fixed point theorems.

Definition 37.[30] *Let (X,G) be a G-metric space. A mapping $T : X \longrightarrow X$ is called a G-weak contraction of type (A) if and only if there exist two constants $\lambda\in(0,1)$ and $L\ge 0$ such that*

$$G(Tx,Ty,Ty) \le \lambda M(x,y,y) + LN(x,y,y),$$

for all $x,y\in X$, where

$$M(x,y,y) = \max\left\{\begin{array}{c} G(x,y,y), G(x,Tx,Tx), G(y,Ty,Ty),\\ \frac{G(x,Ty,Ty)+G(y,Ty,Ty)+G(y,Tx,Tx)}{3}\end{array}\right\},$$

$$N(x,y,y) = \min\left\{ G(x,Tx,Tx), G(y,Ty,Ty), G(y,Tx,Tx)\right\}.$$

Definition 38.[30] *Let (X,G) be a G-metric space and let ψ,ϕ,φ be altering distance functions. A mapping $T:X\longrightarrow X$ is called a (ψ,ϕ,φ,G)-weak contraction of type (A) if and only if there exists a constant $L\geq 0$ such that*

$$\psi(G(Tx,Ty,Ty))\leq\psi(M(x,y,y))-\phi(M_1(x,y,y)) +L\varphi(N(x,y,y)),$$

for all $x,y\in X$, where

$$M(x,y,y)=\max\left\{\begin{array}{c}G(x,y,y),G(x,Tx,Tx),G(y,Ty,Ty),\\ \frac{G(x,Ty,Ty)+G(y,Ty,Ty)+G(y,Tx,Tx)}{3}\end{array}\right\},$$

$$M_1(x,y,y)=\max\left\{G(x,y,y),G(y,Ty,Ty)\right\},$$

$$N(x,y,y)=\min\left\{G(x,Tx,Tx),G(y,Ty,Ty),G(y,Tx,Tx)\right\}.$$

The main result of Shatanawi and Postolache [30] is the following.

Theorem 41.[30] *Let (X,G) be a complete G-metric space and let $T:X\longrightarrow X$ be a (ψ,ϕ,φ,G)-weak contraction of type (A). Then T has a unique fixed point.*

Definition 39.[30] *Let (X,G) be a G-metric space. A mapping $T:X\longrightarrow X$ is called a G-weak contraction of type (B) if and only if there exist two constants $\lambda\in(0,1)$ and $L\geq 0$ such that*

$$G(Tx,Tx,Ty)\leq\lambda m(x,x,y)+Ln(x,x,y),$$

for all $x,y\in X$, where

$$m(x,x,y)=\max\left\{\begin{array}{c}G(x,x,y),G(x,x,Tx),G(y,y,Ty),\\ \frac{G(x,x,Ty)+G(y,y,Ty)+G(y,y,Tx)}{3}\end{array}\right\},$$

$$n(x,x,y)=\min\left\{G(x,x,Tx),G(y,y,Ty),G(y,y,Tx)\right\}.$$

Definition 40.[30] *Let (X,G) be a G-metric space and let ψ,ϕ,φ be altering distance functions. A mapping $T:X\longrightarrow X$ is called a (ψ,ϕ,φ,G)-weak contraction of type (B) if and only if there exists a constant $L\geq 0$ such that*

$$\psi(G(Tx,Tx,Ty))\leq\psi(m(x,x,y))-\phi(m_1(x,x,y)) +L\varphi(n(x,x,y)),$$

for all $x,y\in X$, where

$$m(x,x,y)=\max\left\{\begin{array}{c}G(x,x,y),G(x,x,Tx),G(y,y,Ty),\\ \frac{G(x,x,Ty)+G(y,y,Ty)+G(y,y,Tx)}{3}\end{array}\right\},$$

$$m_1(x,x,y)=\max\left\{G(x,x,y),G(y,y,Ty)\right\},$$

$$n(x,x,y)=\min\left\{G(x,x,Tx),G(y,y,Ty),G(y,y,Tx)\right\}.$$

Theorem 42.[30] *Let (X,G) be a complete G-metric space and let $T:X\longrightarrow X$ be a (ψ,ϕ,φ,G)-weak contraction of type (B). Then T has a unique fixed point.*

3.23 Ding and Karapınar (2012)

Ding and Karapınar [31] demonstrated the inconsistency of Theorem 10 due to Shatanawi [15] and expressed a more natural proof of the theorem.

Example 2.[31] Let$X=[0,1]$. Define $G:X\times X\times X\longrightarrow\mathbb{R}_+$ by

$$G(x,y,z)=|x-y|+|x-z|+|y-z|,$$

for all $x,y,z\in X$. Then (X,G) is a G-metric space. Define a mapping $F:X\times X\longrightarrow X$ by $F(x,y)=\frac{1}{3}x+\frac{1}{8}y$ and $g:X\longrightarrow X$ by $g(x)=\frac{x}{2}$ for all $x,y\in X$. Then for all $x,y,u,v,w,z\in X$ with $y=v=w$, we have

$$G(F(x,y),F(u,v),F(z,w))=G\left(\frac{1}{3}x+\frac{1}{8}y,\frac{1}{3}u+\frac{1}{8}v,\frac{1}{3}z+\frac{1}{8}w\right) =\frac{|x-u|+|x-z|+|u-z|}{3}$$

and

$$G(gx,gu,gz)+G(gy,gv,gw)=G\left(\frac{x}{2},\frac{u}{2},\frac{z}{2}\right)+G\left(\frac{y}{2},\frac{v}{2},\frac{w}{2}\right) =\frac{|x-u|+|x-z|+|u-z|}{2}.$$

Then it is easy to see that there is no $\lambda\in[0,\frac{1}{2})$ such that

$$G(F(x,y),F(u,v),F(z,w))\leq\lambda(G(gx,gu,gz)+G(gy,gv,gw)),$$

for all $x,y,u,v,z,w\in X$. Thus, Theorem 10 cannot be applied to this example. However, it is easy to see that 0 is the unique point $x\in X$ such that $x=gx=F(x,x)$.

Theorem 43.[31] *Let (X,G) be a G-metric space. Let $F:X\times X\longrightarrow X$ and $g:X\longrightarrow X$ be two mappings such that*

$$G(F(x,y),F(u,v),F(u,v))+G(F(y,x),F(v,u),F(v,u))\leq\lambda(G(gx,gu,gu)+$$

for all $x,y,u,v\in X$. Assume that F and g satisfy the following conditions:

(i)$F(X\times X)\subset g(X)$;
(ii)$g(X)$ is G-complete;
(iii)g is G-continuous and commutes with F.

If $\lambda\in[0,1)$, then there is a unique x in X such that $gx=F(x,x)=x$.

Ding and Karapınar [31] noted that Theorem 10 is an immediate corollary of their Theorem 43.

3.24 Karapınar, Kaymakçalan and Taş (2012)

Karapınar et al. [32] improved and generalized the work of Aydi et al. [16] on coupled fixed point in the setting of partially ordered G-metric space. They illustrated the weakness of Theorem 14 and Corollary 2 due to Aydi et al. [16] with the following example.

Example 3.[32] Let $X=\mathbb{R}$. Define $G: X\times X\times X \longrightarrow \mathbb{R}_+$ by

$$G(x,y,z)=|x-y|+|x-z|+|y-z|,$$

for all $x,y,z\in X$. Then (X,G) is a G-metric space. Define a mapping $F: X\times X\longrightarrow X$ by $F(x,y)=\frac{1}{12}x+\frac{7}{12}y$ and $g: X\longrightarrow X$ by $g(x)=\frac{x}{2}$ for all $x,y\in X$. Then for all $x,y,u,v,w,z\in X$ with $x=u=z$, we have

$$\begin{aligned}
&G(F(x,y),F(u,v),F(z,w))\\
&=G\left(\frac{1}{12}x+\frac{7}{12}y,\frac{1}{12}u+\frac{7}{12}v,\frac{1}{12}z+\frac{7}{12}w\right)\\
&=\frac{7}{12}(|v-y|+|w-y|+|w-v|)
\end{aligned}$$

and

$$\begin{aligned}
G(gx,gu,gz)+G(gy,gv,gw)&=G\left(\frac{x}{2},\frac{u}{2},\frac{z}{2}\right)+G\left(\frac{y}{2},\frac{v}{2},\frac{w}{2}\right)\\
&=\frac{1}{2}(|y-v|+|y-w|+|v-w|).
\end{aligned}$$

Then it is clear that there is no $\phi\in\Phi$ that provides the statement of Theorem 14. However, $(0,0)$ is the unique common coincidence point of F and g. In fact, $F(0,0)=g(0)=0$.

Definition 41.[32] *An element $(x,y)\in X\times X$ is called a common coupled coincidence point of the mappings $F: X\times X\longrightarrow X$ and $g: X\longrightarrow X$ if*

$$F(x,y)=g(x)=x \text{ and } F(y,x)=g(y)=y \text{ for all } x,y\in X.$$

Definition 42.[32] *Let $(X,\preceq)$ be a partially ordered set, G be a G-metric on X and $g: X\longrightarrow X$ be a self-mapping on X. Then $(X,G,\preceq)$ is called g-ordered complete if for each convergent sequence $\{x_n\}_{n\in\mathbb{N}}\subset X$, the following conditions hold:*

(i)if $\{x_n\}_{n\in\mathbb{N}}$ is a non-increasing sequence such that $x_n\longrightarrow x^*$ implies $gx^*\preceq gx_n$ for all n;

(ii)if $\{y_n\}_{n\in\mathbb{N}}$ is a non-decreasing sequence such that $y_n\longrightarrow y^*$ implies $gy^*\succeq gy_n$ for all n.

Karapınar et al. [32] stated their result which successively guarantees a coupled fixed point as follows.

Theorem 44.[32] *Let $(X,\preceq)$ be a partially ordered set and G be a G-metric on X such that (X,G) is a complete G-metric space. Suppose that there exist $\phi\in\Phi$, $F: X\times X\longrightarrow X$ and $g: X\longrightarrow X$ such that*

$$\begin{aligned}
&[G(F(x,y),F(u,v),F(w,z))+G(F(y,x),F(v,u),F(z,w))]\\
&\leq\phi(G(gx,gu,gw)+G(gy,gv,gz)),
\end{aligned}$$

for all $x,y,u,v,w,z\in X$ with $gw\preceq gu\preceq gx$ and $gy\preceq gv\preceq gz$. Suppose also that F is continuous and has the mixed g-monotone property, $F(X\times X)\subseteq g(X)$ and g is continuous and commutes with F. If there exist $x_0,y_0\in X$ such that $gx_0\preceq F(x_0,y_0)$ and $F(y_0,x_0)\preceq gy_0$, then F and g have a coupled coincidence point, that is, there exists $(x,y)\in X\times X$ such that $gx=F(x,y)$ and $gy=F(y,x)$.

In the following Theorem 45, Karapınar et al [32] omitted the continuity hypothesis of F.

Theorem 45.[32] *Let $(X,\preceq)$ be a partially ordered set and G be a G-metric on X such that $(X,G,\preceq)$ is g-ordered complete. Suppose that there exist $\phi\in\Phi$, $F: X\times X\longrightarrow X$ and $g: X\longrightarrow X$ such that*

$$\begin{aligned}
&[G(F(x,y),F(u,v),F(w,z))+G(F(y,x),F(v,u),F(z,w))]\\
&\qquad\leq\phi(G(gx,gu,gw)+G(gy,gv,gz)),
\end{aligned}$$

for all $x,y,u,v,w,z\in X$ with $gw\preceq gu\preceq gx$ and $gy\preceq gv\preceq gz$. Suppose also that $(g(X),G)$ is complete, F has the mixed g-monotone property and $F(X\times X)\subseteq g(X)$. If there exist $x_0,y_0\in X$ such that $gx_0\preceq F(x_0,y_0)$ and $F(y_0,x_0)\preceq gy_0$, then F and g have a coupled coincidence point.

3.25 Mohiuddine and Alotaibi (2012)

Mohiuddine and Alotaibi [33] proved some triple fixed point theorems for mixed monotone mappings in the framework of G-metric space endowed with partial order.

Let $\{x_n\}_{n\in\mathbb{N}}$, $\{y_n\}_{n\in\mathbb{N}}$ and $\{z_n\}_{n\in\mathbb{N}}$ be any three sequences of non-negative real numbers. Denote by Θ the set of all functions $\theta:{\mathbb{R}_+}^3\longrightarrow[0,1)$ such that $\theta(x_n,y_n,z_n)\longrightarrow 1$ implies $x_n,y_n,z_n\longrightarrow 0$.

Let $(X,\preceq)$ be a partially ordered set. A mapping $F: X\times X\longrightarrow X$ is said to have mixed monotone property if $F(x,y,z)$ is monotone non-decreasing in x and z and is monotone non-increasing in y, that is, for any $x,y,z\in X$,

$$\begin{aligned}
&x_1,x_2\in X,\ x_1\preceq x_2\Rightarrow F(x_1,y,z)\preceq F(x_2,y,z),\\
&y_1,y_2\in X,\ y_1\preceq y_2\Rightarrow F(x,y_2,z)\preceq F(x,y_1,z),\\
&z_1,z_2\in X,\ z_1\preceq z_2\Rightarrow F(x,y,z_1)\preceq F(x,y,z_2).
\end{aligned}$$

Theorem 46.[33] *Let $(X,\preceq)$ be a partially ordered set and G be a G-metric on X such that (X,G) is a complete G-metric space. Suppose that $F: X\times X\times X\longrightarrow X$ is a continuous mapping having the mixed monotone property. Assume that there exists $\theta\in\Theta$ such that*

$$\begin{aligned}
&G(F(x,y,z),F(s,t,u),F(p,q,r))+G(F(y,x,z),F(t,s,u),F(q,p,r))\\
&\qquad+G(F(z,y,x),F(u,t,s),F(r,q,p))\\
&\leq\theta(G(x,s,p),G(y,t,q),G(z,u,r))(G(x,s,p)+G(y,t,q)+G(z,u,r)),
\end{aligned}$$

for all $x,y,z,s,t,u,p,q,r\in X$ with $x\succeq s\succeq p$ and $y\preceq t\preceq q$ and $z\succeq u\succeq r$, where either $s\neq p$ or $t\neq q$ or $u\neq r$. If there exist $x_0,y_0,z_0\in X$ such that $x_0\preceq F(x_0,y_0,z_0)$, $y_0\succeq F(y_0,x_0,y_0)$ and $z_0\preceq F(z_0,y_0,x_0)$, then F has a tripled fixed point, that is, there exist $x,y,z\in X$ such that $F(x,y,z)=x$, $F(y,x,y)=y$ and $F(z,y,x)=z$.

Theorem 47.[33] *Let $(X,\preceq)$ be a partially ordered set and G be a G-metric on X such that (X,G) is a complete G-metric space. Suppose that there exist $\theta\in\Theta$ and a* *mapping $F : X \times X \times X \longrightarrow X$ having the mixed monotone property such that*

$$\begin{aligned}&G(F(x,y,z),F(s,t,u),F(p,q,r))\\&+G(F(y,x,z),F(t,s,u),F(q,p,r))\\&+G(F(z,y,x),F(u,t,s),F(r,q,p))\\&\leq \theta(G(x,s,p),G(y,t,q),G(z,u,r))(G(x,s,p)\\&+G(y,t,q)+G(z,u,r)),\end{aligned}$$

for all $x,y,z,s,t,u,p,q,r \in X$ with $x \succeq s \succeq p$ and $y \preceq t \preceq q$ and $z \succeq u \succeq r$, where either $s \neq p$ or $t \neq q$ or $u \neq r$. Assume that X has the following properties:

(i)if a non-decreasing sequence $\{x_n\}_{n\in\mathbb{N}}$ is G-convergent to x ($\{z_n\}_{n\in\mathbb{N}}$ is G-convergent to z), then $x_n \preceq x$ ($z_n \preceq z$ respectively) for all n;

(ii)if a non-increasing sequence $\{y_n\}_{n\in\mathbb{N}}$ is G-convergent to y, then $y_n \succeq y$ for all n.

If there exist $x_0,y_0,z_0 \in X$ such that $x_0 \preceq F(x_0,y_0,z_0)$, $y_0 \succeq F(y_0,x_0,y_0)$ and $z_0 \preceq F(z_0,y_0,x_0)$, then F has a tripled fixed point.

3.26 Shatanawi, Abbas, Aydi, Tahat (2012)

Shatanawi et al. [34] obtained common coupled coincidence point results for hybrid pair of two mappings without exploiting the notion of continuity in the setting of a partially ordered generalized metric space.

