Strength Optimisation of Concrete Based on Scheffe's Model using Bida Gravel as Partial Replacement for Crushed Granite

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Abstract
In Nigeria, the need to source for cheap and locally available materials like Bida gravel as a replaceable substitute for granite without necessarily compromising the structural integrity of the concrete arise due to the rapid rise in the cost of crushed granite (which is an important ingredient in concrete. This research work involves the application of Scheffe's optimisation technique to obtain the statistical model of response function for the optimisation of compressive strength of a five-component concrete made with water, cement, sand, granite and Bida gravel. A total of ninety (90) standard 150mm x 150mm x 150 mm cubes were cast, consisting of three cubes per mix ratio and for a total of thirty (30) mix ratios. The first fifteen was used to determine the coefficients of the model, while the other fifteen were used to validate the model. The maximum compressive strength predicted by this model was 29.93N/mm^2 corresponding to mix ratio of 0.45:1:1:1.9:0.1 for water: cement: sand: granite: Bida gravel. A MATLAB program was used to ease the use of the model. The output of the statistical model compared favorably with the corresponding experimental results and the predictions from the response function were tested with statistical Fischer test and found to be adequate at 95% accuracy level. The model delivered in this study can be used for optimising and predicting the future 28th days compressive strength of concrete made from water, cement, sand, granite and Bida gravel as partial replacement for granite when the mix proportions of the concrete is known and vice versa.

Keywords: Bida gravel, compressive strength, concrete, optimisation, Scheffe's, statistical model.

Introduction

One of the basic needs of man is housing and indeed the built environment is of paramount importance to every living creature all over the world. In many developing countries like Nigeria, there is a perpetual problem of accommodation and inadequate housing among other infrastructural facilities deficit. A previous research showed that about seven million Nigerians have no accommodation (Uwe, 2010). It is essential to note that majority of housing units in Nigeria are constructed using concrete, which has coarse aggregate as a basic constituent.

According to Neville and Brook (2010), concrete is a product of water, cement and aggregate, and when sufficiently hardened, is used in carrying various loads. However, in Nigeria due to the rapid rise in the cost of crushed granite (which is an important ingredient in concrete) there is a need to source for cheap and locally available materials like Bida gravel as a replaceable substitutes for granite. Gravel material commonly used as concrete in split (broken stone). However, in certain conditions, it take an action when the aggregate crushed stone is difficult to get due to the limited availability and price is relatively expensive so that the material required replacement of broken stone material. Using the materials available around us, of course, if and only if, the material is still in accordance with the specifications of materials that can be used as a mixture of concrete (Limantara *et al.*, 2018).

Aggregates are granular materials, which are acquired naturally, artificially or as recycled. It is well-known that the mineralogy and properties of aggregate, which are derived from naturally formed bed or crushed rock, significantly influence the strength and stability of concrete. The strength of aggregate, significantly contributes to the strength of concrete, since aggregate constitute more than 75 % of the volume of concrete (Ghambhir, 2013). Porosity, grade, size distribution, moisture content, shape, break strength, surface texture, modulus of elasticity, impurities of aggregates are significant for the technology of concrete. These properties of the aggregate result from mineralogical composition of the host rock or the features of formation (Mehta and Monteiro, 2006).

The Bida natural deposits aggregates occur in middle Niger basin of Nigeria in several million metric tons. The aggregate deposits are greatly used in Bida for building constructions and for domestic dwelling units. Usually, most natural deposits are usually not too deep and in most cases like the deposits in Bida accessible with extreme effort and are usually surfaced mined. The natural deposits aggregates are cheaper than crushed rock aggregates and it is used for concrete production in Bida and localities where the deposits occur. Concrete production in the communities within the basin depends mainly on locally available materials and it is therefore a local material for local application.

The use of statistical method which has found its application in the industries in the area of optimization of products, is a welcome development (Simon, 2003). According to Rajsekaran (2005), the use of statistical experimental design approach in concrete mixture proportioning helps structural engineers to evolve the best possible design in the area of cost, weight, reliability or a combination of these parameter. In their work Kalntari *et al.*, (2009) acknowledged that the selection of mix proportion is a very important process in the selection of suitable components required for concrete production and also the means of maximizing some important parameter like compressive strength, durability and smooth consistency. Scheffe's Optimization Model which is a statistical experimental design approach in concrete mix proportioning, was used to optimise the compressive strength of concrete produced with Bida gravel as a partial replacement for coarse aggregate.

