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SHORT COMMUNICATIONS

# Forecasting of 2-3 Prey-Predator Using Aiyesimi Non-Diffused Mathematical Model

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## ABSTRACT

The model is based on the interactions of three Predators competing for two Preys. The equilibrium points are initially evaluated to arrive at seven equilibrium points. To analyse this using local stability, the Jacobian matrix is reduced using row transformation and then the eigenvalues were obtained. The solution is then examined, to determine the criteria for the stability of the system. The stability of the system was found to depend intrinsically on the level of interaction of the competing species as well as the object of competition in which the absence of one or more species leads to the stagnation of the others. Hence, it is observed that, at is the population interaction is not sustainable over time while at the population interaction is continuous over time.

**Keywords**: Prey-Predator, Non-Diffusive, Local Stability

## INTRODUCTION

Hsu (1981) proposed a mathematical model for the two predator species exploiting a single prey. He found out that when the interspecific interference coefficient is small, the winner competes with rivals successfully. Mitra*et al.* (1992) studied the permanent coexistence and global\newline stability of a simple Lotka-Volterra type mathematical model of a living resource supporting two predators. The study showed that the permanent coexistence of the system depends on the threshold of the ratio between the coefficients of numerical responses of the two predators/consumers. In their investigation Dubeyand Das (2000) proposed a Guass-type model with diffusion of which is analyzed. For the research, they considered a system of two predators competing with interference for a limited prey. They showed that in the absence of intraspecific interaction of the predator series, the interior equilibrium is unstable. Harrison (1979) examined the global stability of competing Predators over a limited Prey. The research shows that for limited Preys, the system is said to be unstable. This present research is therefore intended to analyze the local stability of the model investigated in (Aiyesimi*et al.*, 2016). The model consists of three interacting predators competing for two preys for which both preys are independent and are not competing against each other. To achieve this, we solve the Jacobian matrix using Row Transformation Method.

### The Equilibrium State of Model Without Diffusion

The model in Aiyesimi*et al.* (2016) are based on the general Volterra formulation of competing species as used in Elettreby (2009) which is listed below;

$$\frac{dx_1}{dt} = x_1(r_1 - \alpha_1 y_1 - \alpha_2 y_2) \quad (1)$$

$$\frac{dx_2}{dt} = x_2(r_2 - \beta_1 y_2 - \beta_2 y_3) \quad (2)$$

$$\frac{dy_1}{dt} = -y_1(s_3 - \sigma_1 x_1 - \sigma_2 y_2 - \sigma_3 y_3) \quad (3)$$

$$\frac{dy_2}{dt} = -y_2(s_4 - \delta_1 x_1 - \delta_2 x_2 + \delta_3 y_1) \quad (4)$$

$$\frac{dy_3}{dt} = -y_3(s_5 - \phi_1 x_2 + \phi_2 y_1) \quad (5)$$

in which $y_1$ is Predator1, $y_2$ is Predator2, $y_3$ is Predator3, $x_1$ is Prey1 and $x_2$ is Prey2

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$\alpha_1, \alpha_2, \beta_1, \beta_2, \sigma_1, \sigma_2, \sigma_3, \delta_1, \delta_2, \delta_3, \phi_1, \phi_2$ are interspecies interaction coefficients, $r_1$ and $r_2$ the birth rates of Prey1 and Prey2 respectively, $s_3, s_4, s_5$ are death rates of Predator 1, Predator 2 and Predator 3 respectively.

$t$ is the time

The equilibrium positions of the prey-predator interactions dynamics is given by;

$$\frac{dx_1}{dt} = \frac{dx_2}{dt} = \frac{dy_1}{dt} = \frac{dy_2}{dt} = \frac{dy_3}{dt} = 0 \qquad (6)$$

Therefore, there exist the following equilibria namely $E_0(0,0,0,0,0)$ $E_1(0, x_2', y_1', y_2', y_3')$ $E_2(x_1' 0, y_1', y_2', y_3'), E_3(x_1' x_2' 0, y_2', y_3'), E_4(x_1' x_2', x_3', 0, y_3'),$ $E_5(x_1' x_2', y_1', y_2', 0)$ and $E_6(x_1', x_2', y_1', y_2', y_3')$