Definition 43.*[34] Let $(X,\preceq)$ be a partially ordered set. A mapping $F : X \times X \longrightarrow X$ is said to have g-mixed monotone where $g : X \longrightarrow X$ if for any $x_1,x_2,y_1,y_2 \in X$,*

$$gx_1 \preceq gx_2 \Rightarrow F(x_1,y) \preceq F(x_2,y),$$
$$gy_1 \preceq gy_2 \Rightarrow F(x,y_2) \preceq F(x,y_1) \text{ for all } x,y \in X.$$

Definition 44.*[34] Let X be a nonempty set. Mappings $g : X \longrightarrow X$ and $F : X \times X \longrightarrow X$ are said to be compatible if for some sequences $\{x_n\}_{n\in\mathbb{N}}$ and $\{y_n\}_{n\in\mathbb{N}}$ in X such that $\lim_{n\to\infty} F(x_n,y_n) = \lim_{n\to\infty} g(x_n) = x$ for some x in X implies*

$$\lim_{n\to\infty} F(gx_n,gy_n) = \lim_{n\to\infty} g(F(x_n,y_n)).$$

Let $\psi \in \Psi$ and let $\Lambda = \{\lambda | \lambda : \mathbb{R}_+ \times \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ is continuous and $\lambda(t,s) = 0$ if and only if $t = s = 0\}$.

Theorem 48.*[34] Let $(X,\preceq)$ be a partially ordered set such that there exists a G-metric on X. Let $F : X\times X \longrightarrow X$ and $g : X \longrightarrow X$ be continuous mappings such that F has the mixed g-monotone property and there exist $\psi \in \Psi$ and $\lambda \in \Lambda$ such that*

$$\begin{aligned}&\psi(G(F(x,y),F(u,v),F(w,z))) \leq \psi(\frac{1}{2}(G(gx,gu,gw)\\&+G(gy,gv,gz))) \quad -\lambda(G(gx,gu,gw)\\&+G(gy,gv,gz),G(gx,gu,gw)+G(gy,gv,gz)),\end{aligned}$$

for all $x,y,z,u,v,w \in X$ with $gx \succeq gu \succeq gw$ and $gy \preceq gv \preceq gz$. If $F(X\times X)$ is contained in complete subspace $g(X)$ and $\{F,g\}$ is compatible. Then F and g have a coupled coincidence point provided that there exist $x_0,y_0 \in X$ such that $gx_0 \preceq F(x_0,y_0)$ and $F(y_0,x_0) \preceq gy_0$.

Corollary 4.*[34] Let $(X,\preceq)$ be a partially ordered set such that there exists a complete G-metric on X. Let $F : X\times X \longrightarrow X$ be a continuous mappings such that F has the mixed monotone property and there exist $\psi \in \Psi$ and $\lambda \in \Lambda$ such that*

$$\begin{aligned}&\psi(G(F(x,y),F(u,v),F(w,z))) \leq \psi(\frac{1}{2}(G(x,u,w)+G(y,v,z)))\\&-\lambda(G(x,u,w)+G(y,v,z),G(x,u,w)+G(y,v,z)),\end{aligned}$$

for all $x,y,z,u,v,w \in X$ with $x \succeq u$ and $y \preceq v$. Then F has a coupled fixed point provided that there exist $x_0,y_0 \in X$ such that $x_0 \preceq F(x_0,y_0)$ and $F(y_0,x_0) \preceq y_0$.

Theorem 49.*[34] Let $(X,\preceq)$ be a partially ordered set such that there exists a G-metric on X. Let $F : X\times X \longrightarrow X$ and $g : X \longrightarrow X$ be continuous mappings such that F has the mixed g-monotone property and there exist $\psi \in \Psi$ and $\lambda \in \Lambda$ such that*

$$\begin{aligned}&\psi(G(F(x,y),F(u,v),F(w,z)))\\&\leq \psi(\frac{1}{2}(G(gx,gu,gw)+G(gy,gv,gz)))\\&\quad -\lambda(G(gx,gu,gw)+G(gy,gv,gz),\\&G(gx,gu,gw)+G(gy,gv,gz)),\end{aligned}$$

for all $x,y,z,u,v,w \in X$ with $gx \succeq gu \succeq gw$ and $gy \preceq gv \preceq gz$. If $F(X\times X)$ is contained in complete subspace $g(X)$ and X has the following property:

(i)for a non-decreasing sequence $\{x_n\}_{n\in\mathbb{N}}$ with $x_n \longrightarrow x$, we have $x_n \preceq x$ for all n;

(ii)for a non-increasing sequence $\{y_n\}_{n\in\mathbb{N}}$ with $y_n \longrightarrow y$, we have $y \preceq y_n$ for all n.

Then F and g have a coupled coincidence point provided that there exist $x_0,y_0 \in X$ such that $gx_0 \preceq F(x_0,y_0)$ and $F(y_0,x_0) \preceq gy_0$.

3.27 Aggarwal, Chugh and Kamal (2012)

Aggarwal et al. [35] obtained some Suzuki-type fixed point results in G-metric spaces and discussed the G-continuity of the fixed point.

Theorem 50.*[35] Let (X,G) be a complete G-metric space and let T be a self-mapping on X. Define a strictly decreasing function $\theta : [0,1) \longrightarrow (\frac{1}{2},1]$ by*

$$\theta(r) = \frac{1}{1+r}.$$

Assume there exists $r \in [0,1)$ such that for every $x,y \in X$,

$$\theta(r)G(x,Tx,Tx) \leq G(x,y,y) \Rightarrow G(Tx,Ty,Ty) \leq rG(x,y,y).$$

Then there exists a unique fixed point z of T and $\lim_{n\to\infty} T^n x = z$ for all $x \in X$. Moreover, T is G-continuous at z.

3.28 Ye and Gu (2012)

Ye and Gu [36] introduced a new twice power type contractive condition for three mappings in G-metric spaces and established some common fixed point theorems. Their main result is the following.

Theorem 51.[36] *Let (X,G) be a complete G-metric space. Suppose the three self-mappings $T,S,R : X \longrightarrow X$ satisfy the following condition:*

$$\begin{aligned} G^2(Tx,Sy,Rz) \leq & aG(x,Tx,Tx)G(y,Sy,Sy) \\ & + bG(y,Sy,Sy)G(z,Rz,Rz) \\ & + cG(x,Tx,Tx)G(z,Rz,Rz), \end{aligned}$$

for all $x,y,z \in X$, where $a,b,c \in \mathbb{R}_+$ and $a+b+c < 1$. Then T, S, and R have a unique common fixed point (say u) and T, S, and R are all G-continuous at u.

Theorem 52.[36] *Let (X,G) be a complete G-metric space. Suppose the three self-mappings $T,S,R : X \longrightarrow X$ satisfy the following condition:*

$$\begin{aligned} G^2(Tx,Sy,Rz) \leq & aG(x,Tx,Sy)G(y,Sy,Rz) \\ & + bG(y,Sy,Rz)G(z,Rz,Tx) \qquad (11) \\ & + cG(x,Tx,Sy)G(z,Rz,Tx), \end{aligned}$$

for all $x,y,z \in X$, where $a,b,c \in \mathbb{R}_+$ and $a+b+c < 1$. Then T, S, and R have a unique common fixed point (say u) and T, S, and R are all G-continuous at u.

3.29 Cho, Rhoades, Saadati, Samet, Shatanawi (2012)

Cho et al. [37] studied coupled coincidence and coupled common fixed point theorems in ordered generalized metric spaces for nonlinear contractive condition related to a pair of altering distance functions. Their main result is the following.

Theorem 53.[37] *Let $(X,\preceq)$ be a partially ordered set and (X,G) be a complete G-metric space. Let $F : X \times X \longrightarrow X$ and $g : X \longrightarrow X$ be continuous mappings such that F has the mixed g-monotone property and g commutes with F. Assume that there are altering distance functions ψ and φ such that*

$$\begin{aligned} & \psi(G(F(x,y),F(u,v),F(w,z))) \\ & \leq \psi(\max\{G(gx,gu,gw),G(gy,gv,gz)\}) \\ & -\varphi(\max\{G(gx,gu,gw),G(gy,gv,gz)\}), \end{aligned}$$

for all $x,y,u,v,w,z \in X$ with $gw \leq gu \leq gx$ and $gy \leq gv \leq gz$. Also, suppose that $F(X\times X) \subseteq g(X)$. If there exist $x_0,y_0 \in X$ such that $gx_0 \leq F(x_0,y_0)$ and $F(y_0,x_0) \leq gy_0$, then F and g have a coupled coincidence point.

Theorem 54.[37] *Let $(X,\preceq)$ be a partially ordered set and G be a G-metric on X such that $(X,G,\preceq)$ is regular. Let $F : X\times X \longrightarrow X$ and $g : X \longrightarrow X$ be two mappings, ψ and φ be altering distance functions such that*

$$\begin{aligned} \psi(G(F(x,y),F(u,v),F(w,z))) \leq & \psi(\max\{G(gx,gu,gw),G(gy,gv,gz)\}) \\ & -\varphi(\max\{G(gx,gu,gw),G(gy,gv,gz)\}), \end{aligned}$$

for all $x,y,u,v,w,z \in X$ with $gw \leq gu \leq gx$ and $gy \leq gv \leq gz$. Suppose also that $(g(X),G)$ is G-complete, F has the mixed g-monotone property and $F(X\times X) \subseteq g(X)$. If there exist $x_0,y_0 \in X$ such that $gx_0 \leq F(x_0,y_0)$ and $F(y_0,x_0) \leq gy_0$, then F and g have a coupled coincidence point.

Theorem 55.[37] *In addition to the hypotheses of Theorem 53, suppose that, for any $(x,y),(x^*,y^*) \in X\times X$, there exists $(u,v) \in X\times X$ such that $(F(u,v),F(v,u))$ is comparable with $(F(x,y),F(y,x))$ and $(F(x^*,y^*),F(y^*,x^*))$. Then F and g have a unique coupled common fixed point, that is, there exists a unique $(x,y) \in X\times X$ such that $x = gx = F(x,y)$ and $y = gy = F(y,x)$.*

Theorem 56.[37] *Let $(X,\preceq)$ be a partially ordered set and (X,G) be a complete G-metric space. Let $F : X\times X \longrightarrow X$ and $g : X \longrightarrow X$ be continuous mappings such that F has the mixed g-monotone property and g commutes with F. Assume that there exist $\alpha,\beta \in R$ such that*

$$\begin{aligned} \int_0^{G(F(x,y),F(u,v),F(w,z))} \alpha(s)ds \leq & \int_0^{\max\{G(gx,gu,gw),G(gy,gv,gz)\}} \alpha(s)ds \\ & - \int_0^{\max\{G(gx,gu,gw),G(gy,gv,gz)\}} \beta(s)ds, \end{aligned}$$

for all $x,y,u,v,w,z \in X$ with $gw \leq gu \leq gx$ and $gy \leq gv \leq gz$. Also, suppose that $F(X\times X) \subseteq g(X)$. If there exist $x_0,y_0 \in X$ such that $gx_0 \leq F(x_0,y_0)$ and $F(y_0,x_0) \leq gy_0$, then F and g have a coupled coincidence point.

3.30 Jleli and Samet (2012)

Jleli and Samet [38] discussed the concept of G-metric spaces and the fixed point existence results of contractive mappings defined on such spaces. They observed that most fixed point results in the context of non-symmetric G-metric space can be deduced from results in the setting of quasi-metric space. In fact, they noticed that taking $d(x,y) = G(x,y,y)$ forms a quasi-metric. Therefore, if one can transform the contractive condition of existence results in a G-metric space in terms such as $G(x,y,y)$, then the related fixed point results become the known fixed point results in the context of a quasi-metric space.

Definition 45.[38] *Let X be a nonempty set and $d : X\times X \longrightarrow \mathbb{R}_+$ be a given function which satisfies*

$(i) d(x,y) = 0$ if and only if $x = y$;

(ii) $d(x,y) \leq d(x,z)+d(z,y)$ *for any points* $x,y,z \in X$.

Then d *is called a quasi-metric and the pair* (X,d) *is called a quasi-metric space.*

Taking the following theorem as a model example, Jleli and Samet [38] showed that for a given linear contractive condition, the existing fixed point results on G-metric spaces are immediate consequences of existing fixed point theorems on metric spaces.

Theorem 57. *[38] Let* (X,G) *be a complete* G*-metric space and let* $T : X \longrightarrow X$ *be a mapping that satisfies the following condition:*

$$G(Tx,Ty,Tz) \leq aG(x,Tx,Tx)+bG(y,Ty,Ty) \quad (12)$$
$$+cG(z,Tz,Tz)+dG(x,y,z), \quad (13)$$

for all $x,y,z \in X$*, where* $a,b,c,d > 0$ *such that* $\lambda = a+b+c+d < 1$*. Then* T *has a unique fixed point.*

Setting $z = y$ in (12), we have

$$G(Tx,Ty,Ty) \leq \lambda \max\{G(x,Tx,Tx),G(y,Ty,Ty),G(x,y,y)\}, \quad (14)$$

for all $x,y \in X$. Also from (12), we have

$$G(Ty,Tx,Tx) \leq \lambda \max\{G(y,Ty,Ty),G(x,Tx,Tx),G(y,x,x)\} \quad (15)$$

for all $x,y \in X$. Define the metric space $\delta : X \times X \longrightarrow \mathbb{R}_+$ by

$$\delta(x,y) = \max\{G(x,y,y),G(y,x,x)\}.$$

Then by (14) and (15), we have

$$\delta(Tx,Ty) = \max\{\delta(x,Tx),\delta(y,Ty),\delta(x,y)\}.$$

Hence, from the above contractive condition in the complete metric space (X,δ), T has a unique fixed point.

For nonlinear type of contractive condition, Jleli and Samet [38] considered the following theorem as a model example and deduced a fixed point theorem on a corresponding G-metric space by defining the quasi-metric $\delta(x,y) = G(x,y,y)$ for all $x,y \in X$.

Theorem 58. *[38] Let* (X,d) *be a complete quasi-metric space and let* $T : X \longrightarrow X$ *be a mapping that satisfies the following condition:*

$$d(Tx,Ty) \leq d(x,y) - \psi(d(x,y)),$$

for all $x,y \in X$*, where* $\psi : \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ *is continuous with* $\psi^{-1}(\{0\}) = \{0\}$*. Then* T *has a unique fixed point.*

Theorem 59. *[38] Let* (X,G) *be a complete* G*-metric space and let* $T : X \longrightarrow X$ *be a mapping that satisfies the following condition:*

$$G(Tx,Ty,Ty) \leq G(x,y,y) - \psi(G(x,y,y)),$$

for all $x,y \in X$*, where* $\psi : \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ *is continuous with* $\psi^{-1}(\{0\}) = \{0\}$*. Then* T *has a unique fixed point.*

3.31 Samet, Vetro and Vetro (2013)

Samet et al. [39] established some propositions to show that many fixed point theorems on G-metric spaces given by many authors follow directly from well-known theorems on metric spaces.

In particular, Samet et al. [39] noted that Theorems 2.1, 2.2 and 2.3 in [9] and Theorem 2.1 in [7] are only particular cases of the following theorem due to Ćirić.

Theorem 60. *[39] Let* (X,d) *be a complete metric space and let* $T : X \longrightarrow X$ *be a self-mapping with the property*

$$d(Tx,Ty) \leq \lambda \max\{d(x,y),d(x,Tx),d(y,Ty),d(x,Ty),d(y,Tx)\},$$

for all $x,y \in X$*, where* $\lambda \in [0,1)$*. Then* T *has a unique fixed point.*

In the context of common fixed point theorems for Quasi-contractive condition, Edelstein-type theorems for compact G-metric spaces and ψ-contractive condition-related theorems, Samet et al. [39] demonstrated using the following theorems that results in these respect can equally be attributed to existence results in metric spaces.