Concrete mix design could be carried out using either the statistical or empirical experimental method (Simon *et al.,* 1997). The task of concrete mix optimization implies selecting the most suitable concrete aggregates from the data base (Genadij and Juris, 1998). For example, optimization of mix proportions of mineral aggregates for use in polymer concrete was attempted using statistical methods (Mohan *et al.,* 2002). There have been some advances in statistical experimental design for performing tests on concrete but these do not explicitly take into consideration the chemistry involved (Simon, 2003). Some of the statistical experimental methods include simplex design (Scheffe, 1958) and (Obam, 1998), mixture experiments involving process variables, mixture models with inverse terms (Draper and John, 1997) and K-model (Draper and Pukelsheim, 1997). Empirical methods are prone to trial and error which results in material wastage whenever they are used (Ezeh and Ibearugbulem, 2009). Sequel to this, statistical experimental method could be employed using simplex design. The materials used in such experiments include water, cement, sand and granite.

Some recent studies such as Baoju *et al.* (2020), predicted the capillary water absorption of concrete, using multivariable regression models and Henrique *et al.* (2020), produced self-compacting concrete with the blend based on statistical mixture design and simultaneous optimization. There is a need to formulate mathematical models that will prescribe concrete mix ratios, when the desired compressive strength is known and vice-versa. Similarly, the need to determine the combination of the materials that would give the highest compressive strength should be met.

Materials and Methods

Materials

The various materials used to actualize the aim of this study include cement, sand, granite, Bida gravel and water.

1. Dangote cement, a brand of Ordinary Portland Cement conforming to BS EN 197-1 (2001).

2. The fine aggregate used in this work was river sand free from deleterious matters such as dirts, clay and organic matters. The sand is hard and durable conforming to BS 882: Part 2 (2002).

3. Crushed granite chippings of nominal size ranging between 6.3mm – 20mm obtained from a construction site in Federal University of Technology, Minna, Niger state was used as the main coarse aggregate. The granite is hard and durable conforming to BS 882: Part 2 (2002).

4. The Bida gravel with size range of 6.3mm - 20mm used as partial replacement for coarse aggregate in various mix proportion was obtained from Bida in Niger State. The granite is hard and durable conforming to BS 882: Part 2 (2002).

5. Portable drinking water was used for the production of the concrete specimen tested and was sourced directly from the tap in the laboratory which conforms to BS EN 1008, (2002) requirement.

Methods

Model Development

Simplex lattice design proposed by scheffe (1958) will be used to formulate a mathematical model, which relates compressive strength of concrete and its components ratio of water-cement ratio, cement, sand, granite and Bida gravel.

Simplex Lattice Design formulation for (5,2) System

In mixture experiment involving the study of properties of a q- component mixture which are dependent on the component ratio only, the factor space is a regular, (q - 1) simplex. The relationship that holds for the component of mixture is given as

$$\sum_{X_i}^{q} X_i = 1 \quad (1)$$

Where $X_i \geq 0 =$ the component ratio

q = the number of components

Therefore, for a 5-component mixture, the sum of all the proportions of the components must be unity. That means

$$X_1 + X_2 + X_3 + X_4 + X_5 = 1 \quad (2)$$

Where in this case;

X_1 = proportion of water content

X_2 = proportion of cement

X_3 = proportion of sand

X_4 = proportion of granite

X_5 = proportion of Bida gravel

For quinary system, q = 5, the regular simplex is a tetrahedron. Each point in the pentahedron represents a certain composition of the quinary system.

Scheffe (1958) showed that the response function (property) in multi-component system can be approximated by a polynomial. To describe such function adequately, high degree polynomials are required and hence a great many experimental trials. According to Scheffe (1958), a polynomial of degree n in q variable has $C_q^n + n - 1$ coefficients and is in the form:

$$\hat{y} = \sum_{i=1}^{q} \beta_i x_i + \sum_{1 \leq i \leq j \leq q}^{q} \beta_{ij} x_i x_j + \quad (3)$$

The relationship given in equation (3) allow the equation component to be eliminated and the number of coefficients reduced to

$C_q^n + n - 1$. However, it is relevant that all the q components be introduced into the model.

Scheffe (1958) suggested that mixture properties can be described by the reduced polynomials from Equation (3) subject to the normalization condition of Equation (1) for a sum of independent variables. The reduced second-degree polynomial for a quinary system is derived as follows:

$$\hat{Y} = b_0 + b_1 X_1 + b_2 X_2 + b_3 X_3 + b_4 X_4 + b_5 X_5$$
$$b_{12} X_1 X_2 + b_{13} X_1 X_3 + b_{14} X_1 X_4 + b_{15} X_1$$
$$X_5 + b_{23} X_2 X_3 + b_{24} X_2 X_4 + b_{25} X_2 X_5$$
$$+ b_{34} X_3 X_4 + b_{35} X_3 X_5 + b_{45} X_4 X_5 + b_{11}$$
$$X_1^2 + b_{22} X_2^2 + b_{33} X_3^2 + b_{44} X_4^2 + b_{55} X_5^2 \quad (4)$$

where b is a constant coefficient.