We thus have the following:

$$E_0(0,0,0,0,0) = (0,0,0,0,0) \qquad (7)$$

$$E_1(0, x_2', y_1', y_2', y_3') = \begin{pmatrix} 0, \dfrac{(\phi_2 s_4 - \delta_3 s_5)}{(\phi_2\delta_2 - \phi_1\delta_3)}, \dfrac{(\phi_1 s_4 - s_5\delta_2)}{(\phi_2\delta_2 - \phi_1\delta_3)}, \\ \dfrac{(\beta_2 s_3 - r_2\sigma_3)}{(\sigma_2\beta_2 - \beta_1\sigma_3)}, \dfrac{(\sigma_2 r_2 - \beta_1 s_3)}{(\sigma_2\beta_2 - \beta_1\sigma_3)} \end{pmatrix} \qquad (8)$$

$$E_2(x_1' 0, y_1', y_2', y_3') = \begin{pmatrix} \dfrac{s_4\phi_2 - s_5\delta_3}{\delta_1\phi_2}, 0, \dfrac{-s_5}{\phi_2}, \dfrac{r_1\phi_2 + \alpha_1 s_5}{\alpha_2\phi_2}, \\ \left( \dfrac{s_3\delta_1\phi_2\alpha_2 - \sigma_1\alpha_2(s_4\phi_2 - s_5\delta_3) - \sigma_2\delta_1(r_1\phi_2 + \alpha_1 s_5)}{\alpha_2\phi_2\sigma_3\delta_1} \right) \end{pmatrix} \qquad (9)$$

$$E_3(x_1', x_2', 0, y_2', y_3') = \begin{pmatrix} \dfrac{s_4\phi_1 - \delta_2 s_5}{\delta_1\phi_1}, \dfrac{s_5}{\phi_1} \\ , 0, \dfrac{r_1}{\alpha_2}, \dfrac{r_2\alpha_2 - r_1\beta_1}{\alpha_2\beta_2} \end{pmatrix} \qquad (10)$$

$$E_4(x_1', x_2', y_1', 0, y_3') = \begin{pmatrix} \dfrac{s_3\beta_2 - \sigma_3 r_2}{\sigma_1\beta_2}, \\ \dfrac{\alpha_1 s_5 + r_1\phi_3}{\alpha_1\phi_1}, \dfrac{r_1}{\alpha_1}, 0, \dfrac{r_2}{\beta_2} \end{pmatrix} \qquad (11)$$

$$E_5(x_1', x_2', y_1', y_2', 0) = \begin{pmatrix} \dfrac{s_3\beta_1 - \sigma_3 r_2}{\sigma_1\beta_1}, \dfrac{\begin{pmatrix} \alpha_1\beta_1 s_5 + \\ \phi_2(\beta_1 r_1 - \alpha_2 r_2) \end{pmatrix}}{\alpha_1\phi_1\beta_1}, \\ \dfrac{\beta_1 r_1 - \alpha_2 r_2}{\alpha_1\beta_1}, \dfrac{r_2}{\beta_1}, 0 \end{pmatrix} \qquad (12)$$

$$E_6(x_1', x_2', y_1', y_2', y_3') = \begin{pmatrix} \dfrac{\begin{pmatrix} -\beta_2\begin{pmatrix} \alpha_3 B \\ +\sigma_2 A \end{pmatrix} \\ -\sigma_3\begin{pmatrix} r_2 B \\ -\beta_1 A \end{pmatrix} \end{pmatrix}}{\sigma_1\beta_2 B}, \dfrac{\begin{pmatrix} -\alpha_5\alpha_1 B \\ +\phi_2\begin{pmatrix} r_1 B \\ -\alpha_2 A \end{pmatrix} \end{pmatrix}}{\alpha_1\phi_1 B} \\ \dfrac{r_1 B - \alpha_2 A}{\alpha_1 B}, \dfrac{A}{B}, \dfrac{r_2 B - \beta_1 A}{\beta_2 B} \end{pmatrix} \qquad (13)$$