Theorem 61. *[39] Let* (X,G) *be a* G*-metric space and let* $T,S : X \longrightarrow X$ *be weakly compatible self-mappings of* X*. Suppose that the mappings* S *and* T *satisfy one of the following conditions:*

$$G(Tx,Ty,Ty) \leq \lambda \max\{G(Sx,Sy,Sy),G(Sx,Tx,Tx),G(Sy,Ty,Ty)\}$$

or

$$G(Tx,Ty,Ty) \leq \lambda \max\{G(Sx,Sy,Sy),G(Sx,Sx,Tx),G(Sy,Sy,Ty)\},$$

for all $x,y \in X$ *where* $\lambda \in [0,1)$*. If the range of* S *contains the range of* T *and* SX *is a* G*-complete subspace of* X*, then* T *and* S *have a unique common fixed point.*

Theorem 62. *[39] Let* (X,G) *be a sequentially* G*-compact* G*-metric space and let* $T : X \longrightarrow X$ *be a self-mapping such that*

$$G(Tx,Ty,Ty) < G(x,y,y),$$

for all $x,y \in X$ *with* $x \neq y$*. Then* T *has a unique fixed point.*

Theorem 63. *[39] Let* (X,G) *be a* G*-metric space and let* $T,S : X \longrightarrow X$ *be self-mappings of* X *such that* $TX \subseteq SX$*. Suppose that there exist* $F,\psi : \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ *such that:*

(i) F *is non-decreasing, continuous and* $F(0) = 0 < F(t)$ *for every* $t > 0$*;*

(ii) ψ *is non-decreasing, right continuous and* $\psi(t) < t$ *for every* $t > 0$*;*

(iii) $F(G(Tx,Ty,Ty)) \leq \psi(F(\max\{G(Sx,Sy,Sy),G(Sx,Tx,Tx),G(Sy,Ty,Ty)\}))$ *for all* $x,y \in X$.

If one of TX and SX is a G-complete subspace of X, then T and S have a coincidence point. Further, if T and S are weakly compatible, then T and S have a unique common fixed point.

In essence, Samet et al. [39] showed that some fixed point generalizations in fixed point theory are not real generalizations as they could easily be obtained from the corresponding fixed point theorems in metric spaces. They therefore, recommended that researchers in fixed point theory should be careful in their efforts to generalize results.

3.32 Nashine and Kadelburg (2013)

Nashine and Kadelburg [40] introduced generalized cyclic contractions in G-metric spaces and established some fixed point theorems.

Definition 46.[40] *Let (X,G) be a G-metric space. Let p be a positive integer, $\mathscr{A}_1,\mathscr{A}_2,\dots,\mathscr{A}_p$ be non-empty subsets of X and $Y=\bigcup_{i=1}^{p}\mathscr{A}_i$. An operator $f:Y\longrightarrow Y$ satisfies a generalized cyclic contraction, if*

(i)$Y=\bigcup_{i=1}^{p}\mathscr{A}_i$ is a cyclic representation of Y with respect to f;

(ii)for any $(x,y,z)\in\mathscr{A}_i\times\mathscr{A}_{i+1}\times\mathscr{A}_{i+1}$, $i=1,2,\dots,p$ (with $\mathscr{A}_{p+1}=\mathscr{A}_1$),

$$\psi(G(fx,fy,fz))\leq \lambda\psi(\max\{G(x,y,z),G(x,fx,fx),G(y,fy,fy),G(z,fz,fz), \frac{1}{3}(G(x,fy,fy)+G(y,fz,fz)+G(z,fx,fx))\}),$$

where $\lambda\in[0,1)$ and ψ is an altering distance function.

The main result of Nashine and Kadelburg [40] is the following.

Theorem 64.[40] *Let (X,G) be a complete G-metric space, $p\in\mathbb{N}$, $\mathscr{A}_1,\mathscr{A}_2,\dots,\mathscr{A}_p$ non-empty subsets of X and $Y=\bigcup_{i=1}^{p}\mathscr{A}_i$. Suppose $f:Y\longrightarrow Y$ is a generalized cyclic contraction mapping. Then f has a unique fixed point. Moreover, the fixed point of f belongs to $\bigcap_{i=1}^{p}A_i$.*

3.33 Alghamdi and Karapınar(2013)

Alghamdi and Karapınar [41] introduced G-β-ϕ-contractive mappings in the context of G-metric spaces and proved existence and uniqueness of fixed points for such contractive mappings.

Let $\phi\in\Phi$ be a Φ-map as described by [6] such that there exist $n_0\in\mathbb{N}$, $\lambda\in(0,1)$ and a convergent series of non-negative terms $\sum_{n=1}^{\infty}v_n$ satisfying

$$\phi^{n+1}(t)\leq\lambda\phi^n(t)+v_n,$$

for $n\geq n_0$ and any $t>0$. Then ϕ is called a (c)-comparison function [41].

Definition 47.[41] *Let (X,G) be a G-metric space and let $T:X\longrightarrow X$ be a given mapping. Then T is said to be a G-β-ϕ-contractive mapping of type I if there exist two functions $\beta:X\times X\times X\longrightarrow\mathbb{R}_+$ and $\phi\in\Phi$ such that for all $x,y,z\in X$,*

$$\beta(x,y,z)G(Tx,Ty,Tz)\leq\phi(G(x,y,z)).$$

Definition 48.[41] *Let (X,G) be a G-metric space and let $T:X\longrightarrow X$ be a given mapping. Then T is said to be a G-β-ϕ-contractive mapping of type II if there exist two functions $\beta:X\times X\times X\longrightarrow\mathbb{R}_+$ and $\phi\in\Phi$ such that for all $x,y\in X$,*

$$\beta(x,y,y)G(Tx,Ty,Ty)\leq\phi(G(x,y,y)).$$

Definition 49.[41] *Let (X,G) be a G-metric space and let $T:X\longrightarrow X$ be a given mapping. Then T is said to be a G-β-ϕ-contractive mapping of type A if there exist two functions $\beta:X\times X\times X\longrightarrow\mathbb{R}_+$ and $\phi\in\Phi$ such that for all $x,y\in X$,*

$$\beta(x,y,Tx)G(Tx,Ty,T^2x)\leq\phi(G(x,y,T^2x)).$$

Definition 50.[41] *Let $T:X\longrightarrow X$ and $\beta:X\times X\times X\longrightarrow\mathbb{R}_+$. We say that T is β-admissible if for all $x,y,z\in X$, we have*

$$\beta(x,y,z)\geq1\Rightarrow\beta(Tx,Ty,Tz)\geq1$$

The main result of Alghamdi and Karapınar [41] is the following.

Theorem 65.[41] *Let (X,G) be a complete G-metric space. Suppose that $T:X\longrightarrow X$ is a G-β-ϕ-contractive mapping of type A and satisfies the following conditions:*

(i)T is β-admissible;

(ii)there exists $x_0\in X$ such that $\beta(x_0,Tx_0,Tx_0)\geq1$;

(iii)T is G-continuous.

Then there exists $u\in X$ such that $Tu=u$.

In the following theorem, continuity condition is not necessary.

Theorem 66.[41] *Let (X,G) be a complete G-metric space. Suppose that $T:X\longrightarrow X$ is a G-β-ϕ-contractive mapping of type A and satisfies the following conditions:*

(i)T is β-admissible;

(ii)there exists $x_0\in X$ such that $\beta(x_0,Tx_0,Tx_0)\geq1$;

(iii)if $\{x_n\}_{n\in\mathbb{N}}$ is a sequence in X such that $\beta(x_n,x_{n+1},x_{n+1})\geq1$ for all n and $\{x_n\}_{n\in\mathbb{N}}$ is G-convergent to $x\in X$,then $\beta(x_n,x,x_{n+1})\geq1$ for all n.

Then there exists $u\in X$ such that $Tu=u$.

Consequently, Alghamdi and Karapınar [41] proved some results for cyclic contractive mappings.

Following [1], a nonempty subset A in the G-metric space (X,G) is G-closed if $\bar{A}=A$. Note that

$$x\in\bar{A}\iff B_G(x,\varepsilon)\cap A\neq0,\ \text{for all}\ \varepsilon>0.$$

Theorem 67.[41] *Let A, B be non-empty G-closed subsets of a complete G-metric (X,G) space, let $Y = A \cup B$, and let $T : Y \longrightarrow Y$ be a given self-mapping satisfying $T(A) \subset B$ and $T(B) \subset A$. If there exists a function $\phi \in \Phi$ such that*

$$G(Tx,Ty,Ty) \leq \phi(G(x,y,y)),$$

for all $x \in A, y \in B$, then T has a unique fixed point $u \in A \cap B$.

Alghamdi and Karapınar further proved some coupled fixed point results in the following manner.

Theorem 68.[41] *Let (X,G) be a complete G-metric space and let $F : X \times X \longrightarrow X$ be a given mapping. Suppose there exist $\phi \in \Phi$ and a function $\beta : X^2 \times X^2 \times X^2 \longrightarrow \mathbb{R}_+$ such that*

$$\beta((x,y),(u,v),(u,v))G(F(x,y),F(u,v),F(u,v)) \leq \frac{1}{2}\phi(G(x,u,u)+G(y,v,v)),$$

for all $(x,y),(u,v) \in X \times X$. Suppose also that

(i)for all $(x,y),(u,v) \in X \times X$, we have $\beta((x,y),(u,v),(u,v)) \geq 1 \Rightarrow \beta((F(x,y),F(y,x)),(F(u,v),F(v,u)),(F(u,v),F(v,u))) \geq 1$;
(ii)there exists $(x_0,y_0) \in F$ such that $\beta((x_0,y_0),(F(x_0,y_0),F(y_0,x_0)),(F(x_0,y_0),F(y_0,x_0))) \geq 1$ and $\beta((F(y_0,x_0),F(x_0,y_0)),(F(y_0,x_0),F(x_0,y_0)),(y_0,x_0)) \geq 1$;
(iii)F is continuous.

Then F has a coupled fixed point.

3.34 Asadi, Karapınar and Salimi (2013)

Asadi et al [42] proved some fixed point theorems in the framework of G-metric space that cannot be obtained in the manner of Jleli and Samet [38] and Samet et al. [39]. Their main result is the following.

Theorem 69.[42] *Let (X,G) be a complete G-metric space and $T : X \longrightarrow X$ be a mapping satisfying the following condition for all $x,y \in X$:*

$$G(Tx,Ty,Ty) \leq \lambda G(x,Tx,y),$$

where $\lambda \in [0,1)$. Then T has a unique fixed point.

Theorem 70.[42] *Let (X,G) be a complete G-metric space and $T : X \longrightarrow X$ be a mapping satisfying*

$$\psi(G(Tx,T^2x,Ty)) \leq \psi(G(x,Tx,y)) - \varphi(G(x,Tx,y)),$$

for all $x,y \in X$, where $\psi \in \Psi$ and $\varphi \in \Upsilon$. Then T has a unique fixed point.

Theorem 71.[42] *Let (X,G) be a complete G-metric space and $T : X \longrightarrow X$ be an onto mapping satisfying*

$$G(Tx,T^2x,Ty) \geq \lambda G(x,Tx,y),$$

for all $x,y \in X$, where $\lambda > 1$. Then T has a unique fixed point.

3.35 Mustafa, Aydi and Karapınar (2013)

Mustafa et al. [43] introduced generalized Meir-Keeler type contractions in G-metric space and showed that every orbitally continuous generalized Meir-Keeler type contraction has a unique fixed point on complete G-metric space.

Definition 51.[43] *Let (X,G) be a G-metric space. A mapping $T : X \longrightarrow X$ is said to be orbitally G-continuous whenever*

$$\lim_{i\to\infty} G(T^{n_i}x,z,z) = 0 \Rightarrow \lim_{i\to\infty} G(TT^{n_i}x,Tz,Tz) = 0,$$

for each $x \in X$ and $n_i \in \mathbb{N}$.

Definition 52.[43] *Let (X,G) be a G-metric space and $T : X \longrightarrow X$ be a self-mapping on X. Then T is called a generalized Meir-Keeler type contraction whenever for each $\varepsilon > 0$, there exists $\delta > 0$ such that*

$$\varepsilon \leq M(x,y,z) < \varepsilon + \delta \Rightarrow G(Tx,Ty,Tz) < \varepsilon,$$

for all $x,y,z \in X$, where

$$M(x,y,z) = \max\{G(x,y,z),G(Tx,x,x),G(Ty,y,y),G(Tz,z,z)\}.$$

The main result of Mustafa et al. [43] is the following.

Theorem 72.[43] *Let (X,G) be a complete G-metric space and $T : X \longrightarrow X$ be an orbitally continuous generalized Meir-Keeler type contraction. Then T has a unique fixed point, say $u \in X$. Moreover, $\lim_{n\to\infty} G(T^nx,u,u) = 0$ for all $x \in X$.*

Remark.[43] The above Theorem 72 remains true if the hypothesis that T is a generalized Meir-Keeler type contraction is replaced by the fact that for each $\varepsilon > 0$, there exists $\delta > 0$ such that

$$\varepsilon \leq N(x,y,z) < \varepsilon + \delta \Rightarrow G(Tx,Ty,Tz) < \varepsilon,$$

for all $x,y,z \in X$, where

$$N(x,y,z) = \max\{G(x,y,z),G(Tx,Tx,x),G(Ty,Ty,y),G(Tz,Tz,z)\}.$$

3.36 Ding and Karapınar (2013)

Ding and Karapınar [44] established several fixed point theorems for Meir-Keeler type contractions in partially ordered G-metric spaces.

Definition 53.[44] *Let $(X,\preceq)$ be a partially ordered set, G be a G-metric on X. Then $(X,G,\preceq)$ is called ordered complete if for each convergent sequence $\{x_n\}_{n\in\mathbb{N}}\subset X$, the following conditions hold:*

(i)if $\{x_n\}_{n\in\mathbb{N}}$ is a non-increasing sequence such that $x_n\longrightarrow x^*$ implies $x^*\preceq x_n$ for all n;
(ii)if $\{y_n\}_{n\in\mathbb{N}}$ is a non-decreasing sequence such that $y_n\longrightarrow y^*$ implies $y^*\succeq y_n$ for all n.

Definition 54.[44] *Let $(X,G,\preceq)$ be a partially ordered G-metric space. Suppose that $T:X\longrightarrow X$ is a self-mapping such that for each $\varepsilon>0$, there exists $\delta>0$ such that for any $x,y,z\in X$ with $x\preceq y\preceq z$,*

$$\varepsilon\le G(x,y,z)<\varepsilon+\delta\Rightarrow G(Tx,Ty,Tz)<\varepsilon.$$

Then T is called G-Meir-Keeler contractive.

Remark.[44] Notice that if $T:X\longrightarrow X$ is G-Meir-Keeler contractive on a G-metric space (X,G), then T is contractive, that is,

$$G(Tx,Ty,Tz)<G(x,y,z),$$

for all distinct tripled $(x,y,z)\in X^3$ with $x\preceq y\preceq z$.

Definition 55.[44] *Let $(X,\preceq)$ be a partially ordered set and $T:X\longrightarrow X$ is a self-mapping on X. Then T is said to be non-decreasing if for $x,y\in X$, $x\preceq y\Rightarrow Tx\preceq Ty$.*

Definition 56.[44] *Let $(X,G,\preceq)$ be a partially ordered G-metric space. Suppose that $T:X\longrightarrow X$ is a self-mapping such that for each $\varepsilon>0$, there exists $\delta>0$ such that for any $x,y\in X$ with $x\preceq y$,*

$$\varepsilon\le G(x,y,y)<\varepsilon+\delta\Rightarrow G(Tx,Ty,Ty)<\varepsilon.$$

Then T is called G-Meir-Keeler contractive of second type.

The main result of Ding and Karapınar [44] is the following.

Theorem 73.[44] *Let$(X,\preceq)$ be a partially ordered set endowed with a G-metric and $T:X\longrightarrow X$ be a given mapping. Suppose that the following conditions hold:*

(i)(X,G) is G-complete;
(ii)T is non-decreasing (with respect to $\preceq$);
(iii)there exists $x_0\in X$ such that $x_0\preceq Tx_0$;
(iv)T is G-continuous;
(v)$T:X\longrightarrow X$ is G-Meir-Keeler contractive of second type.

Then T has a fixed point. Moreover, if for all $(x,y)\in X\times X$ there exists $w\in X$ such that $x\preceq w$ and $y\preceq w$, we obtain the uniqueness of the fixed point.

Theorem 74.[44] *Let$(X,\preceq)$ be a partially ordered set endowed with a G-metric and $T:X\longrightarrow X$ be a given mapping. Suppose that there exists a function $\mu:\mathbb{R}_+\longrightarrow\mathbb{R}_+$ satisfying following conditions:*

(i)$\mu(0)=0$ and $\mu(t)>0$ for all $t>0$;
(ii)μ is non-decreasing and right continuous;
(iii)for every $\varepsilon>0$, there exists $\delta>0$ such that

$$\varepsilon\le\mu(G(x,y,y))<\varepsilon+\delta\Rightarrow\mu(G(Tx,Ty,Ty))<\mu(\varepsilon)$$

for all $(x,y)\in X\times X$ with $x\preceq y$. Then T is G-Meir-Keeler contractive of second type.