Multiplying equation (2) by b_0

$$b_0 X_1 + b_0 X_2 + b_0 X_3 + b_0 X_4 + b_0 X_5 = b_0 \quad (5)$$

Multiplying equation (2) successively by x_1, x_2, x_3, x_4 and x_5, and rearranging the products

$$X_1^2 = X_1 - X_1 X_2 - X_1 X_3 - X_1 X_4 - X_1 X_5$$
$$X_2^2 = X_2 - X_1 X_2 - X_2 X_3 - X_2 X_4 - X_2 X_5$$
$$X_3^2 = X_3 - X_1 X_3 - X_2 X_3 - X_3 X_4 - X_3 X_5$$
$$X_4^2 = X_4 - X_1 X_4 - X_2 X_4 - X_3 X_4 - X_4 X_5$$
$$X_5^2 = X_5 - X_1 X_5 - X_2 X_5 - X_3 X_5 - X_4 X_5 \quad (6)$$

Substituting equation (5) and (6) into equation (4)

$$\hat{Y} = (b_0 + b_1 + b_{11}) X_1 + (b_0 + b_2 + b_{22}) X_2$$
$$+ (b_0 + b_3 + b_{33}) X_3 + (b_0 + b_4 + b_{44}) X_4$$
$$+ (b_0 + b_5 + b_{55}) X_5 + (b_{12} - b_{11} - b_{22}) X_1 X_2$$
$$+ (b_{13} - b_{11} - b_{33}) X_1 X_3 + (b_{14} - b_{11} - b_{44}) X_1$$
$$+ (b_{15} - b_{11} - b_{55}) X_1 X_5 + (b_{23} - b_{22} - b_{33})$$
$$X_2 X_3 + (b_{24} - b_{22} - b_{44}) X_2 X_4 + (b_{25} - b_{22}$$
$$- b_{55})\ X_2 X_5 + (b_{34} - b_{33} - b_{44}) X_3 X_4 + (b_{35}$$
$$- b_{33} - b_{55}) X_3 X_5 + (b_{45} - b_{44} - b_{55}) X_4 X_5 \quad (7)$$

The reduced second degree polynomial for a quinary system is as follows:

$$\hat{Y} = \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5$$
$$\beta_{12} X_1 X_2 + \beta_{13} X_1 X_3 + \beta_{14} X_1 X_4 + \beta_{15} X_1$$
$$X_5 + \beta_{23} X_2 X_3 + \beta_{24} X_2 X_4 + \beta_{25} X_2 X_5$$
$$+ \beta_{34} X_3 X_4 + \beta_{35} X_3 X_5 + \beta_{45} X_4 X_5 \quad (8)$$

The coefficients of polynomial given by Scheffe (1958) is

$$\beta_i = Y_i \text{ and } \beta_{ij} = 4Y_{ij} - 2Y_i - 2Y_j \quad (9)$$

Where, $\beta_i = \beta_1, \beta_2, \beta_3, \beta_4, \beta_5$

$\beta_{ij} = \beta_{12}, \beta_{13}, \beta_{14}, \beta_{15}, \beta_{23}, \beta_{24}, \beta_{25}, \beta_{34}, \beta_{35},$

β_{45}

Y_i and Y_{ij} is the response (property)

nEquation (8) is the response function for optimization of concrete using Bida gravel as partial replacement for crushed granite consisting of five components. The terms Y_i and Y_{ij} are the response (i.e. compressive strengths) at the points i and ij. The values of these response are determined by carrying out compression tests on cube obtained using Bida gravel as a partial replacement for coarse aggregate for concrete. Scheffe's simplex lattice designs provide a uniform scatter of points over the (q – 1) – simplex. The points form a (q -1) – lattice on the simplex where q is the number of mixture components, 'n' is the degree of polynomial. Scheffe (1958) showed that for each component, there exist (n + 1) similar levels, $X_i = 0, \frac{1}{n}, \frac{2}{n}, ..., 1$, and all possible mixtures are derived with such values of component concentration. So for (5, 2) – lattice the proportion of every component that must be used are 0, ½ and 1. He also revealed that the number of points in (q, n) lattice is given as:

Number of point = $\frac{q(q+1)...(q+n-1)}{n!}$ (10)

Where "n" is a digit number. This means that for a (5,2) lattice, the number of points (coefficients) $= \frac{5(5+1)}{2!} = 15$ points

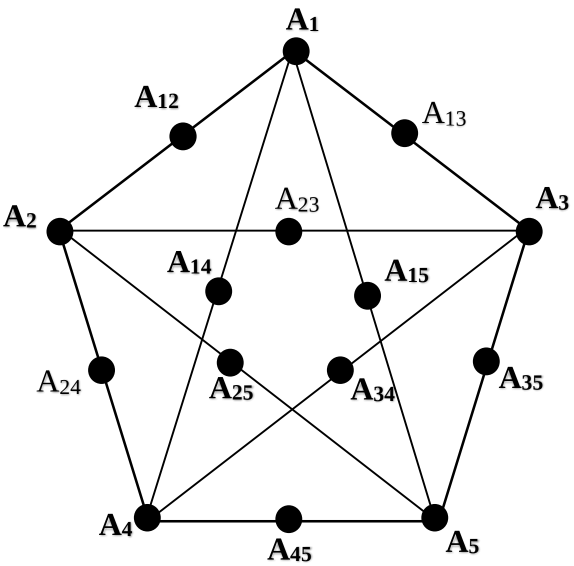

Fig 1: A (5, 2) Simplex lattice, representing five-component concrete mix

Concrete Mix Design

According to Osadebe and Ibearugbulem (2009), the actual mix ratios relate with pseudo mix ratios in the mathematical form:

$$\{Z\} = [A]\{X\} \quad (11)$$

Where Z represents the actual components while X represents the pseudo components, where A is the constant: a five by five matrix. The matrix A will be obtained from the first five mix ratios whereby the actual components for the first five points are chosen arbitrarily for the pentahedron vertices. The mix ratios are

Z_1[0.45:1:1:1.9:0.1], Z_2[0.50:1:1.5:1.8:0.2], Z_4[0.60:1:2:2.4:0.6], Z_5[0.65:1:2:3:1] and the corresponding pseudo mix ratios are X_1[1:0:0:0:0], X_2[0:1:0:0:0], X_3[0:0:1:0:0], X_4[0:0:0:1:0], X_5[0:0:0:0:1].

X_1 = fraction of water content

X_2 = fraction of cement

X_3 = fraction of sand

X_4 = fraction of granite

X_5 = fraction of Bida gravel

The matrix A can be taken to be the transpose of the first five actual mix ratios shown in Table 1 and this resulted to:

$$[A] = \begin{bmatrix} 0.45 & 0.5 & 0.55 & 0.60 & 0.65 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1.5 & 1.5 & 2 & 2 \\ 1.9 & 1.8 & 2.55 & 2.4 & 3 \\ 0.1 & 0.2 & 0.45 & 0.6 & 1 \end{bmatrix} \quad (12)$$

The five actual and pseudo mix ratios in Table 1 correspond to points of observations, Y_1, Y_2, Y_3, Y_4 and Y_5 are located at the five vertices of the pentahedron. For a (5, 2) simplex design,

Table 1: Actual and Pseudo Components of the Model (Fifteen Experimental Point)

S/N	Pseudo Components (X_i)						Actual Components (Z_i)				
	Water	Cement	Sand	Granite	Bida gravel	Points	Water	Cement	Sand	Granite	Bida gravel
	X_1	X_2	X_3	X_4	X_5		Z_1	Z_2	Z_3	Z_4	Z_5
1	1	0	0	0	0	Y_1	0.45	1	1	1.9	0.1
2	0	1	0	0	0	Y_2	0.50	1	1.5	1.8	0.2
3	0	0	1	0	0	Y_3	0.55	1	1.5	2.55	0.45
4	0	0	0	1	0	Y_4	0.60	1	2	2.4	0.6
5	0	0	0	0	1	Y_5	0.65	1	2	3.0	1
6	0.5	0.5	0	0	0	Y_{12}	0.475	1	1.25	1.85	0.15
7	0.5	0	0.5	0	0	Y_{13}	0.50	1	1.25	2.225	0.275
8	0.5	0	0	0.5	0	Y_{14}	0.525	1	1.5	2.15	0.35
9	0.5	0	0	0	0.5	Y_{15}	0.55	1	1.5	2.45	0.55
10	0	0.5	0.5	0	0	Y_{23}	0.525	1	1.5	2.175	0.325
11	0	0.5	0	0.5	0	Y_{24}	0.55	1	1.75	2.1	0.4
12	0	0.5	0	0	0.5	Y_{25}	0.575	1	1.75	2.4	0.6
13	0	0	0.5	0.5	0	Y_{34}	0.575	1	1.75	2.475	0.525
14	0	0	0.5	0	0.5	Y_{35}	0.60	1	1.75	2.775	0.725
15	0	0	0	0.5	0.5	Y_{45}	0.625	1	2	2.7	0.8