Where,

$$A = r_1\sigma_1\beta_2(\delta_2\phi_2 - \delta_3\phi_1) - \delta_1\alpha_1\phi_1(s_3\beta_2 - r_2\sigma_3) + s_4\sigma_1\beta_2\alpha_1\phi_1 + \delta_2\sigma_1\beta_2\alpha_1 s_5 \qquad (14)$$

$$B = \delta_1\alpha_1\phi_1(\sigma_3\beta_1 - \beta_2\sigma_2) - \delta_2\beta_2\sigma_1\alpha_2\phi_2 + \delta_3\beta_2\sigma_1\alpha_2\phi_1 \qquad (15)$$

**Local stability of the equilibrium state of model without diffusion**

The Jacobian matrix based on (Bicout, 2013) of this modelis given as:

$$J(X) = \begin{bmatrix} \begin{pmatrix} r_1 - \alpha_1 y_1 \\ -\alpha_2 y_2 \end{pmatrix} & 0 & -\alpha_1 x_1 & -\alpha_2 x_1 & 0 \\ 0 & \begin{pmatrix} r_2 - \beta_1 y_2 \\ -\beta_2 y_3 \end{pmatrix} & 0 & -\beta_1 x_2 & -\beta_2 x_2 \\ -\ \sigma_1 y_1 & 0 & \begin{pmatrix} r_3 - s_3 + \sigma_1 x_1 \\ +\sigma_2 y_2 + \sigma_3 y_3 \end{pmatrix} & \sigma_2 y_1 & \sigma_3 y_1 \\ \delta_1 y_2 & \delta_2 y_2 & -\delta_2 y_3 & \begin{pmatrix} r_4 - s_4 + \delta_1 x_1 \\ +\delta_2 x_2 - \delta_3 y_1 \end{pmatrix} & 0 \\ 0 & \phi_1 y_3 & -\phi_2 y_3 & 0 & \begin{pmatrix} r_3 - s_3 + \phi_1 x_2 \\ -\phi_2 y_1 \end{pmatrix} \end{bmatrix} \qquad (16)$$

Using the row reduction operations this is transformed into

$$J(X) = \begin{bmatrix} a_1 & 0 & a_2 & a_3 & 0 \\ 0 & b_1 & 0 & b_2 & b_3 \\ 0 & 0 & \left(\dfrac{a_1c_2 - a_2c_1}{a_1}\right) & \left(\dfrac{a_1c_3 - a_3c_1}{a_1}\right) & c_4 \\ 0 & 0 & 0 & M_1 & M_2 \\ 0 & 0 & 0 & 0 & \left(\dfrac{M_1N_2 - M_2N_1}{M_1}\right) \end{bmatrix} \qquad (17)$$

Therefore, we have the following as the characteristic equation of the system:

$$|J - \lambda I| = \begin{vmatrix} a_1 - \lambda & 0 & a_2 & a_3 & 0 \\ 0 & b_1 - \lambda & 0 & b_2 & b_3 \\ 0 & 0 & \left(\dfrac{a_1c_2 - a_2c_1}{a_1}\right) - \lambda & \left(\dfrac{a_1c_3 - a_3c_1}{a_1}\right) & c_4 \\ 0 & 0 & 0 & M_1 - \lambda & M_2 \\ 0 & 0 & 0 & 0 & \left(\dfrac{M_1N_2 - M_2N_1}{M_1}\right) - \lambda \end{vmatrix} = 0 \qquad (18)$$

Which results in

$$(a_1-\lambda)(b_1-\lambda)\left(\left(\frac{a_1c_2-a_2c_1}{a_1}\right)-\lambda\right)(M_1-\lambda)\left(\left(\frac{M_1N_2-M_2N_1}{M_1}\right)-\lambda\right)=0 \quad (19)$$

At Equilibrium point $E_0(0,0,0,0,0)=(0,0,0,0,0)$, we obtain

$$\lambda_1=r_1,\lambda_2=r_2,\lambda_3=-s_3,\lambda_4=-s_4,\lambda_5=-s_5 \quad (20)$$

which is considered to be unstable in view of the appear of positive eigenvalues.