3.37 Shatanawi and Pitea (2013)

Following the definition of Ω-distance by [14], Shatanawi and Pitea [45] established some common coupled fixed point results. Their result is the following.

Theorem 75.[45] *Let (X,G) be a G-metric space and Ω an Ω-distance on X such that X is Ω-bounded. $g:X\longrightarrow X$ and $F:X\times X\longrightarrow X$ are mappings. Suppose there exists $\lambda\in[0,1)$ such that for each x,y,z,x^*,y^*,z^* in X,*

$$\Omega(F(x,y),F(x^*,y^*),F(z,z^*))+\Omega(F(y,x),F(y^*,x^*),F(z^*,z))$$

$$\le\lambda\max\left\{\begin{array}{c}\Omega(gx,gx^*,gz)+\Omega(gy,gy^*,gz^*),\\ \Omega(gx^*,gx,gz)+\Omega(gy^*,gy,gz^*),\\ \Omega(gx,F(x^*,y^*),gz)+\Omega(gy,F(y^*,x^*),gz^*),\\ \Omega(F(x,y),gx^*,gz)+\Omega(F(y,x),gy^*,gz^*),\\ \Omega(gx^*,F(x,y),gz)+\Omega(gy^*,F(y,x),gz^*),\\ \Omega(F(x,y),F(x^*,y^*),gz)+\Omega(F(y,x),F(y^*,x^*),gz^*)\end{array}\right\}.$$

Consider also that the following conditions hold:

(i)$F(X\times X)\subseteq gX$;
(ii)gX is a complete subspace of X with respect to the topology, induced by G;
(iii)If $F(u,v)\neq gu$ or $F(v,u)\neq gv$, then

$$\inf\{\Omega(gx,F(x,y),gu)+\Omega(gy,F(y,x),gv)+\Omega(gx,gu,F(x,y))+\Omega(gy,gv,F(y,x))\}>0. \tag{16}$$

Then, F and g have a unique coupled coincidence point (u,v). Moreover, $F(u,v)=gu=gv=F(v,u)$.

Theorem 76.[45] *Let (X,G) be a G-metric space and Ω an Ω-distance on X. Consider $g:X\longrightarrow X$, $F:X\times X\longrightarrow X$ and $\pi:gX\longrightarrow\mathbb{R}_+$ such that for all x,y,z,z^* in X,*

$$\Omega(gx,F(x,y),F(z,z^*))+\Omega(gy,F(y,x),F(z^*,z))\le\pi(gx)+\pi(gy)+\pi(gz)+\pi(gz^*)-\pi(F(x,y))-\pi(F(y,x))-\pi(F(z,z^*))-\pi(F(z^*,z)).$$

Consider also that the following conditions hold:

(i)$F(X\times X)\subseteq gX$;

(ii)gX is a complete subspace of X with respect to the topology induced by G;

(iii)there exists $\lambda > 0$ such that $\Omega(x,x,y) \leq \lambda\Omega(x,y,y)$ holds for all $x,y \in X$;

(iv)if $F(u,v) \neq gu$ or $F(v,u) \neq gv$, then

$$\inf\{\Omega(gx,F(x,y),gu)+\Omega(gy,F(y,x),gv) \\ +\Omega(gx,gu,F(x,y))+\Omega(gy,gv,F(y,x))\} > 0.$$

Then, F and g have a coupled coincidence point (u,v).

3.38 Vats, Kumar and Sihag (2013)

Vats et al. [46] introduced the concept of compatible and compatible mapping of type (A) in G-metric space and proved a common fixed point theorem for two pair of expansive mappings.

Definition 57.[46] *Two self-mappings S and T of a G-metric space (X,G) are said to be compatible if*

$\lim_{n\to\infty} G(TSx_n,STx_n,STx_n) = 0$ and $\lim_{n\to\infty} G(STx_n,TSx_n,TSx_n) = 0$ whenever $\{x_n\}_{n\in\mathbb{N}}$ is a sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t$, for some $t \in X$.

Definition 58.[46] *Two self-mappings S and T of a G-metric space (X,G) are said to be compatible mappings of type (A) if*

$\lim_{n\to\infty} G(TSx_n,SSx_n,SSx_n) = 0$ and $\lim_{n\to\infty} G(STx_n,TTx_n,TTx_n) = 0$ whenever $\{x_n\}_{n\in\mathbb{N}}$ is a sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t$, for some $t \in X$.

Theorem 77.[46] *Let A, B, S and T be mappings of a complete G-metric space (X,G) into itself and $\phi \in \Phi$ satisfying the following conditions:*

(i)A and B are surjective;

(ii)one of the mappings A, B, S and T is sequentially continuous;

(iii)the pair $\{A,S\}$ and $\{B,T\}$ are compatible mappings of type (A);

(iv)$\phi(G(Ax,By,Bz)) \geq G(Sx,Ty,Tz)$,

for all $x,y,z \in X$. Then A, B, S and T have a unique common fixed point.

Corollary 5.[46] *Let A, B, S and T be mappings of a complete G-metric space (X,G) into itself satisfying the following conditions:*

(i)A and B are surjective;

(ii)one of the mappings A, B, S and T is sequentially continuous;

(iii)the pair $\{A,S\}$ and $\{B,T\}$ are compatible mappings of type (A);

(iv)$G(Ax,By,Bz) \geq \lambda G(Sx,Ty,Tz)$.

for all $x,y,z \in X$, where $\lambda > 1$. Then A, B, S and T have a unique common fixed point.

3.39 Aydi, Chauhan and Radenović (2013)

Aydi et al. [47] proved some integral type fixed point theorems for a pair of weakly compatible mappings in G-metric space, employing the notion of common limit range property.

Definition 59.[47] *A pair $\{f,g\}$ of self-mappings of a G-metric space (X,G) is said to satisfy the common limit range property with respect to mapping g (denoted by (CLRg) property) if there exists a sequence $\{x_n\}_{n\in\mathbb{N}}$ such that $\{fx_n\}$ and $\{gx_n\}$ are G-convergent to gu for some $u \in X$, that is,*

$$\lim_{n\to\infty} G(fx_n,fx_n,gu) = \lim_{n\to\infty} G(gx_n,gx_n,gu) = 0.$$

Definition 60.[47] *Two families of self-mappings $\{f_i\}_{i=1}^m$ and $\{g_k\}_{k=1}^n$ are said to be pairwise commuting if*

(i)$f_if_j = f_jf_i$ for all $i,j \in \{1,2,\ldots,m\}$;

(ii)$g_kg_l = g_lg_k$ for all $k,l \in \{1,2,\ldots,n\}$;

(iii)$f_ig_k = g_kf_i$ for all $i \in \{1,2,\ldots,m\}$ and $k \in \{1,2,\ldots,n\}$.

Theorem 78.[47] *Let (X,G) be a G-metric space and the pair $\{f,g\}$ of self-mappings is weakly compatible such that:*

$$\int_0^{G(fx,fy,fz)} \alpha(t)dt \leq \lambda \int_0^{G(gx,gy,gz)} \alpha(t)dt,$$

for all $x,y,z \in X$, where $\lambda \in [0,1)$ and $\alpha : \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ is a Lebesgue integrable mapping which is summable, non-negative and such that for each $\varepsilon > 0$, $\int_0^{\varepsilon} \alpha(t)dt > 0$. If the pair $\{f,g\}$ satisfies the (CLRg) property, then f and g have a unique common fixed point in X.

Theorem 79.[47] *Let (X,G) be a G-metric space and the pair $\{f,g\}$ of self-mappings is weakly compatible such that:*

$$\int_0^{G(fx,fy,fz)} \alpha(t)dt \leq \phi \int_0^{M(x,y,z)} \alpha(t)dt,$$

for all $x,y,z \in X$, where $\phi \in \Phi$, $\alpha : \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ is a Lebesgue integrable mapping which is summable, non-negative and such that for each $\varepsilon > 0$, $\int_0^{\varepsilon} \alpha(t)dt > 0$ and

$$M(x,y,z) = \max\left\{\begin{array}{c} G(gx,gy,gz), G(gx,fx,fx), \\ G(gy,fy,fy), G(gz,fz,fz) \end{array}\right\},$$

or

$$M(x,y,z) = \max\left\{\begin{array}{c} G(gx,gy,gz), G(gx,gx,fx), \\ G(gy,gy,fy), G(gz,gz,fz) \end{array}\right\}.$$

If the pair $\{f,g\}$ satisfies the (CLRg) property, then f and g have a unique common fixed point in X.

3.40 Manro (2013)

Manro [48] introduced new concepts of subcompatibility and subsequential continuity and established common fixed point theorem for four mappings.

Definition 61.[48] *Let (X,G) be a G-metric space. Two self-mappings f and g on X are said to be subcompatible if and only if there exists a sequence $\{x_n\}_{n\in\mathbb{N}}$ in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z$, where $z \in X$ and satisfy*

$$\lim_{n\to\infty} G(fgx_n, gfx_n, gfx_n) = 0.$$

Definition 62.[48] *Let (X,G) be a G-metric space. Two self-mappings f and g on X are said to be reciprocally continuous if and only if $\lim_{n\to\infty} fgx_n = ft$ and $\lim_{n\to\infty} gfx_n = gt$, whenever the sequence $\{x_n\}_{n\in\mathbb{N}}$ in X is such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t$, where $t \in X$.*

Definition 63.[48] *Let (X,G) be a G-metric space. Two self-mappings f and g on X are said to be subsequentially continuous if and only if there exists a sequence $\{x_n\}_{n\in\mathbb{N}}$ in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t$, where $t \in X$ and satisfy $\lim_{n\to\infty} fgx_n = ft$ and $\lim_{n\to\infty} gfx_n = gt$.*

The main result of Manro [48] is the following.

Theorem 80.[48] *Let f,g,h and k be four self-mappings of a G-metric space (X,G). If the pairs $\{f,h\}$ and $\{g,k\}$ are subcompatible and subsequentially continuous, then*

(i) f and h have a coincidence point;
(ii) g and k have a coincidence point.

Further, let $\lambda : (\mathbb{R}_+)^6 \longrightarrow \mathbb{R}$ be an upper semi-continuous function satisfying $\lambda(u,u,0,0,u,u) > 0$, for all $u > 0$. Suppose that $\{f,h\}$ and $\{g,k\}$ satisfy

$$\lambda(G(fx,gy,gy), G(hx,ky,ky), G(fx,hx,hx), G(gy,ky,ky), G(hx,gy,gy), G(fx,ky,ky)) \le 0,$$

for all $x,y \in X$. Then f,g,h and k have a unique common fixed point.

3.41 Gu and Ye (2013)

Gu and Ye [49] introduced a new third power type contractive condition in G-metric spaces and established some fixed point theorems. Their main result is the following.

Theorem 81.[49] *Let (X,G) be a complete G-metric space. Suppose the self-mapping $T : X \longrightarrow X$ satisfies*

$$G^3(Tx,Ty,Tz) \le \lambda G(x,Tx,Tx)G(y,Ty,Ty)G(z,Tz,Tz),$$

for all $x,y,z \in X$, where $0 \le \lambda < 1$. Then T has a unique fixed point (say u) and T is G-continuous at u.

Theorem 82.[49] *Let (X,G) be a complete G-metric space. Suppose the self-mapping $T : X \longrightarrow X$ satisfies*

$$G^3(Tx,T^2x,T^3x) \le \lambda G(x,Tx,Tx)G(Tx,T^2x,T^2x)G(T^2x,T^3x,T^3x),$$

for all $x \in X$, where $0 \le \lambda < 1$. Then T has a unique fixed point.

3.42 Karapınar and Agarwal (2013)

Jleli and Samet [38] and Samet et al. [39] observed that most fixed point results in the context of non-symmetric G-metric space can be deduced from existence results in the setting of quasi-metric space. However, Karapınar and Agarwal [50] noted that the approach of [38] is inapplicable unless the contractive condition in the statement of the theorem can be reduced into two variables. They proved some fixed point theorems for contractive conditions that cannot be reduced into two variables.

Theorem 83.[50] *Let (X,G) be a G-metric space. Let $T : X \longrightarrow X$ be a mapping such that*

$$G(Tx,Ty,Tz) \le \lambda M(x,y,z),$$

for all $x,y,z \in X$, where $\lambda \in [0,\frac{1}{2})$ and

$$M(x,y,z) = \max \left\{ \begin{array}{c} G(x,Tx,y), G(y,T^2x,Ty), G(Tx,T^2x,Ty), G(y,Tx,Ty), \\ G(x,Tx,z), G(z,T^2x,Tz), G(Tx,T^2x,Tz), G(z,Tx,Ty), \\ G(x,y,z), G(x,Tx,Tx), G(y,Ty,Ty), G(z,Tz,Tz), \\ G(z,Tx,Tx), G(x,Ty,Ty), G(y,Tz,Tz) \end{array} \right\}.$$

Then there is a unique $u \in X$ such that $Tu = u$.

Theorem 84.[50] *Let (X,G) be a G-metric space. Let $T : X \longrightarrow X$ be a mapping such that*

$$G(Tx,T^2x,Ty) \le G(x,Tx,y) - \varphi(G(x,Tx,y))$$

for all $x,y \in X$, where $\varphi : \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ is continuous with $\varphi^{-1}(\{0\}) = 0$. Then there is a unique $u \in X$ such that $Tu = u$.

3.43 Gupta and Yadava (2014)

Gupta and Yadava [51] introduced the ρ-contraction on G-metric space and proved some fixed point theorems in ordered G-metric spaces.

Definition 64.[51] *Let $(X,\preceq,G)$ be an ordered G-metric space. A function $\rho : X \times X \times X \longrightarrow \mathbb{R}$ is called a ρ-function in X if it satisfies the following conditions:*

(i) $\rho(x,y,z) \ge 0$ for every comparable $x,y,z \in X$;
(ii) for any sequence $\{x_n\}_{n\in\mathbb{N}}$, $\{y_n\}_{n\in\mathbb{N}}$ and $\{z_n\}_{n\in\mathbb{N}}$ in X such that x_n, y_n and z_n are comparable at each $n \in N$, if $\lim_{n\to\infty} x_n = x$, $\lim_{n\to\infty} y_n = y$ and $\lim_{n\to\infty} z_n = z$, then $\lim_{n\to\infty} \rho(x_n,y_n,z_n) = \rho(x,y,z)$;

(iii)for any sequence $\{x_n\}_{n\in\mathbb{N}}$, $\{y_n\}_{n\in\mathbb{N}}$ and $\{z_n\}_{n\in\mathbb{N}}$ in X such that x_n, y_n and z_n are comparable at each $n \in N$, if $\lim_{n\to\infty} \rho(x_n,y_n,z_n) = 0$, then $\lim_{n\to\infty} G(x_n,y_n,z_n) = 0$.

If in addition, the following condition is also satisfied:

(iv)for any sequence $\{x_n\}_{n\in\mathbb{N}}$, $\{y_n\}_{n\in\mathbb{N}}$ and $\{z_n\}_{n\in\mathbb{N}}$ in X such that x_n, y_n and z_n are comparable at each $n \in N$, if $\lim_{n\to\infty} G(x_n,y_n,z_n)$ exists, then $\lim_{n\to\infty} \rho(x_n,y_n,z_n)$ also exists,

then ρ is said to be a ρ-function of type (A) with respect to $\preceq$ in X.

Definition 65.[51] *Let $(X,\preceq,G)$ be an ordered G-metric space. A mapping $f : X \longrightarrow X$ is called ρ-contraction with respect to $\preceq$ if there exists a ρ-function with respect to $\preceq$ in X such that*

$$G(fx, fy, fz) \leq G(x,y,z) - \rho(x,y,z), \tag{17}$$

for any comparable $x,y,z \in X$.

Naturally, if there exists a ρ-function of type (A) with respect to $\preceq$ in X such that inequality 17 holds for any comparable $x,y,z \in X$, then f is said to be a ρ-contraction of type (A) with respect to $\preceq$.

Theorem 85.[51] *Let $(X,\preceq,G)$ be a complete ordered G-metric space and $f : X \longrightarrow X$ be continuous and nondecreasing ρ-contraction of type (A) with respect to $\preceq$. If there exists $x_0 \in X$ with $x_0 \preceq fx_0$, then $\{f^n x_0\}_{n\in\mathbb{N}}$ converges to a fixed point of f in X.*

Theorem 86.[51] *Let $(X,\preceq,G)$ be a complete ordered G-metric space and $f : X \longrightarrow X$ be nondecreasing ρ-contraction of type (A) with respect to $\preceq$. If there exists $x_0 \in X$ with $x_0 \preceq fx_0$, then $\{f^n x_0\}_{n\in\mathbb{N}}$ converges to a fixed point of f in X.*

3.44 Bilgili, Erhan, Karapınar, Türkoğlu, (2014)

Following observation by Jleli and Samet [38] and Samet et al. [39] that most fixed point results in the context of non-symmetric G-metric space can be deduced from existing results in the setting of quasi-metric space, Bilgili et al. [52] proved the existence and uniqueness of fixed points of certain cyclic mappings in the context of G-metric spaces that can not be obtained by usual fixed point results via techniques used in [38] and [39].