ten other observations are needed to add up to the first five to get a total of fifteen observations. This will be used to formulate the model. The remaining ten points were located at the mid points of the lines joining the five vertices. On substitution of these ten pseudo mix ratios, one after the other into equation (13), the real mix ratios corresponding to the pseudo ones will be obtained. The actual and the corresponding pseudo mix ratios for the first fifteen experimental points are shown in Table 1. In order to validate the optimization function (model), extra fifteen control points was provided. The actual components (Z_i) for the fifteen control points were calculated by choosing the psedo-components (X_i) arbitrarily. The actual and the corresponding mix ratios for

Table 2: Actual and Pseudo Components for the Fifteen Control Point

S/N	Pseudo Components (X_i)					Actual Components (Z_i)					
	Water	Cement	Sand	Granite	Bida gravel	Points	Water	Cement	Sand	Granite	Bida gravel
	X_1	X_2	X_3	X_4	X_5		Z_1	Z_2	Z_3	Z_4	Z_5
1	0.25	0.25	0.25	0.25	0	C_1	0.525	1	1.5	2.613	0.338
2	0.25	0.25	0.25	0	0.25	C_2	0.538	1	1.5	2.313	0.438
3	0.25	0.25	0	0.25	0.25	C_3	0.55	1	1.625	2.275	0.475
4	0.25	0	0.25	0.25	0.25	C_4	0.563	1	1.625	2.463	0.538
5	0	0.25	0.25	0.25	0.25	C_5	0.575	1	1.75	2.438	0.563
6	0.4	0.2	0.2	0.2	0	C_6	0.51	1	1.4	2.11	0.29
7	0.4	0.2	0.2	0	0.2	C_7	0.52	1	1.4	2.23	0.37
8	0.4	0.2	0	0.2	0.2	C_8	0.53	1	1.5	2.20	0.40
9	0.2	0.2	0.4	0.2	0	C_9	0.53	1	1.5	2.24	0.36
10	0.3	0.2	0.2	0.15	0.15	C_{10}	0.533	1	1.5	2.25	0.40
11	0.3	0.15	0.2	0.15	0.2	C_{11}	0.54	1	1.525	2.31	0.44
12	0.2	0.15	0.2	0.15	0.3	C_{12}	0.56	1	1.625	2.42	0.53
13	0.333	0.333	0.333	0	0	C_{13}	0.50	1	1.332	2.081	0.25
14	0.333	0	0.333	0.333	0	C_{14}	0.533	1	1.499	2.281	0.383
15	0.333	0.333	0	0.333	0	C_{15}	0.516	1	1.499	2.031	0.30

the fifteen control points are shown in Table 2.

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{pmatrix} = \begin{bmatrix} 0.45 & 0.5 & 0.55 & 0.60 & 0.65 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1.5 & 1.5 & 2 & 2 \\ 1.9 & 1.8 & 2.55 & 2.4 & 3 \\ 0.1 & 0.2 & 0.45 & 0.6 & 1 \end{bmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{pmatrix} \quad (13)$$

Compressive Strength Test

The materials were batched by weight in their dried state. Mixing was done manually. The moulds used for the concrete cubes were 150mm x 150mm x 150mm. After mixing properly, the concrete was cast into the moulds, and tamped very well to ensure proper compaction and to minimize the void space in the concrete. The cubes were de-moulded after 24 hours from the period of casting and transferred immediately to curing water tank at room temperature. The concrete cubes were immersed in the curing water tank for 27 days to make up 28 days after which they were tested for compressive strength using compression machine. The concrete cubes immediately after the removal from the curing tank was left for some time to dry and then the masses were measured and recorded. The volume of the concrete cube was also measured and recorded and then subjected to crushing using compression testing machine. The maximum load applied at crushing was recorded. Three replicates of each of the mix ratios were made. Therefore, for the fifteen experimental points and fifteen control points, a total of 90 cubes were tested. The compressive strength of cubes was obtained using equation (13) below

Compressive Strength =

$$\frac{\text{Crushing Load (N)}}{\text{Cross section area}(\text{mm}^2)} \qquad (14)$$

Results and Discussion

The results of the 28th days cubes strengths of the concrete for formulation of model and control points are shown in Table 3 and 4 respectively. The regression equation was used to predict cubes strengths of different mixtures and the results of the model or predicted values are shown in column 6 of table 3 and 4 respectively. A MATLAB program was developed to ease the use of the model as shown in Appendix. In the program, any mix ratio can be specified as an input and the computer processes and prints out possible compressive strength that match the mix proportions.