On the other hand at the equilibrium point $E_1(0,x_2',y_1',y_2',y_3')$ the we have eigenvalues as

$$\left.\begin{aligned}
&\lambda_1=r_1-\alpha_1\gamma_2-\alpha_2\gamma_3\\
&\lambda_2=r_2-\beta_1\gamma_3-\beta_2\gamma_4,\\
&\lambda_3=-s_3+\sigma_2\gamma_3+\sigma_3\gamma_4,\\
&\lambda_4=d_4-\frac{b_2d_2}{b_1}-\frac{c_3d_3}{c_2},\\
&\lambda_5=\frac{M_{1A}N_{2A}-M_{2A}N_{1A}}{M_{1A}}
\end{aligned}\right\} \quad (21)$$

Where

$$\left.\begin{aligned}
\gamma_1&=\frac{(\phi_2\alpha_4-\delta_3\alpha_5)}{(\phi_1\delta_3-\phi_2\delta_2)}\\
\gamma_2&=\frac{(\phi_1\alpha_4-\alpha_5\delta_2)}{(\phi_1\delta_3-\phi_2\delta_2)}\\
\gamma_3&=\frac{(\beta_2\alpha_3-r_2\sigma_3)}{(\sigma_2\beta_2-\beta_1\sigma_3)}\\
\gamma_4&=\frac{(\sigma_2r_2-\beta_1\alpha_3)}{(\sigma_2\beta_2-\beta_1\sigma_3)}\\
M_{1A}&=\frac{b_1c_2d_4-b_2c_2d_2-b_1c_3d_3}{b_1c_2}\\
M_{2A}&=\frac{-c_4d_3}{c_2}\\
N_{1A}&=\frac{-b_2c_2e_1-b_1c_3e_2}{b_1c_2}\\
N_{2A}&=\frac{b_1c_2e_3-b_3c_2e_1-b_1c_4e_2}{b_1c_2}
\end{aligned}\right\} \quad (22)$$

The conditions for a stable system are,

$$\left.\begin{aligned}
&\beta_1\gamma_3+\beta_2\gamma_4>r_2\\
&s_3>\sigma_2\gamma_3+\sigma_3\gamma_4\\
&\frac{b_2d_2}{b_1}+\frac{c_3d_3}{c_2}>d_4\\
&M_{2A}N_{1A}>M_{1A}N_{2A}
\end{aligned}\right\} \quad (23)$$

At the equilibrium point, $E_2(x_1',0,y_1',y_2',y_3')$ we obtain;

$$\left.\begin{aligned}
&\lambda_1=r_1-\alpha_1\gamma_6-\alpha_2\gamma_7=a_{1B}\\
&\lambda_2=r_2-\beta_1\gamma_7-\beta_2\gamma_8=b_{1B}\\
&\lambda_3=\frac{a_{1B}c_{2B}-a_{2B}c_{1B}}{a_{1B}},\\
&\lambda_4=d_{4B}-\frac{a_{3B}d_{1B}}{a_{1B}}-\left(\frac{a_{1B}d_{3B}-a_{3B}d_{1B}}{a_{1B}c_{2B}-a_{2B}c_{1B}}\right)\left(\frac{a_{1B}c_{3B}-a_{3B}c_{1B}}{a_{1B}}\right),\\
&\lambda_5=\frac{M_{1B}N_{2B}-M_{2B}N_{1B}}{M_{1B}}
\end{aligned}\right\} \quad (24)$$