The following is their main result. Accordingly [52], let $\psi \in \Psi$ and $\varphi \in \Upsilon$.

Theorem 87.[52] *Let (X,G) be a complete G-metric space and $(A_j)_{j=1}^m$ be a family of nonempty G-closed subsets of X with $Y = \bigcup_{j=1}^m A_j$. Let $T : Y \longrightarrow Y$ be a mapping satisfying*

$$T(A_j) \subseteq A_{j+1},\ j = 1,2,\ldots,m,\ \text{where } A_{m+1} = A_1.$$

Suppose that there exist functions $\psi \in \Psi$ and $\varphi \in \Upsilon$ such that the mapping T satisfies the inequality

$$\psi(G(Tx,T^2x,Ty)) \leq \psi(M(x,y,y)) - \varphi(M(x,y,y)),$$

for all $x \in A_j$ and $y \in A_{j+1}$, $j = 1,2,\ldots,m$ where

$$M(x,y,y) = \max\left\{\begin{array}{c} G(x,y,y), G(x,Tx,Tx), G(y,Ty,Ty), G(x,Tx,y), \\ \frac{1}{2}G(x,T^2x,Ty), \frac{1}{2}G(y,Ty,Tx), \\ \frac{1}{2}G(y,T^2x,Ty), \frac{1}{2}(G(x,Ty,Ty)+G(y,Tx,Tx)), \\ \frac{1}{2}(G(x,T^2x,Ty)+G(y,Tx,Tx)) \end{array}\right.$$

Then T has a unique fixed point in $\bigcap_{j=1}^m A_j$.

3.45 Chen and Huang (2015)

Chen and Huang [53] introduced a new concept of $(\alpha,\phi)_g$-contractive type mappings and established coupled coincidence and coupled common fixed point theorems for such mappings in partially ordered G-metric spaces.

Definition 66.[53] *Let $(X,G,\preceq)$ be a partially ordered G-metric space and $F : X\times X \longrightarrow X$ and $g : X \longrightarrow X$ be two mappings. Then F is said to be $(\alpha,\phi)_g$-contractive if there exist two functions $\alpha : X^2\times X^2\times X^2 \longrightarrow \mathbb{R}_+$ and $\phi \in \Phi$ such that*

$$\alpha((gx,gy),(gu,gv),(gz,gw))G(F(x,y),F(u,v),F(z,w)) \leq \phi\left(\frac{G(gx,gu,gz)+G(gy,gv,gw)}{2}\right),$$

for all $x,y,u,v,z,w \in X$ with $gx \succeq gu \succeq gz$ and $gy \preceq gv \preceq gw$.

Definition 67.[53] *Let $F : X\times X \longrightarrow X$, $g : X \longrightarrow X$ and $\alpha : X^2\times X^2\times X^2 \longrightarrow \mathbb{R}_+$ be three mappings. Then F is said to be g-α-admissible if*

$$\alpha((gx,gy),(gu,gv),(gz,gw)) \geq 1 \Rightarrow \alpha((F(x,y),F(y,x)),(F(u,v)F(v,u)),(F(z,w),F(w,z))) \geq 1,$$

for all $x,y,u,v,z,w \in X$.

The main result of Chen and Huang [53] is the following.

Theorem 88.[53] *Let $(X,\preceq)$ be a partially ordered set and suppose there is a G-metric G on X such that (X,G) is a complete G-metric space. Let $F : X\times X \longrightarrow X$ and $g : X \longrightarrow X$ be such that F has the mixed g-monotone property. Assume there exists a function $\phi \in \Phi$ and $\alpha : X^2\times X^2\times X^2 \longrightarrow \mathbb{R}_+$ such that for all $x,y,u,v,z,w \in X$, the following hold:*

$$\alpha((gx,gy),(gu,gv),(gz,gw))G(F(x,y),F(u,v),F(z,w)) \leq \phi\left(\frac{G(gx,gu,gz)+G(gy,gv,gw)}{2}\right),$$

for all $gw \preceq gu \preceq gx$ and $gy \preceq gv \preceq gz$. Suppose also that $F(X\times X) \subseteq g(X)$, g is continuous and commutes with F and

(i)F is g-α-admissible;
(ii)there exist $x_0, y_0 \in X$ such that

$$\alpha((gx_0, gy_0), (F(x_0, y_0), (F(y_0, x_0)), (F(x_0, y_0), F(y_0, x_0))) \geq 1 \quad (18)$$

and

$$\alpha((gy_0, gx_0), (F(y_0, x_0), (F(x_0, y_0)), (F(y_0, x_0), F(x_0, y_0))) \geq 1; \quad (19)$$

(iii)F is continuous.

If there exist $x_0, y_0 \in X$ such that $gx_0 \preceq F(x_0, y_0)$ and $gy_0 \succeq F(y_0, x_0)$, then F and g have a coupled coincidence point, that is, there exists $(x, y) \in X \times X$ such that $gx = F(x, y)$ and $gy = F(y, x)$.

3.46 Gajić and Stojaković (2015)

Gajić and Stojaković [54] obtained some fixed point theorems for Matkowski type mapping in G-metric spaces.

Accordingly [54], let $\alpha : [0,\infty)^5 \longrightarrow [0,\infty)$ be a nondecreasing function with respect to each variable and let $\lambda(t) = \alpha(t,t,2t,3t,3t)$ for $t \geq 0$. Let Λ be the set of all functions $\lambda : [0,\infty) \longrightarrow [0,\infty)$ such that

(i) $\lim\limits_{n\to\infty} \lambda^n(t) = 0, t > 0$;
(ii) $\lim\limits_{t\to\infty}(t - \lambda(t)) = \infty$.

Definition 68.*[54] Let (X, G) be a G-metric space and $T : X \longrightarrow X$ be a self-mapping on X. If for every $x \in X$, there exists a positive integer $n = n(x)$ such that for all $y \in X$,*

$$G(T^{n(x)}x, T^{n(x)}x, T^{n(x)}y) \leq \alpha(G(x,x,y), G(x,x,T^{n(x)}y), G(x,T^{n(x)}x,T^{n(x)}x), G(y,T^{n(x)}x,T^{n(x)}x), G(y,y,T^{n(x)}y)), \quad (20)$$

then T is said to be a weak contraction in X.

The main result of Gajić and Stojaković [54] is the following.

Theorem 89.*[54] Let (X, G) be a complete G-metric space and $T : X \longrightarrow X$ be a weak contraction in X. Then T has a unique fixed point $a \in X$ and for every $x \in X$, $\lim\limits_{k\to\infty} T^k x = a$ and $T^{n(a)}$ is continuous at a.*

3.47 Abd-Elhamed (2015)

Abd-Elhamed [55] proved some fixed point theorems for contractions in compact G-metric spaces. The main result of Abd-Elhamed [55] is the following.

Theorem 90.*[55] Let (X, G) be a compact G-metric space. If $T : X \longrightarrow X$ satisfies*

$$G(Tx_1, Tx_2, Tx_3) < G(x_1, x_2, x_3),$$

where $x_1 \neq x_2 \neq x_3$ in X, then T has a unique fixed point in X and the fixed point can be found as the limit of $T^n(x_0)$ as $n \to \infty$ for any $x_0 \in X$.

Theorem 91.*[55] Let (X, G) be a compact G-metric space and let $T : X \longrightarrow X$ be a self-mapping on X. Assume that*

$$\alpha G(x, Tx, Tx) < G(x,y,z) \Rightarrow G(Tx, Ty, Tz) < G(x,y,z),$$

for $\alpha \in (0, \frac{1}{2}]$, $x, y, z \in X$. Then T has a unique fixed point.

3.48 Zada, Shah and Li (2016)

Zada et al. [56] introduced the idea of integral type contraction in G-metric space and proved some coupled coincidence fixed point results for two pairs of mapping. Their main result is the following.

Theorem 92.*[56] Let (X, G) be a G-metric space. Let $F, S : X \times X \longrightarrow X$ and $g, h : X \longrightarrow X$ be mappings such that*

$$\int_0^{G(F(a,b),S(p,q),S(c,r))} \alpha(t)dt \leq \lambda \int_0^{(G(ha,gp,gc)+G(hb,gq,gr))} \alpha(t)dt,$$

for all $a, b, c, p, q, r \in X$ and $\alpha : \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ is a Lebesgue integrable mapping which is summable such that for each $\varepsilon > 0$, $\int_0^\varepsilon \alpha(t)dt > 0$. Assume that F, S and g, h satisfy the following conditions:

(i)$F(X \times X) \subset g(X)$ and $S(X \times X) \subset h(X)$;
(ii)$g(X)$ or $h(X)$ is complete;
(iii)g and h are G-continuous and the pairs $\{F, h\}$ and $\{S, g\}$ are commuting mappings.

If $\lambda \in [0, \frac{1}{8})$, then there is a unique $u \in X$ such that $F(u,u) = S(u,u) = g(u) = h(u) = u$.

Corollary 6.*[56] Let (X, G) be a G-metric space. Let $F, S : X \times X \longrightarrow X$ and $g, h : X \longrightarrow X$ be two mappings such that*

$$\int_0^{G(F(a,b),S(p,q),S(p,q))} \alpha(t)dt \leq \lambda \int_0^{(G(ha,gp,gp)+G(hb,gq,gq))} \alpha(t)dt,$$

for all $a, b, p, q \in X$ and $\alpha : \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ is a Lebesgue integrable mapping which is summable such that for each $\varepsilon > 0$, $\int_0^\varepsilon \alpha(t)dt > 0$. Assume that F, S and g, h satisfy the following conditions:

(i)$F(X \times X) \subset g(X)$ and $S(X \times X) \subset h(X)$;
(ii)$g(X)$ or $h(X)$ is complete;
(iii)g and h are G-continuous and the pairs $\{F, h\}$ and $\{S, g\}$ are commuting mappings.

If $\lambda \in [0, \frac{1}{8})$, then there is a unique $u \in X$ such that $F(u,u) = S(u,u) = g(u) = h(u) = u$.

3.49 Abbas, Hussain, Popović, Radenović (2016)

Abbas et al. [57] presented some fixed point results of convex contraction mappings in the framework of G-metric spaces.

Definition 69.*[57] Let (X,G) be a G-metric space, T a self-mapping on X and $\varepsilon>0$ a given number. A point x in X is called*

(i)an ε-fixed point of T, if $G(x,Tx,T^2x)<\varepsilon$;
(ii)approximate fixed point of T, if T has an ε-fixed point for all $\varepsilon>0$.

Definition 70.*[57] Let (X,G) be a G-metric space. A self-mapping T on X is called asymptotic regular if for any x in X, we have $G(T^nx,T^{n+1}x,T^{n+2}x)\longrightarrow 0$ as $n\to\infty$.*

Definition 71.*[57] Let X be a nonempty set and $\alpha,\eta:X\times X\times X\longrightarrow\mathbb{R}_+$ be two mappings. A self-mapping T on X is said to be α-admissible with respect to η if for any $x,y,z\in X$,*

$$\alpha(x,y,z)\geq\eta(x,y,z)\Rightarrow\alpha(Tx,Ty,Tz)\geq\eta(Tx,Ty,Tz).$$

Definition 72.*[57] Let X be a nonempty set and $\alpha,\eta:X\times X\times X\longrightarrow\mathbb{R}_+$ be two mappings. A self-mapping T on X is said to be convex contraction, if for any $x,y,z\in X$,*

$$\eta(x,Tx,Ty)\leq\alpha(x,y,z)\Rightarrow G(T^2x,T^2y,T^2z)\\ \leq aG(Tx,Ty,Tz)+bG(x,y,z),$$

where $a,b\geq 0$ with $a+b<1$.

Definition 73.*[57] Let X be a nonempty set and $\alpha,\eta:X\times X\times X\longrightarrow\mathbb{R}_+$ be two mappings. A self-mapping T on X is said to be convex contraction, if for any $x,y,z\in X$,*

$$\begin{aligned}&\eta(x,Tx,Ty)\leq\alpha(x,y,z)\\&\Rightarrow G(T^2x,T^2y,T^2z)\\&\leq a_1G(x,Tx,Tx)+a_2G(Tx,T^2x,T^2x)+b_1G(y,Ty,Ty)\\&\quad+b_2G(Ty,T^2y,T^2y)+c_1G(z,Tz,Tz)+c_2G(Tz,T^2z,T^2z),\end{aligned}$$

where $a_1,a_2,b_1,b_2,c_1,c_2\geq 0$ with $a_1+a_2+b_1+b_2+c_1+c_2<1$.

Definition 74.*[57] Let T be a self-mapping on a nonempty set X and $\alpha,\eta:X\times X\times X\longrightarrow\mathbb{R}_+$. We say that the set X has H^*-property if for any $x,y\in Fix(T)=\{x\in X:Tx=x\}$ with $\alpha(x,y,y)<\eta(x,Tx,Tx)$, there exists $z\in X$ such that $\alpha(x,z,z)\geq\eta(x,z,z)$ and $\alpha(y,z,z)\geq\eta(y,z,z)$. Also for any $x,y\in X$, we have $\eta(x,Tx,Tx)\leq\eta(x,y,z)$.*

We now present the main result of Abbas et al. [57].

Theorem 93.*[57] Let (X,G) be a complete G-metric space and T, an α-admissible convex contraction with respect to η. If $\alpha(x,Tx,Tx)\geq\eta(x,Tx,Tx)$ for any $x\in X$, then T has an approximate fixed point.*

Theorem 94.*[57] Let (X,G) be a complete G-metric space and T, a continuous convex contraction and α-admissible mapping with respect to η. Suppose that there exists a point x_0 in X such that*

$$\alpha(x_0,Tx_0,Tx_0)\geq\eta(x_0,Tx_0,Tx_0).$$

Then T has a fixed point. Moreover, T has a unique fixed point provided that X has H^*-property.

Theorem 95.*[57] Let (X,G) be a complete G-metric space and T, a convex contraction of order 2 α-admissible with respect to η. If $\alpha(x,Tx,Tx)\geq\eta(x,Tx,Tx)$ for any $x\in X$, then T has an approximate fixed point.*

Definition 75.*[57] Let T be a self-mapping on a G-metric space (X,G) and $a,b\in\mathbb{R}_+$ with $0<a\leq b$. If there exists a mapping $\lambda:X\times X\times X\longrightarrow[0,1]$ with $\theta(a,b):=\sup\{\lambda(x,y,z):a\leq G(x,y,z)\leq b\}<1$ such that for any $x,y,z\in X$, $\eta(x,Tx,Tx)\leq\alpha(x,y,z)$ implies that*

$$G(Tx,Ty,Tz)\leq\lambda(x,y,z)\\ \times\max\left\{\begin{array}{c}\frac{1}{2}(G(x,Tx,Tx)+G(y,Ty,Ty)+G(z,Tz,Tz)),\\ \frac{1}{2}(G(x,Ty,Tz)+G(y,Tz,Tx)+G(z,Tx,Ty)),\\ G(x,y,z)\end{array}\right\},$$

then T is α-η-weakly Zamfirescu mapping.

Theorem 96.*[57] Let (X,G) be a G-metric space and T a self-mapping on X. If T is an α-η-weakly Zamfirescu mapping and α-admissible with respect to η with $\alpha(x,Tx,Tx)\geq\eta(x,Tx,Tx)$ for any $x\in X$, then T has an approximate fixed point.*

Theorem 97.*[57] Let (X,G) be a complete G-metric space and T a continuous, α-η-weakly Zamfirescu mapping and α-admissible with respect to η. If there exists $x_0\in X$ such that $\alpha(x_0,Tx_0)\geq\eta(x_0,Tx_0)$, then T has a fixed point.*

3.50 Ansari, Chandok, Hussain, Mustafa, Jaradat (2017)

Ansari et al. [58] presented some coincidence and common fixed point results for generalized contractive mappings using the concept of partially weakly G-α-admissibility in the framework of G-metric space.