The model gave an optimum compressive strength of 29.93N/mm^2 corresponding to mix ratio of 0.45:1:1:1.9:0.1 for water-cement ratio, cement, fine aggregate (sand), granite and Bida gravel respectively. This further showed that the optimum compressive strength was achieved by replacing 5% of the coarse aggregate (granite) with Bida gravel in the concrete mix. The minimum strength was found to be 16.18N/mm^2 corresponding to mix ratio of 0.575:1:1.75:2.4:0.6. this further showed that the minimum value of compressive strength was achieved by replacing 20% of the coarse aggregate (granite) with Bida gravel in the concrete mix.

Development of the Model

From equation (8), The general form of Scheffe's (5,2) – lattice polynomial is given by

$$\hat{Y} = \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5$$
$$\beta_{12} X_1 X_2 + \beta_{13} X_1 X_3 + \beta_{14} X_1 X_4 + \beta_{15} X_1$$
$$X_5 + \beta_{23} X_2 X_3 + \beta_{24} X_2 X_4 + \beta_{25} X_2 X_5$$
$$+ \beta_{34} X_3 X_4 + \beta_{35} X_3 X_5 + \beta_{45} X_4 X_5 \qquad (8)$$

The coefficients of polynomial given by Scheffe (1958) is

$$\beta_i = Y_i \text{ and } \beta_{ij} = 4Y_{ij} - 2Y_i - 2Y_j \qquad (9)$$

Where, $\beta_i = \beta_1, \beta_2, \beta_3, \beta_4, \beta_5$

$\beta_{ij} = \beta_{12}, \beta_{13}, \beta_{14}, \beta_{15}, \beta_{23}, \beta_{24}, \beta_{25}, \beta_{34}, \beta_{35}, \beta_{45}$

Y_i and Y_{ij} are experimental compressive strength at each points/coefficients (Y_1, Y_2, Y_3, Y_4, Y_5, Y_{12}, Y_{13}, Y_{14}, Y_{15}, Y_{23}, Y_{24}, Y_{25}, Y_{34}, Y_{35} and Y_{45}).

Thus, $\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_{12}, \beta_{13}, \beta_{14}, \beta_{15}, \beta_{23}, \beta_{24}, \beta_{34}, \beta_{35}, \beta_{45}$ are determined as follows;

$\beta_i = Y_i$

$\beta_1 = Y_1 = 29.93$

$\beta_2 = Y_2 = 21.28$

$\beta_3 = Y_3 = 24.09$

$\beta_4 = Y_4 = 19.96$

$\beta_5 = Y_5 = 16.53$

$\beta_{ij} = 4Y_{ij} - 2Y_i - 2Y_j$

$\beta_{12} = 4Y_{12} - 2Y_1 - 2Y_2 = 4(27.04) - 2(29.93) - 2(21.28) = 5.74$

$\beta_{13} = 4Y_{13} - 2Y_1 - 2Y_3 = 4(21.08) - 2(29.93) - 2(24.09) = -23.72$

$\beta_{14} = 4Y_{14} - 2Y_1 - 2Y_4 = 4(26.3) - 2(29.93) - 2(19.96) = 5.42$

$\beta_{15} = 4Y_{15} - 2Y_1 - 2Y_5 = 4(21.16) - 2(29.93) - 2(16.53) = -8.28$

$\beta_{23} = 4Y_{23} - 2Y_2 - 2Y_3 = 4(26.07) - 2(21.28) - 2(24.09) = 13.54$

$\beta_{24} = 4Y_{24} - 2Y_2 - 2Y_4 = 4(26.37) - 2(21.28) - 2(19.96) = 23$

$\beta_{25} = 4Y_{25} - 2Y_2 - 2Y_5 = 4(16.18) - 2(21.28) - 2(16.53) = -10.9$

$\beta_{34} = 4Y_{34} - 2Y_3 - 2Y_4 = 4(18.28) - 2(24.09) - 2(19.96) = -14.98$

$\beta_{35} = 4Y_{35} - 2Y_3 - 2Y_5 = 4(17.9) - 2(24.09) - 2(16.53) = -9.64$

$\beta_{45} = 4Y_{45} - 2Y_4 - 2Y_5 = 4(20.7) - 2(19.96) - 2(16.53) = 9.82$

By substituting all the coefficients into equation (8), thus

$$\hat{Y} = 29.93X_1 + 21.28X_2 + 24.09X_3 + 19.96X_4 + 16.53X_5 + 5.74X_1X_2 - 23.72X_1X_3 + 5.42X_1X_4 - 8.28X_1X_5 + 13.54X_2X_3 + 23X_2X_4 - 10.9X_2X_5 - 14.98X_3X_4 - 9.64X_3X_5 + 9.82X_4X_5 \quad (15)$$

Equation (15) is the statistical model for the optimisation of compressive strength of concrete based on Scheffe's (5, 2) factor space using Bida gravel as a partial replacement for coarse aggregate.