Where

$$\left.\begin{aligned}
\gamma_5&=\frac{\alpha_5\delta_3-\alpha_4\phi_2}{\delta_1\phi_2}\\
\gamma_6&=\frac{\alpha_5}{\phi_2}\\
\gamma_7&=\frac{r_1\phi_2-\alpha_1\alpha_5}{\alpha_2\phi_2}\\
\gamma_8&=\frac{(-\alpha_3\delta_1\phi_2\alpha_2-\sigma_1\alpha_2(\alpha_5\delta_3-\alpha_4\phi_2)-\sigma_4\delta_1(r_1\phi_2-\alpha_1\alpha_5))}{\sigma_3\phi_2\delta_1\alpha_2}\\
M_{1B}&=d_{4B}-\frac{a_{3B}d_{1B}}{a_{1B}}-\left(\frac{a_{1B}d_{3B}-a_{3B}d_{1B}}{a_{1B}c_{2B}-a_{2B}c_{1B}}\right)\left(\frac{a_{1B}c_{3B}-a_{3B}c_{1B}}{a_{1B}}\right)\\
M_{2B}&=-c_{4B}\left(\frac{a_{1B}d_{3B}-a_{3B}d_{1B}}{a_{1B}}\right)\\
N_{1B}&=\frac{-b_{2B}e_{1B}}{b_{1B}}-\frac{e_{2B}(a_{1B}c_{3B}-a_{3B}c_{1B})}{(a_{1B}c_{2B}-a_{2B}c_{1B})}\\
N_{2B}&=\left(\frac{b_{1B}e_{3B}-b_{3B}e_{1B}}{b_{1B}}\right)-\frac{a_{1B}e_{2B}c_{4B}(a_{1B}c_{3B}-a_{3B}c_{1B})}{(a_{1B}c_{2B}-a_{2B}c_{1B})}
\end{aligned}\right\} \quad (25)$$

The conditions for a stable system are,

$$\left.\begin{aligned}
&\alpha_1\gamma_6+\alpha_2\gamma_7>r_1\\
&a_{2B}c_{1B}>a_{1B}c_{2B}\\
&M_{1B}<0\\
&M_{2B}N_{1B}>M_{1B}N_{2B}
\end{aligned}\right\} \quad (26)$$

Also at the equilibrium point $E_3(x_1',x_2',0,y_2',y_3')$ we deduce that;

$$\left.\begin{aligned} \lambda_1 &= 0 = a_{1C} \\ \lambda_2 &= r_2 - \beta_1\gamma_{11} - \beta_2\gamma_{12} = b_{1C} \\ \lambda_3 &= \frac{a_{1C}c_{2C} - a_{2C}c_{1C}}{a_{1C}} = Lim_{a_{1C}\to 0} = c_{2c}, \\ \lambda_4 &= d_{4C} - \frac{b_{2C}d_{2C}}{b_{1c}} = M_{1C}, \\ \lambda_5 &= N_{2B} \end{aligned}\right\} \quad (27)$$

Where

$$\left.\begin{aligned} \gamma_9 &= \frac{s_5\phi_1 - \delta_2 s_5}{\delta_1\phi_1} \\ \gamma_{10} &= \frac{s_5}{\phi_1} \\ \gamma_{11} &= \frac{r_1}{\alpha_2} \\ \gamma_{12} &= \frac{r_2\alpha_2 - r_1\beta_1}{\alpha_2\beta_2} \\ M_{1C} &= d_{4C} - \frac{b_{2C}d_{2C}}{b_{1C}} \\ M_{2C} &= 0 \\ N_{1C} &= \frac{-b_{2C}e_{1C}}{b_{1C}} \\ N_{2C} &= \left(\frac{b_{1C}e_{3C} - b_{3C}e_{1C}}{b_{1C}}\right) \end{aligned}\right\} \quad (28)$$

The conditions for a stable system are,

$$\left.\begin{aligned} &\beta_1\gamma_{11} + \beta_2\gamma_{12} > r_2 \\ &s_3 > \sigma_1\gamma_9 + \sigma_2\gamma_{11} + \sigma_3\gamma_{12} \\ &\frac{b_{2C}d_{2C}}{b_{1C}} > d_{4C} \\ &N_{2B} < 0 \end{aligned}\right\} \quad (29)$$

At the equilibrium point $E_4(x_1'x_2', x_3', 0, y_3')$, we have

$$\left.\begin{aligned} \lambda_1 &= 0 = a_{1D} \\ \lambda_2 &= 0 = b_{1D} \\ \lambda_3 &= s_3 + \sigma_1\gamma_{13} + \sigma_3\gamma_{16} = c_{2D}, \\ \lambda_4 &= d_{4D} = M_{1D}, \\ \lambda_5 &= N_{2D} \end{aligned}\right\} \quad (30)$$