Definition 76.*[58] Let (X,G) be a G-metric space and let f be a self-mapping on X and $\alpha:X\times X\times X\longrightarrow\mathbb{R}_+$ be a function. Then f is said to be a $G-\alpha-$admissible (or α-admissible of rank 3) mapping if for all $x,y,z\in X$,*

$$\alpha(x,y,z)\geq 1\Rightarrow\alpha(fx,fy,fz)\geq 1.$$

Definition 77.[58] *Let (X,G) be a G-metric space and let f be a self-mapping on X and $\eta : X\times X\times X \longrightarrow \mathbb{R}_+$ be a function. Then f is said to be a $G-\eta-$subadmissible (or η-subadmissible of rank 3) mapping if for all $x,y,z\in X$,*

$$\eta(x,y,z)\leq 1 \Rightarrow \eta(fx,fy,fz)\leq 1.$$

Definition 78.[58] *Let X be an arbitrary set, $\alpha : X\times X\times X \longrightarrow \mathbb{R}_+$ and $f: X\longrightarrow X$. Then f is called an α-dominating mapping on X if*

$$\alpha(x,fx,fx)\geq 1 \text{ or } \alpha(x,x,fx)\geq 1 \text{ for each } x\in X.$$

Definition 79.[58] *Let X be an arbitrary set, $\eta : X\times X\times X \longrightarrow \mathbb{R}_+$ and $f: X\longrightarrow X$. Then f is called an η-subdominating mapping on X if*

$$\eta(x,fx,fx)\leq 1 \text{ or } \eta(x,x,fx)\leq 1 \text{ for each } x\in X.$$

Definition 80.[58] *Let (X,G) be a G-metric space and $\alpha : X\times X\times X \longrightarrow \mathbb{R}_+$ be a function. Then X is said to be α-regular (or α-regular of rank 3) mapping if and only if the following hypothesis holds:*

For any convergent sequence $\{x_n\}\longrightarrow z$ in X with $\alpha(x_n,x_{n+1},x_{n+2})\geq 1$ it follows that $\alpha(x_n,z,z)\geq 1$, or $\alpha(z,x_n,z)\geq 1$, or $\alpha(z,z,x_n)\geq 1$ for all $n\in\mathbb{N}$.

Definition 81.[58] *Let (X,G) be a G-metric space. Let $f,g : X\longrightarrow X$ be self-mappings on X and $\alpha : X\times X\times X \longrightarrow \mathbb{R}_+$ be a function. Then the pair $\{f,g\}$ is said to be partially weakly $G-\alpha-$admissible (or α-admissible of rank 3) mapping if and only if for all $x\in X$,*

$$\alpha(fx,gfx,gfx)\geq 1.$$

Definition 82.[58] *Let (X,G) be a G-metric space. Let $f,g : X\longrightarrow X$ be self-mappings on X and $\alpha : X\times X\times X \longrightarrow \mathbb{R}_+$ be a function. Then the pair $\{f,g\}$ is said to be partially weakly $G-\alpha-$admissible (or α-admissible of rank 3) mapping with respect to a self-mapping R on X if and only if for all $x\in X$,*

$$\alpha(fx,gy,gy)\geq 1,$$

where $y\in R^{-1}(fx)$.

Definition 83.[58] *Let (X,G) be a G-metric space and $\eta : X\times X\times X \longrightarrow \mathbb{R}_+$ be a function. Then X is said to be η-regular (or η-regular of rank 3) mapping if and only if the following hypothesis holds:*

For any convergent sequence $\{x_n\}\longrightarrow z$ in X with $\eta(x_n,x_{n+1},x_{n+2})\leq 1$, it follows that $\eta(x_n,z,z)\leq 1$, or $\eta(z,x_n,z)\leq 1$, or $\eta(z,z,x_n)\leq 1$ for all $n\in\mathbb{N}$.

Definition 84.[58] *Let (X,G) be a G-metric space. Let $f,g : X\longrightarrow X$ be self-mappings on X and $\eta : X\times X\times X \longrightarrow \mathbb{R}_+$ be a function. Then the pair $\{f,g\}$ is said to be partially weakly $G-\eta-$subadmissible (or η-subadmissible of rank 3) mapping if and only if for all $x\in X$,*

$$\eta(fx,gfx,gfx)\leq 1.$$

Definition 85.[58] *Let (X,G) be a G-metric space. Let $f,g : X\longrightarrow X$ be self-mappings on X and $\eta : X\times X\times X \longrightarrow \mathbb{R}_+$ be a function. Then the pair $\{f,g\}$ is said to be partially weakly $G-\eta-$subadmissible (or η-subadmissible of rank 3) mapping with respect to a self-mapping R on X if and only if for all $x\in X$,*

$$\eta(fx,gy,gy)\leq 1,$$

where $y\in R^{-1}(fx)$.

Definition 86.[58] *A function $\varphi : \mathbb{R}_+\longrightarrow\mathbb{R}_+$ is called an Ultra-altering distance function if the following properties are satisfied.*

(i)φ is continuous;
(ii)$\varphi(t)\neq 0$ if and only if $t\neq 0$.

Definition 87.[58] *A function $f : \mathbb{R}_+\times\mathbb{R}_+\longrightarrow\mathbb{R}$ is called a $\mathscr{C}$-class function if it is continuous function and satisfies the following properties for all $s,t\in\mathbb{R}$;*

(i)$f(s,t)\leq s$;
(ii)$f(s,t)=s$ implies that either $s=0$ or $t=0$.

Definition 88.[58] *Let $H : R\times\mathbb{R}_+\longrightarrow\mathbb{R}$ be a function. H is of subclass of type I if it is continuous and for*

$$x\geq 1\Rightarrow H(1,y)\leq H(x,y), \text{ for all } y\in\mathbb{R}+.$$

Definition 89.[58] *Let $\mathscr{F} : \mathbb{R}_+\times\mathbb{R}_+\longrightarrow R$ and $H : R\times\mathbb{R}_+\longrightarrow\mathbb{R}$ be two mappings. Then the pair $\{\mathscr{F},H\}$ is said to be an upper class of type I if $\mathscr{F}$ is continuous, H is a function of subclass of type I and satisfies*

(i)$0\leq x\leq 1\Rightarrow\mathscr{F}(x,y)\leq\mathscr{F}(1,y)$;
(ii)$H(1,y_1)\leq\mathscr{F}(x,y_2)\Rightarrow y_1\leq xy_2$, for all $y_1,y_2\in\mathbb{R}_+$, $x\in[0,1]$.

Definition 90.[58] *Let $\mathscr{F} : \mathbb{R}_+\times\mathbb{R}_+\longrightarrow R$ and $H : R\times\mathbb{R}_+\longrightarrow\mathbb{R}$ be two mappings. Then the pair $\{\mathscr{F},H\}$ is said to be a special upper class of type I if $\mathscr{F}$ is continuous, H is a function of subclass of type I and satisfies*

(i)$0\leq s\leq 1\Rightarrow\mathscr{F}(s,t)\leq\mathscr{F}(1,t)$;
(ii)$H(1,y)\leq\mathscr{F}(1,t)\Rightarrow y\leq t$, for all $s,t,y\in\mathbb{R}_+$.

Definition 91.[58] *A function $F : \mathbb{R}_+\times\mathbb{R}_+\longrightarrow\mathbb{R}$ is called strong $\mathscr{C}$-class function if it is a continuous function and satisfies the following properties for all $s,t\in\mathbb{R}$;*

(i)$F(s,t)\leq s$;
(ii)$F(s,t)=s$ implies that either $s=0$ or $t=0$;
(iii)F is increasing in the first component and decreasing in the second component.

Theorem 98.[58] *Let (X,G) be a complete G-metric space. Let $f,g,h,R,S,T : X\longrightarrow X$ be six continuous mappings such that*

(i)$f(X)\subseteq R(X)$, $g(X)\subseteq S(X)$ and $h(X)\subseteq T(X)$;
(ii)pairs $\{f,T\}$, $\{g,R\}$ and $\{h,S\}$ are compatible;

(iii)pairs $\{f,g\}$, $\{g,h\}$ and $\{h,f\}$ are partially weakly α-admissible with respect to R, S and T respectively;

(iv)for any $x,y,z \in X$,
$H(\alpha(Tx,Ry,Sz),\psi(G(fx,gy,hz))) \leq \mathscr{F}(1,F(\psi(M(x,y,z)),\varphi(M(x,y,z))))$,
where

$$M(x,y,z) = \max\left\{\begin{array}{c}\frac{1}{6}(G(Tx,gy,gy)+G(Ry,hz,hz)+G(Sz,fx,fx)),\\ \frac{1}{6}(G(Tx,Tx,gy)+G(Ry,Ry,hz)+G(Sz,Sz,fx)),\\ \frac{1}{6}(G(Tx,fx,fx)+G(Ry,gy,gy)+G(Sz,Sz,hz)),\\ \frac{1}{6}(G(Tx,Tx,fx)+G(Ry,Ry,gy)+G(Sz,hz,hz)),\\ G(Tx,Ry,Sz)\end{array}\right\},$$

ψ is an altering distance function, φ is an Ultra-altering distance function, the pair $\{\mathscr{F},H\}$ is an upper class of type I, and F is a strong $\mathscr{C}$-class function.

Then, the pairs $\{f,T\}$, $\{g,R\}$ and $\{h,S\}$ have a coincidence point z in X. Moreover, if $\alpha(Tz,Rz,Sz) \geq 1$, then z is a coincidence point of f, g, h, R, S and T.

Theorem 99.*[58] Let (X,G) be a complete G-metric space. Let $f,g,h,R,S,T : X \longrightarrow X$ be six mappings such that*

(i)$f(X) \subseteq R(X)$, $g(X) \subseteq S(X)$ and $h(X) \subseteq T(X)$;

(ii)RX, SX and TX are G-complete;

(iii)pairs $\{f,T\}$, $\{g,R\}$ and $\{h,S\}$ are weakly compatible;

(iv)pairs $\{f,g\}$, $\{g,h\}$ and $\{h,f\}$ are partially weakly α-admissible with respect to R, S and T respectively;

(v)for any $x,y,z \in X$,
$H(\alpha(Tx,Ry,Sz),\psi(G(fx,gy,hz))) \leq \mathscr{F}(1,F(\psi(M(x,y,z)),\varphi(M(x,y,z))))$,
where

$$M(x,y,z) = \max\left\{\begin{array}{c}\frac{1}{6}(G(Tx,gy,gy)+G(Ry,hz,hz)+G(Sz,fx,fx)),\\ \frac{1}{6}(G(Tx,Tx,gy)+G(Ry,Ry,hz)+G(Sz,Sz,fx)),\\ \frac{1}{6}(G(Tx,fx,fx)+G(Ry,gy,gy)+G(Sz,Sz,hz)),\\ \frac{1}{6}(G(Tx,Tx,fx)+G(Ry,Ry,gy)+G(Sz,hz,hz)),\\ G(Tx,Ry,Sz)\end{array}\right\},$$

ψ is an altering distance function, φ is an Ultra-altering distance function, the pair $\{\mathscr{F},H\}$ is an upper class of type I, and F is a strong $\mathscr{C}$-class function.

Then, the pairs $\{f,T\}$, $\{g,R\}$ and $\{h,S\}$ have a coincidence point z in X. Moreover, if $\alpha(Tz,Rz,Sz) \geq 1$, then z is a coincidence point of f, g, h, R, S and T.

3.51 Aydi (2017)

Aydi [59] established some fixed point theorems in generalized metric spaces involving generalized cyclic contractions.

Lemma 1.*[59] If $\phi \in \Phi$, then the function $s : (0,\infty) \longrightarrow (0,\infty)$ defined by*

$$s(t) = \sum_{n=0}^{\infty} \phi^n(t) \quad t > 0,$$

is non-decreasing and continuous at 0.

The main result of Aydi [59] is the following.

Theorem 100.*[59] Let (X,G) be a complete G-metric space. Let $(A_i)_{i=1}^m$ be a family of non-empty G-closed subsets of X, m a positive integer and $Y = \bigcup_{i=1}^m (A_i)$. Let $T : Y \longrightarrow Y$ be a mapping such that $T(A_i) \subseteq A_{i+1}$ for all $i = 1,\ldots,m$ with $A_{m+1} = A_i$. Suppose also that there exists $\phi \in \Phi$ such that*

$$G(Tx,Ty,Tz) \leq \phi(G(x,y,z)),$$

for all $(x,y,z) \in A_i \times A_{i+1} \times A_{i+1}$ where $i = 1,\ldots,m$. Then

(i)T has a unique fixed point, say u, that belongs to $\bigcap_{i=1}^m (A_i)$;

(ii)the following estimates hold:

$$G(x_n,u,u) \leq s(\phi^n(G(x_0,x_1,x_1))), \quad n \geq 1,$$
$$G(x_n,u,u) \leq s(G(x_n,x_{n+1},x_{n+1})), \quad n \geq 1,$$

(iii)for any $x \in Y$, $G(x,u,u) \leq s(G(x,Tx,Tx))$,

where s is as defined in Lemma 1.

Theorem 101.*Let (X,G) be a complete G-metric space and $T : X \longrightarrow X$. Suppose there exist $\psi,\phi \in \Psi$ such that*

$$\psi(G(Tx,Ty,Tz)) \leq \psi(G(x,y,z)) - \phi(G(x,y,z)),$$

for all $x,y,z \in X$. Then T has a unique fixed point.

Aydi [59] extended Theorem 101 to more general classes of mappings involving cyclic (ψ,φ)-contractions, relaxing the monotone property of the function $\phi \in \Psi$ and replacing its continuity property by lower semi-continuity, $\varphi \in \Upsilon$.

Theorem 102.*[59] Let (X,G) be a complete G-metric space. Let $(A_i)_{i=1}^m$ be a family of non-empty G-closed subsets of X, m a positive integer and $Y = \bigcup_{i=1}^m (A_i)$. Let $T : Y \longrightarrow Y$ be a mapping such that $T(A_i) \subseteq A_{i+1}$ for all $i = 1,\ldots,m$ with $A_{m+1} = A_i$. Suppose also that there exist $\psi \in \Psi$ and $\varphi \in \Upsilon$ such that*

$$\psi(G(Tx,Ty,Tz)) \leq \psi(G(x,y,z)) - \varphi(G(x,y,z))$$

for all $(x,y,z) \in A_i \times A_{i+1} \times A_{i+1}$ with $i = 1,2,\ldots,m$. Then T has a unique fixed point that belongs to $\bigcap_{i=1}^m (A_i)$.

3.52 Singh, Joshi, Kumam, Singh (2017)

Singh et al. [60] defined generalized F-contraction and Roger Hardy type F-contractive mappings in the framework of G-metric spaces and obtained some fixed point results. Interestingly, their results in G-metric spaces cannot be deduced from any existence results in metric spaces in the manner of Jleli and Samet [38] and Samet et al. [39].

Consequently [60], denote by Δ_F the family of all functions $F : \mathbb{R}_+ \longrightarrow \mathbb{R}$ which satisfy the following conditions.

(i)F is strictly increasing;
(ii)$\inf F = -\infty$;
(iii)F is continuous on $(0,1)$.

Definition 92.[60] *A mapping $T : X \longrightarrow X$ is said to be a generalized F-contraction in G-metric spaces if $F \in \Delta_F$ and there exists $\tau > 0$ such that for all $x,y \in X$,*

$$G(Tx,T^2x,Ty) > 0 \Rightarrow$$
$$\tau + F(G(Tx,T^2x,Ty)) \le F(G(x,Tx,y)).$$

Theorem 103.[60] *Let (X,G) be a G-complete metric space and $T : X \longrightarrow X$ be a generalized F-contraction. Then T has a unique fixed point $x^* \in X$ and for every $x_0 \in X$, the sequence $\{T^n x_0\}_{n\in\mathbb{N}}$ converges to x.*

Definition 93.[60] *A mapping $T : X \longrightarrow X$ is said to be a generalized Hardy-Rogers type F-contraction in G-metric spaces if $F \in \Delta_F$ and there exists $\tau > 0$ such that*

$$G(Tx,Ty,T^2y) > 0 \Rightarrow$$
$$\tau + F(G(Tx,Ty,T^2y)) \le F(\alpha G(x,y,Ty) + \beta G(x,Tx,Ty) \quad (21)$$
$$+ \gamma G(y,Ty,T^2y) + \delta G(y,T^2x,T^2y) + \eta G(x,Tx,T^2x)),$$

for all $x,y \in X$ and $\alpha,\beta,\gamma,\delta,\eta \ge 0$ with $\alpha+\beta+\gamma+\delta+\eta < 1$.