Test for Adequacy of the Model

The test for adequacy of the model was done using statistical ANOVA (fisher test) at 95% accuracy level on compressive strength at the control points (that is, C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9, C_{10}, C_{11}, C_{12}, C_{13}, C_{14}, C_{15},). In this test, two hypotheses were set as follows:

i. **Null Hypothesis**
There is no significant difference between the laboratory concrete cube strength and model predicted compressive strength results at 95% accuracy level.

ii. **Alternative Hypothesis**
There is a significant difference between the laboratory concrete cube strength and model predicted compressive strength results accuracy level.

The test was carried out as presented in Table 5.

Legend: YE = Experimental compressive strength; YAE = Average of the experimental compressive strength; YT = Model compressive strength; V = Degree of freedom;

Table 3: Compressive Strength of Concrete Cubes Based on 28 days Strength

Points	Replicate 1 (N/mm^2)	Replicate2 (N/mm^2)	Replicate 3 (N/mm^2)	Mean Values (N/mm^2)	Model (Predicted) values (N/mm^2)
Y_1	26.67	32.44	30.67	29.93	29.93
Y_2	21.07	21.69	21.07	21.28	21.28
Y_3	27.33	22.04	22.89	24.09	24.09
Y_4	18.76	19.33	21.78	19.96	19.96
Y_5	16.36	15.73	17.51	16.53	16.53
Y_{12}	25.33	28.22	27.56	27.04	27.04
Y_{13}	20.22	21.24	21.78	21.08	21.08
Y_{14}	28.22	26.00	24.67	26.30	26.30
Y_{15}	25.11	18.00	20.36	21.16	21.16
Y_{23}	24.00	24.44	29.78	26.07	26.07
Y_{24}	27.11	28.00	24.00	26.37	26.37
Y_{25}	17.42	15.33	15.78	16.18	16.18
Y_{34}	18.49	13.47	22.89	18.28	18.28
Y_{35}	15.29	19.47	18.93	17.90	17.90
Y_{45}	27.07	15.02	20.00	20.70	20.70

YAT = Average of the Model compressive strength; N = Number of points of observation;

$\propto$ = Significant level; S_e^2 = Experimental mean square; S_m^2 = Model mean square

Table 4: Compressive Strength of Concrete Cubes Based on 28 days Strength (Control)

Points	Replicate 1 (N/mm^2)	Replicate 2 (N/mm^2)	Replicate 3 (N/mm^2)	Mean Values (N/mm^2)	Model (Predicted) values (N/mm^2)
C_1	20.44	22.89	22.67	22.00	24.38
C_2	20.22	23.56	21.33	21.70	20.88
C_3	22.67	21.11	20.89	21.56	23.48
C_4	16.89	19.33	20.71	18.98	20.04
C_5	24.44	16.80	27.11	22.78	21.14
C_6	27.33	24.22	29.33	26.96	24.90
C_7	17.33	23.11	22.89	21.11	21.97
C_8	21.78	24.89	22.67	23.11	24.63
C_9	21.33	22.22	24.08	22.54	23.22
C_{10}	23.02	22.67	20.89	22.19	22.71
C_{11}	22.22	22.00	20.67	21.63	21.93
C_{12}	20.22	24.89	17.56	20.89	20.69
C_{13}	27.11	24.67	24.89	25.56	24.58
C_{14}	24.00	20.89	22.22	22.37	20.95
C_{15}	31.11	30.22	24.00	28.44	27.49

$\sum YE = 341.82$; $\sum YT = 342.99$; $YAE = \frac{\sum YE}{N} = \frac{341.82}{15} = 22.79$;