Where

$$\left.\begin{aligned} \gamma_{13} &= \frac{s_3\beta_2 - \sigma_3 r_2}{\sigma_1\beta_2} \\ \gamma_{14} &= \frac{\alpha_1 s_5 + r_1\phi_3}{\alpha_1\phi_1} \\ \gamma_{15} &= \frac{r_1}{\alpha_1} \\ \gamma_{16} &= \frac{r_2}{\beta_2} \\ M_{1D} &= d_{4D} \\ M_{2C} &= 0 \\ N_{1D} &= \frac{-b_{2D}e_{1D}}{b_{1D}} \\ N_{2D} &= \left(\frac{b_{1D}e_{3D} - b_{3D}e_{1D}}{b_{1D}}\right) \end{aligned}\right\} \quad (31)$$

The conditions for a stable system are,

$$\left.\begin{aligned} &s_3 > \sigma_1\gamma_{13} + \sigma_3\gamma_{16} \\ &d_{4D} < 0 \\ &N_{2D} < 0 \end{aligned}\right\} \quad (32)$$

At the equilibrium point $E_5(x_1'x_2', y_1', y_2', 0)$, we have

$$\left.\begin{aligned} \lambda_1 &= 0 = a_{1D} \\ \lambda_2 &= 0 = b_{1D} \\ \lambda_3 &= -s_3 + \sigma_1\gamma_{17} + \sigma_2\gamma_{20} = c_{2E} \\ \lambda_4 &= M_{1E}, \\ \lambda_5 &= 0 \end{aligned}\right\} \quad (33)$$

Where

$$\left.\begin{aligned} \gamma_{13} &= \frac{s_3\beta_2 - \sigma_3 r_2}{\sigma_1\beta_2} \\ \gamma_{14} &= \frac{\alpha_1 s_5 + r_1\phi_3}{\alpha_1\phi_1} \\ \gamma_{15} &= \frac{r_1}{\alpha_1} \\ \gamma_{16} &= \frac{r_2}{\beta_2} \\ M_{1E} &= d_{4E} \\ M_{2E} &= 0 \\ N_1 &= 0 \\ N_{2D} &= 0 \end{aligned}\right\} \quad (34)$$

The conditions for a stable system are,

$$\left.\begin{aligned} &s_3 > \sigma_2\gamma_{17} + \sigma_2\gamma_{20} \\ &d_{4E} > 0 \end{aligned}\right\} \quad (35)$$

**DISCUSSION**

At $E_0$ equilibrium point, the system is said to be unstable, and as such the population of the predator vanishes over time as the Population of the Prey continues to rise. This explains the phenomenon in Deka and Dubey (2016), where the predators naturally goes into extinction.

At $E_1$ equilibrium Prey 1 is not present; hence the condition of stability is based on the interaction of the remaining Predators against the remaining Prey.

At $E_2$ equilibrium Prey 2 is not present; hence the condition of stability is based on the interaction of the remaining Predators against the remaining Prey.

At $E_3$ equilibrium Predator 1 is not present; hence the condition of stability is based on the interaction of the remaining Predators against the remaining Prey.

At $E_4$ equilibrium Predator 2 is not present; hence the condition of stability is based on the interaction of the remaining Predators against the remaining Prey.

At $E_5$ equilibrium Predator 3 is not present; hence the condition of stability is based on the interaction of the remaining Predators against the remaining Prey.

At $E_6$ equilibrium all species are present; hence the condition of stability is based on the interaction of the three Predators against two Prey.

Equilibria points $E_1$-$E_6$ explains the nature of how the population dynamics of both preys and predators in the environment specified in Kissui (2008) will behave given the different senario it is predicated upon, hence shows the right approach in maintaining the prey-predator dynamics in a Wildlife environment.

**Conclusion**

In Prey-Predator System, the stability of the system is based on the level of interaction of competing speciesas well as what they are competing for. At equilibrium points often the absence of one or two specie leads to the stagnation of the population of other species.Multiple Prey-Predator interactions often exhibit similar growth pattern among the prey and also among the predators. Hence, we note that the stability of the system is dynamic depending on the state of interaction between all the species that are involved (i.e. Prey or Predator).

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