Theorem 104.[60] *Let (X,G) be a G-complete metric space and $T : X \longrightarrow X$ be a Hardy-Rogers type generalized F-contractive mapping, that is, if $F \in \Delta_F$ and there exists $\tau > 0$, such that*

$$\tau + F(G(Tx,Ty,T^2y)) \le F(\alpha G(x,y,Ty) + \beta G(x,Tx,Ty)$$
$$+ \gamma G(y,Ty,T^2y) + \delta G(y,T^2x,T^2y) + \eta G(x,Tx,T^2x)),$$

for all $x,y \in X$, $G(Tx,Ty,T^2y) > 0$ and $\alpha,\beta,\gamma,\delta,\eta \ge 0$ with $\alpha+\beta+\gamma+\delta+\eta < 1$. Then T has a fixed point in X. Furthermore, if $\alpha+2\beta+\delta \le 1$, then fixed point of T is unique.

3.53 Sepet and Aydin (2018)

Sepet and Aydin [61] introduced a new type of F-contraction and proved some fixed point theorems.

Definition 94.[61] *Let (X,G) be a G-metric space. A mapping $f,g : X \longrightarrow X$ is said to be type 1 F-contraction on (X,G) if there exists a number $\tau > 0$ such that for all $x,y,z \in X$ satisfying $G(fx,fy,fz) > 0$, the following holds:*

$$\tau + F(G(fx,fy,fz)) \le F(G(gx,gy,gz)).$$

Definition 95.[61] *Let (X,G) be a G-metric space. A mapping $f,g : X \longrightarrow X$ is said to be type 2 F-contraction on (X,G) if there exists a number $\tau > 0$ such that for all $x,y,z \in X$ and $\beta \in [0,\frac{1}{3}]$ satisfying $G(fx,fy,fz) > 0$, the following holds:*

$$\tau + F(G(fx,fy,fz)) \quad (22)$$
$$\le F(\beta(G(gx,fx,fx) + G(gy,fy,fy) + G(gz,fz,fz))). \quad (23)$$

The main result of Sepet and Aydin [61] is the following.

Theorem 105.[61] *Let (X,G) be a complete G-metric space and $f,g : X \longrightarrow X$ be type 1 F-contraction. Then f and g have a unique common fixed point.*

Theorem 106.[61] *Let (X,G) be a complete G-metric space and $f,g : X \longrightarrow X$ be type 2 F-contraction. Then f and g have a unique common fixed point.*

3.54 Kumar and Sharma (2019)

Kumar and Sharma [62] introduced the simulation function ξ and the notion of $\mathscr{Z}$-contraction with respect to ξ in the setting of G-metric space and obtained some related fixed point results.

Definition 96.[62] *A simulation function ξ is a function $\xi : \mathbb{R}_+ \times \mathbb{R}_+ \longrightarrow \mathbb{R}$ satisfying the following:*

(i)$\xi(0,0) = 0$;
(ii)$\xi(a,b) < b-a$, for all $a,b > 0$;
(iii)if $\{a_n\},\{b_n\} \subseteq (0,\infty)$ satisfying $\lim_{n\to\infty}\{a_n\} = \lim_{n\to\infty}\{b_n\} = l$, then $\lim_{n\to\infty}\sup \xi(a_n,b_n) < 0$.

Denote by $\mathscr{Z}$ the set of all simulation functions, ξ.

Definition 97.[62] *Let (X,G) be a G-metric space, $T : X \longrightarrow X$ a mapping and $\xi \in \mathscr{Z}$. Then T is called a $\mathscr{Z}$-contraction with respect to ξ if for all $x,y,z \in X$,*

$$\xi(G(Tx,Ty,Tz),G(x,y,z)) \ge 0.$$

The main result of Kumar and Sharma [62] is the following.

Theorem 107.[62] *Let (X,G) be a complete G-metric space and $T : X \longrightarrow X$ be a $\mathscr{Z}$-contraction with respect to ξ. Then T has a unique fixed point u in X and for every $x_0 \in X$ the Picard sequence $\{x_n\}_{n\in\mathbb{N}}$ where $x_n = Tx_{n-1}$ for all $n \in \mathbb{N}$ converges to the fixed point of T.*

3.55 Kumar, Arora, Imdad, Alfaqih (2019)

Kumar et al. [63] established some coincidence and common fixed point theorems in symmetric G-metric space using simulation functions.

The main result of Kumar et al. [63] is the following.

Theorem 108.[63] *Let (X,G) be a symmetric complete G-metric space and $S,T : X \longrightarrow X$ be self-mappings on X. Suppose that*

(i)$S(X) \subseteq T(X)$;
(ii)$T(X)$ is closed;
(iii)S is T-non-decreasing;
(iv)there exists $x_0 \in X$ with $Tx_0 \leq Sx_0$;
(v)if $\{Tx_n\} \subset X$ is a non-decreasing sequence (w.r.t. $\leq$) with $Tx_n \longrightarrow Tz$ in $T(X)$, then $Tu \leq T(Tu)$ and $Tx_n \leq Tu$, for all $n \in \mathbb{N}$;
(vi)there exists a simulation function ξ such that for every $(x,y) \in X \times X$ with $Tx \leq Ty$, we have

$$\xi(G(Sx,Sy,Sz),H(S,T,x,y,z)) > 0,$$

where ξ is given by Definition 96 and

$$H(S,T,x,y,z) = \max\left\{ \begin{array}{c} G(Tx,Ty,Tz),G(Tx,Sy,Tz),G(Ty,Sx,Tz), \\ G(Tx,Sx,Tz),G(Ty,Sy,Tz) \end{array} \right\}.$$

Then S and T have a coincidence point. Further, if S and T commute at their coincidence points, then S and T have a common fixed point.

Definition 98.[63] *A function $\xi : \mathbb{R}_+ \times \mathbb{R}_+ \longrightarrow \mathbb{R}$ is said to be right monotone simulation function if it satisfies the following:*

(i)$\xi(0,0) = 0$;
(ii)$\xi(a,b) < b - a$, for all $a,b > 0$;
(iii)if $\{a_n\}$, $\{b_n\}$ are sequences in $(0,\infty)$ satisfying $\lim_{n\to\infty}\{a_n\} = \lim_{n\to\infty}\{b_n\} = l$, then $\lim_{n\to\infty}\sup \xi(a_n,b_n) < 0$;
(iv)if $b_1 \leq b_2$, then $\xi(a,b_1) \leq \xi(a,b_2)$, for all $a,b_1,b_2 \geq 0$.

Theorem 109.[63] *Let (X,G) be a symmetric G-metric space and $S,T : X \longrightarrow X$ be self-mappings on X. Suppose that*

(i)$S(X) \subseteq T(X)$;
(ii)$T(X)$ is closed;
(iii)S is T-non-decreasing;
(iv)there exists $x_0 \in X$ with $Tx_0 \leq Sx_0$;
(v)if $\{Tx_n\} \subset X$ is a non-decreasing sequence (w.r.t. $\leq$) with $Tx_n \longrightarrow Tz$ in $T(X)$, then $Tu \leq T(Tu)$ and $Tx_n \leq Tu$, for all $n \in \mathbb{N}$;
(vi)there exists a right monotone simulation function ξ such that for every $(x,y) \in X \times X$ with $Tx \leq Ty$, we have

$$\xi(G(Sx,Sy,Sz),G(Tx,Ty,Tz)) \geq 0.$$

Then S and T have a coincidence point. Further, if S and T commute at their coincidence points, then S and T have a common fixed point.

3.56 Ansari, Jain and Salim (2020)

Ansari et al. [64] introduced the concept of inverse $\mathscr{C}$-class function in G-metric setting and established some fixed point theorems.

Definition 99.[64] *Two self-mappings f and g are called f-compatible of type (E) if $\lim_{n\to\infty} ffx_n = \lim_{n\to\infty} fgx_n = gt$, whenever a sequence $\{x_n\}_{n\in\mathbb{N}}$ in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t$, for some t in X. Similarly, self-mappings f and g are called g-compatible of type (E) if $\lim_{n\to\infty} ggx_n = \lim_{n\to\infty} gfx_n = ft$, whenever a sequence $\{x_n\}_{n\in\mathbb{N}}$ in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t$, for some t in X.*

Definition 100.[64] *Two self-mappings f and g on X are called f-compatible if $\lim_{n\to\infty} d(fgx_n, ggx_n) = 0$, whenever a sequence $\{x_n\}_{n\in\mathbb{N}}$ in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t$, for some t in X. Similarly, self-mappings f and g are called g-compatible if $\lim_{n\to\infty} d(gfx_n, ffx_n) = 0$, whenever a sequence $\{x_n\}_{n\in\mathbb{N}}$ in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t$, for some t in X.*

Definition 101.[64] *A function $F : \mathbb{R}_+ \times \mathbb{R}_+ \longrightarrow \mathbb{R}$ is called inverse-$\mathscr{C}$-class function if it is continuous function and satisfies the following properties for all $s,t \in \mathbb{R}$;*

(i)$F(s,t) \geq s$;
(ii)$F(s,t) = s$ implies that either $s = 0$ or $t = 0$.

We denote inverse $\mathscr{C}$-class functions as $\mathscr{C}_{inv}$.

The main result of Ansari et al. [64] is the following. Accordingly, let $\psi \in \Psi$ and $\varphi \in \Upsilon$.

Theorem 110.[64] *Let f and g be weak semi compatible, R-weakly commuting of type A_f self-mappings of complete G-metric space (X,G) and suppose that $f : \bigcup_{i=1}^m Ai \longrightarrow \bigcup_{i=1}^m Ai$ satisfies the following conditions, where $A_{m+1} = A_1$:*

(i)$f(X) \subseteq g(X)$;
(ii)For all $x,y,z \in X$, we have $\psi(G(gx,gy,gz)) \geq F(\psi(G(fx,fy,fz)),\varphi(G(fx,fy,fz)))$, where $\psi \in \Psi$, $\varphi \in \Upsilon$ and $F \in \mathscr{C}_{inv}$;
(iii)f and g are either f-compatible of type (E) or g-compatible of type (E).

Then f and g have a common fixed point in X.

Theorem 111.[64] *Let f and g be weak semi compatible, R-weakly commuting of type A_f self-mappings of G-metric space (X,G) satisfying the following conditions:*

(i)$f(X) \subseteq g(X)$;
(ii)For all $x,y,z \in X$, we have $\psi(G(gx,gy,gz)) \geq F(\psi(G(fx,fy,fz)),\varphi(G(fx,fy,fz)))$, where $\psi \in \Psi$, $\varphi \in \Upsilon$ and $F \in \mathscr{C}_{inv}$;
(iii)f and g are either f-compatible or g-compatible;
(iv)f and g satisfy E.A. property.

Then f and g have a common fixed point in X.

3.57 Kumar, Arora and Mishra (2020)

Kumar et al. [65] defined the concept of almost $\mathscr{Z}$-contraction and proved some related fixed point theorems in the framework of G-metric spaces.

Let $\mathscr{Z}$ denote the family of all simulation functions ξ defined by (96).

Definition 102.[65] *Let (X,G) be a G-metric space and $\xi\in\mathscr{Z}$. Then the self-mapping $T:X\longrightarrow X$ is said to be almost $\mathscr{Z}$-contraction if for each $x,y,z\in X$, we can find a positive constant λ such that*

$$\xi(G(Tx,Ty,Tz),G(x,y,z)+\lambda M(x,y,z))\leq 0,$$

where

$$M(x,y,z)=\min\left\{\begin{array}{c}G(x,Ty,Ty),G(y,Tx,Tx),G(y,Tz,Tz),\\ G(z,Ty,Ty),G(z,Tx,Tx),G(x,Tz,Tz)\end{array}\right\}.$$

The main result of Kumar et al. [65] is the following.

Theorem 112.[65] *Let (X,G) be a complete G-metric space and $T:X\longrightarrow X$ be an almost $\mathscr{Z}$-contraction with respect to ξ. Then T has a fixed point u in X. Moreover, the sequence $\{T^nx_0\}$ converges to the fixed point of T for each $x_0\in X$.*

Theorem 113.[65] *Let (X,G) be a complete G-metric space and $T:X\longrightarrow X$ be an almost $\mathscr{Z}$-contraction with respect to ξ. If T has a fixed point, then it is unique.*

3.58 Chen, Zhu and Zhu (2021)

Chen et al. [66] proved some fixed point theorems in the framework of G-metric spaces that cannot be obtained from the existence results in the context of quasi-metric spaces as proposed by Jleli and Samet [38] and Samet et al. [39].

Definition 103.[66] *Let (X,G) be a G-metric space. A mapping $T:X\longrightarrow X$ is said to be a $G\phi$-contraction if there exists a (c)-comparison function ϕ such that for all $x,y\in X$,*

$$G(Tx,Ty,T^2y)\leq\phi(G(x,y,Ty)).$$

Definition 104.[66] *Let (X,G) be a G-metric space. A mapping $T:X\longrightarrow X$ is said to be a weak $G\phi$-contraction if there exists a (c)-comparison function ϕ such that for all $x\in X$,*

$$G(Tx,T^2x,T^3x)\leq\phi(G(x,Tx,T^2x)).$$

Denote by $\Omega(X,G\phi)$, the collection of all $G\phi$-contraction mappings and by $\Omega(X,WG\phi)$, the collection of all weak $G\phi$-contraction mappings on a G-metric space (X,G). Then clearly, $\Omega(X,G\phi)\subseteq\Omega(X,WG\phi)$.

The main result of Chen et al. [66] is the following.

Theorem 114.[66] *Let (X,G) be a complete G-metric space and $T:X\longrightarrow X$ be a G-continuous mapping. If T is a weak $G\phi$-contraction mapping, then T has a fixed point.*

Theorem 115.[66] *Let (X,G) be a complete G-metric space and $T:X\longrightarrow X$ be a G-continuous and onto mapping satisfying the following condition for all $x\in X$:*

$$G(Tx,T^2x,T^3x)\geq\lambda G(x,Tx,T^2x),$$

where $\lambda>1$. Then T has a fixed point.

Theorem 116.[66] *Let (X,G) be a complete G-metric space and $T:X\longrightarrow X$ be a G-continuous mapping satisfying the following condition for all $x\in X$:*

$$G(Tx,T^2x,T^2x)\leq\lambda G(x,Tx,T^2x),$$

or

$$G(Tx,Tx,T^2x)\leq\lambda G(x,Tx,T^2x),$$

where $0\leq\lambda<\frac{1}{3}$. Then T has a fixed point.

Theorem 117.[66] *Let (X,G) be a complete G-metric space and $T:X\longrightarrow X$ be a G-continuous mapping satisfying the following condition for all $x\in X$:*

$$G(Tx,T^2x,T^3x)\leq\frac{G(x,Tx,T^2x)+G(Tx,T^2x,T^3x)}{G(x,Tx,T^2x)+G(Tx,T^2x,T^3x)+\lambda}G(x,Tx,T^2x),$$

where $\lambda>0$. Then T has a fixed point.

3.59 Priyobarta, Khomdram, Rohen, Saleem (2021)

Priyobarta et al. [67] extended the concept of α-admissibility to generalized rational α-Geraghty contraction in G-metric space and established some related fixed point theorems.

Accordingly [67], denote by Γ, the collection of all functions $\rho:\mathbb{R}_+\longrightarrow[0,1)$ such that $\rho(t_n)\longrightarrow 1$ implies $t_n\longrightarrow 0$, where $\{t_n\}$ is a bounded sequence of positive real numbers.

Let $T:X\longrightarrow X$ and $\alpha:X\times X\times X\longrightarrow\mathbb{R}_+$ satisfy the properties of Definition (50). Then T is said to be α-admissible for all $x,y,z\in X$.

Definition 105.[67] *Let $T,S:X\longrightarrow X$ and $\alpha:X\times X\times X\longrightarrow\mathbb{R}_+$. Then the pair $\{T,S\}$ is said to be α-admissible if for all $x,y,z\in X$ we have*

$$\alpha(x,y,z)\geq 1\ \Rightarrow\ \alpha(Tx,Sy,Sz)\geq 1 \text{ and } \alpha(Sx,Ty,Tz)\geq 1.$$

Definition 106.[67] *Let $T:X\longrightarrow X$ and $\alpha:X\times X\times X\longrightarrow\mathbb{R}_+$. Then T is said to be triangular α-admissible if for all $x,y,z,a\in X$ we have:*

(i) $\alpha(x,y,z) \geq 1 \Rightarrow \alpha(Tx,Ty,Tz) \geq 1$;
(ii) $\alpha(x,a,a) \geq 1$ and $\alpha(a,y,z) \geq 1 \Rightarrow \alpha(x,y,z) \geq 1$.

Definition 107.[67] *Let (X,G) be a G-metric space and let $\alpha : X \times X \times X \longrightarrow \mathbb{R}_+$ be a function. Then the self-mappings $S,T : X \longrightarrow X$ are said to be a pair of generalized rational α-Geraghty contraction of type I if for all $x,y,z \in X$ and $\rho \in \Gamma$,*

$$\alpha(x,y,z)G(Tx,Sy,Sz) \leq \rho(M(x,y,z))M(x,y,z),$$

where

$$M(x,y,z) = \max\left\{ \begin{array}{c} \frac{G(x,Tx,Tx)G(y,Sy,Sy)}{1+G(x,y,z)+G(Tx,Sy,Sz)}, \frac{G(y,Sy,Sy)G(z,Sz,Sz)}{1+G(x,y,z)+G(Tx,Sy,Sz)}, \\ \frac{G(z,Sz,Sz)G(x,Tx,Tx)}{1+G(x,y,z)+G(Tx,Sy,Sz)}, G(x,y,z), G(Tx,Sy,Sz) \end{array} \right\}.$$

Definition 108.[67] *Let (X,G) be a G-metric space and let $\alpha : X \times X \times X \longrightarrow \mathbb{R}_+$ be a function. Then the self-mappings $S,T : X \longrightarrow X$ are said to be a pair of generalized rational α-Geraghty contraction of type II if for all $x,y,z \in X$ and $\rho \in \Gamma$,*

$$\alpha(x,y,y)G(Tx,Sy,Sy) \leq \rho(N(x,y,y))N(x,y,y),$$

where

$$N(x,y,y) = \max\left\{ \begin{array}{c} \frac{G(x,Tx,Tx)G(y,Sy,Sy)}{1+G(x,y,y)+G(Tx,Sy,Sy)}, \frac{G(y,Sy,Sy)G(y,Sy,Sy)}{1+G(x,y,y)+G(Tx,Sy,Sy)}, \\ G(x,y,y), G(Tx,Sy,Sy) \end{array} \right\}.$$

The main result of Priyobarta et al. [67] is the following.

Theorem 118.[67] *Let (X,G) be a complete G-metric space and let $\alpha : X \times X \times X \longrightarrow \mathbb{R}_+$ be a function. Let $S,T : X \longrightarrow X$ be two self-mappings satisfying the following conditions:*

(i) $\{T,S\}$ is pair of generalized rational α-Geraghty contraction mappings of type I;
(ii) $\{T,S\}$ is a pair of triangular α-admissible mappings;
(iii) there exists $x_0 \in X$ such that $\alpha(x_0,Tx_0,Tx_0) \geq 1$;
(iv) T and S are continuous.

Then a common fixed point exists for the pair $\{T,S\}$.

Theorem 119.[67] *Let (X,G) be a complete G-metric space and let $\alpha : X \times X \times X \longrightarrow \mathbb{R}_+$ be a function. Let $S,T : X \longrightarrow X$ be two self-mappings satisfying the following conditions:*

(i) $\{T,S\}$ is pair of generalized rational α-Geraghty contraction mappings of type I;
(ii) $\{T,S\}$ is a pair of triangular α-admissible mappings;
(iii) there exists $x_0 \in X$ such that $\alpha(x_0,Tx_0,Tx_0) \geq 1$;
(iv) if $\{x_n\}_{n\in\mathbb{N}}$ is a sequence in X such that $\alpha(x_n,x_{n+1},x_{n+1}) \geq 1$ for all $n \in \mathbb{N}\cup\{0\}$ and $x_n \longrightarrow a \in X$ as $n \to \infty$, then a subsequence $\{x_{n_k}\}$ of $\{x_n\}_{n\in\mathbb{N}}$ exists, satisfying $\alpha(x_{n_k},a,a) \geq 1$ for all k.

Then a common fixed point exists for the pair $\{T,S\}$.

3.60 Chary, Reddy, Işik, Aydi and Chary (2021)

Chary et al. [68] initiated the concept of rectangular α-G-admissibility with respect to β and considered related type of contractions in the setting of G-metric spaces.

Definition 109.[68] *Let $\psi \in \Phi$. Then ψ is said to an almost perfect function if it is non-decreasing and satisfies the following:*

(i) $\psi(t) = 0$ if and only if $t = 0$;
(ii) if $\{t_n\}_{n\in\mathbb{N}}$ is a sequence in $\mathbb{R}_+$ such that $\psi(t_n) \longrightarrow 0$, then $\{t_n\}_{n\in\mathbb{N}}$ converges to 0.

Definition 110.[68] *Let $\alpha,\beta : X \times X \times X \longrightarrow \mathbb{R}_+$ be two functions. Then a self-mapping $T : X \longrightarrow X$ is said to be rectangular α-G-admissible with respect to β if the following conditions are satisfied for all $x,y,z,a \in X$:*

(i) if $\alpha(x,y,z) \geq \beta(x,y,z)$, then $\alpha(Tx,Ty,Tz) \geq \beta(Tx,Ty,Tz)$;
(ii) if $\alpha(x,a,a) \geq \beta(x,a,a)$ and $\alpha(a,y,z) \geq \beta(a,y,z)$, then $\alpha(x,y,z) \geq \beta(x,y,z)$.

Definition 111.[68] *Let (X,G) be a G-metric space and let $\alpha,\beta : X \times X \times X \longrightarrow \mathbb{R}_+$ be two functions. Then a self-mapping $T : X \longrightarrow X$ is said to be an α-β-G-contraction if there exist $\lambda \in [0,1)$ and an almost perfect function ψ such that for all $x,y,z \in X$ with $\alpha(x,y,z) \geq \beta(x,y,z)$, we have*

$$\psi(G(Tx,Ty,Tz)) \leq \max\left\{ \begin{array}{c} \lambda\psi(G(x,y,z)), \lambda\psi(G(x,Tx,Tx)), \lambda\psi(G(y,Ty,Ty)), \\ \lambda\psi(G(z,Tz,Tz)), \lambda\psi\left(\frac{1}{4}[G(Tx,y,z)+G(x,Ty,Tz)]\right) \end{array} \right\}.$$

The main result of Chary et al. [68] is the following.

Theorem 120.[68] *Let (X,G) be a G-metric space and $\alpha,\beta : X \times X \times X \longrightarrow \mathbb{R}_+$ be two functions. Let $T : X \longrightarrow X$ be a self-mapping satisfying the following conditions:*

(i) (X,G) is α-β-G-complete;
(ii) T is an α-β-G-contraction;
(iii) T is rectangular α-G-admissible with respect to β;
(iv) T is α-β-G-continuous;
(v) there exists $x_0 \in X$ such that $\alpha(x_0,Tx_0,Tx_0) \geq \beta(x_0,Tx_0,Tx_0)$.

Then a fixed point exists for T.

3.61 Kumar and Arora (2022)

Kumar and Arora [69] introduced new notions of generalized F-contractions of type (S) and type (M) in G-metric spaces and established related fixed point theorems.

Consequently [69], denote by S_G the family of all functions $F : (0,\infty) \longrightarrow \mathbb{R}$ which satisfy the following conditions:

(i) F is strictly increasing;
(ii) $\lim_{n\to\infty} x_n = 0$ if and only if $\lim_{n\to\infty} F(x_n) = -\infty$, for every sequence $\{x_n\}_{n\in\mathbb{N}}$;
(iii) F is continuous on $(0,\infty)$.

Also, denote by M_G the family of all functions $F:(0,\infty)\longrightarrow\mathbb{R}$ which satisfy the following conditions:

(i) F is strictly increasing;
(ii) $\lim_{n\to\infty} x_n = 0$ if and only if $\lim_{n\to\infty} F(x_n) = -\infty$, for every sequence $\{x_n\}_{n\in\mathbb{N}}$;
(iii) there exists $m\in(0,1)$ such that $\lim_{x\to 0^+} x^m F(x) = 0$.

Definition 112.[69] *Let (X,G) be a G-metric space. A mapping $T:X\longrightarrow X$ is said to be a modified generalized F-contraction of type (S) if $F\in S_G$ and there exists $\tau>0$ such that for all $x,y,z\in X$,*

$$G(Tx,Ty,Tz)>0 \Rightarrow \tau+F(G(Tx,Ty,Tz))\leq F(S(x,y,z)),$$

where

$$S(x,y,z)=\max\left\{\begin{array}{c} G(x,Ty,Ty),G(y,Tx,Tx),G(y,Tz,Tz),\\ G(z,Ty,Ty),G(z,Tx,Tx),G(x,Tz,Tz)\end{array}\right\}.$$

Definition 113.[69] *Let (X,G) be a G-metric space. A mapping $T:X\longrightarrow X$ is said to be a modified generalized F-contraction of type (M) if $F\in M_G$ and there exists $\tau>0$ such that for all $x,y,z\in X$,*

$$G(Tx,Ty,Tz)>0 \Rightarrow \tau+F(G(Tx,Ty,Tz))\leq F(S(x,y,z)),$$

where

$$S(x,y,z)=\max\left\{\begin{array}{c} G(x,Ty,Ty),G(y,Tx,Tx),G(y,Tz,Tz),\\ G(z,Ty,Ty),G(z,Tx,Tx),G(x,Tz,Tz)\end{array}\right\}.$$

Kumar and Arora [69] obtained the following main results.

Theorem 121.[69] *Let (X,G) be a complete G-metric space and $T:X\longrightarrow X$ be a modified generalized F-contraction of type (S). Then T has a unique fixed point $u\in X$ and the sequence $\{T^n x_0\}_{n\in\mathbb{N}}$ converges to u.*

Theorem 122.[69] *Let (X,G) be a complete G-metric space and $T:X\longrightarrow X$ be a modified generalized F-contraction of type (M). Then T has a unique fixed point $u\in X$ and the sequence $\{T^n x_0\}_{n\in\mathbb{N}}$ converges to u.*

3.62 Jiddah, Alansari, Mohamed, Shagari, Bakery (2022)

Jiddah et al. [70] introduced a new family of hybrid contractions in the framework of G-metric space and obtained related fixed point results that are not deducible from their corresponding ones in metric space. The preeminence of this class of contractions is that it subsumes some well-known results in the literature and its contractive inequality can be extended in a variety of manners, depending on the given parameters.

Definition 114.[70] *Let (X,G) be a G-metric space. A self-mapping $T:X\longrightarrow X$ is called a Jaggi-type hybrid $(G\text{-}\phi)$-contraction, if there exists $\phi\in\Phi$ such that*

$$G(Tx,Ty,T^2y)\leq\phi(M(x,y,Ty)),$$

for all $x,y\in X\backslash Fix(T)$, where

$$M(x,y,Ty)=\begin{cases}\left[\lambda_1\left(\frac{G(x,Tx,T^2x)\cdot G(y,Ty,T^2y)}{G(x,y,Ty)}\right)^q+\lambda_2 G(x,y,Ty)^q\right]^{\frac{1}{q}}, & \\ \qquad\qquad\qquad for \quad q>0; & \\ G(x,Tx,T^2x)^{\lambda_1}\cdot G(y,Ty,T^2y)^{\lambda_2}, \quad for \quad q=0, & \end{cases}$$

$\lambda_1,\lambda_2\geq 0$ with $\lambda_1+\lambda_2=1$ and $Fix(T)=\{x\in X: Tx=x\}$.

The main results of Jiddah et al. [70] is the following.

Theorem 123.[70] *Let (X,G) be a complete G-metric space and let $T:X\longrightarrow X$ be a continuous Jaggi-type hybrid $(G\text{-}\phi)$-contraction on (X,G). Then T has a fixed point in X (say z), and for any $z_0\in X$, the sequence $\{T^n z_0\}_{n\in\mathbb{N}}$ converges to z.*

Theorem 124.[70] *Let (X,G) be a complete G-metric space and let $T:X\longrightarrow X$ be a Jaggi-type hybrid $(G\text{-}\phi)$-contraction. If for some integer $i>2$, we have that T^i is continuous, then T has a unique fixed point in X.*

3.63 Jiddah, Noorwali, Shagari, Rashid, Jarad (2022)

Jiddah et al. [71] obtained some fixed point results of a new family of hybrid contractions in G-metric space which complement some well-known contractions, including that of Reich and Istrăţescu. Their results cannot be reduced to any existence result in the manner of Jleli and samet [38] or Samet et al. [39].

Definition 115.[71] *Let $\alpha: X\times X\times X\longrightarrow\mathbb{R}_+$ be a function. A self-mapping $T:X\longrightarrow X$ is called $(G\text{-}\alpha)$-orbital admissible if for all $x\in X$,*

$$\alpha(x,Tx,T^2x)\geq 1 \;\Rightarrow\; \alpha(Tx,T^2x,T^3x)\geq 1.$$

Definition 116.[71] *Let (X,G) be a G-metric space and let $\alpha: X\times X\longrightarrow\mathbb{R}_+$ be a function. A self-mapping $T: X\longrightarrow X$ is called hybrid-interpolative Reich-Istrăţescu-type $(G\text{-}\alpha\text{-}\mu)$-contraction if for some $q\in[0,\infty)$, there exist constants $\mu\in(0,1)$, $\delta\geq 0$ and $\lambda_i\geq 0$ with $i=1,2,\ldots,5$ such that for all $x,y\in X\backslash Fix(T)$,*

$$\alpha(x,y,Ty)G(T^2x,T^2y,T^3y)\leq\mu\mathscr{R}_{\mathscr{I}}(x,y,Ty),$$

where

$$\mathscr{R}_{\mathscr{G}}(x,y,Ty) = \begin{cases} \Big[\lambda_1 G(x,y,Ty)^q + \lambda_2 G(x,Tx,T^2x)^q + \lambda_3 G(y,Ty,T^2y)^q \\ +\lambda_4 G(Tx,Ty,T^2y)^q + \lambda_5 G(Tx,T^2x,T^3x)^q + \delta G(Ty,T^2y,T^3y)^q\Big]^{\frac{1}{q}}, \\ \qquad \text{for } q>0, \text{ with } \sum_{i=1}^{5}\lambda_i + \delta \le 1; \\ [G(x,y,Ty)]^{\lambda_1}\cdot[G(x,Tx,T^2x)]^{\lambda_2}\cdot[G(y,Ty,T^2y)]^{\lambda_3} \\ \cdot[G(Tx,Ty,T^2y)]^{\lambda_4}\cdot[G(Tx,T^2x,T^3x)]^{\lambda_5}\cdot[G(Ty,T^2y,T^3y)]^{\delta}, \\ \qquad \text{for } q=0, \text{ with } \sum_{i=1}^{5}\lambda_i + \delta = 1. \end{cases}$$

Jiddah et al. [71] obtained the following main results.

Theorem 125.*[71] Let (X,G) be a complete G-metric space and let $T : X \longrightarrow X$ be a hybrid-interpolative Reich-Istrăţescu-type $(G\text{-}\alpha\text{-}\mu)$-contraction satisfying the following conditions:*

(i)T is $(G\text{-}\alpha)$-orbital admissible;
(ii)T is continuous;
(iii)there exists $x_0 \in X$ such that $\alpha(x_0,Tx_0,T^2x_0) \ge 1$.

Then T has at least a fixed point in X.

Theorem 126.*[71] Let (X,G) be a complete G-metric space and let $T : X \longrightarrow X$ be a hybrid-interpolative Reich-Istrăţescu-type $(G\text{-}\alpha\text{-}\mu)$-contraction satisfying the following conditions:*

(i)T is $(G\text{-}\alpha)$-orbital admissible;
(ii)there exists $x_0 \in X$ such that $\alpha(x_0,Tx_0,T^2x_0) \ge 1$;
(iii)T^3 is continuous and $\alpha(x,Tx,T^2x) \ge 1$ for any $x \in Fix(T^3)$.

Then T has at least a fixed point in X.

Theorem 127.*[71] If in addition to the hypotheses of Theorem 126, we assume supplementary that $\alpha(x,y,Ty) \ge 1$ for any $x,y \in Fix(T)$, then the fixed point of T is unique.*

4 Conclusion

In this project, some important advancements in invariant results of G-metric spaces are surveyed. It is observed herein that the earliest versions of fixed point results in G-metric spaces along with t hose ones in parallel directions are deducible from their counterparts in quasi-metric spaces. However, a more robust techniques have recently been developed which make it impossible to collapse many fixed point theorems in G-metric spaces to their analogues in metric and quasi-metric spaces. Consequently, the latter ideas so established make the research in fixed point concepts of generalized metric spaces worthhile.

Competing Interests

The authors declare that they have no competing interests.

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