Table 5: Fisher-Statistical Test Computations for the Model

Control Points	YE	YT	YE –YAE	YT –YAT	(YE –YAE)^2	(YT –YAT)^2
C_1	22.00	24.38	-0.79	1.51	0.62	2.28
C_2	21.70	20.88	-1.09	-1.99	1.19	3.96
C_3	21.56	23.48	-1.23	0.61	1.51	0.37
C_4	18.98	20.04	-3.81	-2.83	14.52	8.01
C_5	22.78	21.14	-0.01	-1.73	0.00	2.99
C_6	26.96	24.90	4.170	2.03	17.39	4.12
C_7	21.11	21.97	-1.68	-0.90	2.82	0.81
C_8	23.11	24.63	0.32	1.76	0.10	3.10
C_9	22.54	23.22	-0.25	0.35	0.063	0.12
C_{10}	22.19	22.71	-0.60	-0.16	0.36	0.03
C_{11}	21.63	21.93	-1.16	-0.94	1.35	0.88
C_{12}	20.89	20.69	-1.90	-2.18	3.61	4.75
C_{13}	25.56	24.58	2.77	1.71	7.67	2.92
C_{14}	22.37	20.95	-0.42	-1.92	0.18	3.69
C_{15}	28.44	27.49	5.65	4.62	31.92	21.34
Sum	341.82	342.99			83.30	59.37
Mean	YAE = 22.79	YAT = 22.87				

$$YAT = \frac{\sum YT}{N} = \frac{342.99}{15} = 22.87; \qquad \sum (YE - YAE)^2 = 83.30$$

$$\sum (YT - YAT)^2 = 59.37$$

$$S_e^2 = \frac{\sum (YE - YAE)^2}{N-1} = \frac{83.30}{15-1} = 5.95$$

$$S_m^2 = \frac{\sum (YT - YAT)^2}{N-1} = \frac{59.37}{15-1} = 4.24$$

$$F_{calculated} = \frac{S_1^2}{S_2^2}$$

Where S_1^2 is the greater of the S_e^2 and S_m^2, while S_2^2 is the smaller of the two;

Here $S_1^2 = S_e^2 = 5.95$ and $S_2^2 = S_m^2 = 4.24$

$$F_{calculated} = \frac{5.95}{4.24} = 1.403$$

The model is acceptable at 95% accuracy level if:

$$\frac{1}{F_{\alpha(V_1,V_2)}} < \frac{S_1^2}{S_2^2} < F_{\alpha(V_1,V_2)}$$

Where, Significant level
$\alpha = 1 - 0.95 = 0.05$;
Degree of freedom $= N - 1 = 15 - 1 = 14$
From standard F-statistic table,
$F_{\alpha(V_1,V_2)} = F_{0.05(14,14)} = 2.443$ and

$$\frac{1}{F_{\alpha(V_1,V_2)}} = \frac{1}{2.443} = 0.4093$$

Consequently, the condition

$\frac{1}{F_{\alpha(V_1,V_2)}} < \frac{S_1^2}{S_2^2} < F_{\alpha(V_1,V_2)}$ which is

$0.4093 < 1.403 < 2.443,$ is satisfied.

Which simply means that $F_{calculated} < F_{critical}$.

Therefore, the null hypothesis that "there is no significant difference between the experimental and the model result" is accepted. This implies that the model is adequate for use in predicting the probable compressive strength when the mix ratio is known and vice-versa.

4.5 Regression Statistic

The graphical relationship between the experimental and model predicted values of 28 days' compressive strength of the concrete mix considered in this study are shown in Fig. 2. The closeness of the data points to the trendline shows that the values of the predicted strength are in good agreement the experimental values. This is evidenced by the value of R^2 of 0.6907 and R of 0.831.

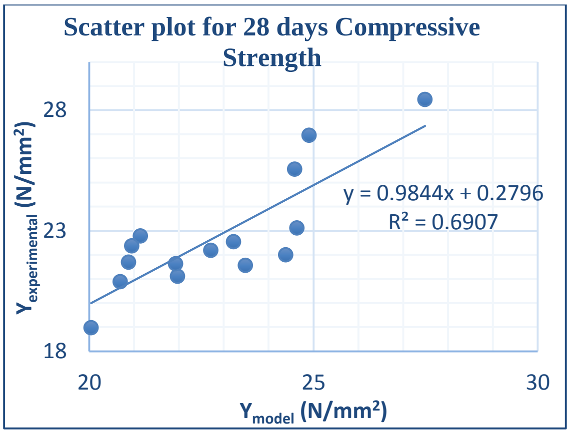

Fig. 2: Correlation of Experimental and Predicted 28 days Compressive Strength

Conclusion

Scheffe's five-component second degree polynomial regression equation was used to develop statistical model for a five-component concrete made with Bida gravel as a partial replacement for crushed granite. This model predicts the compressive strength of concrete made with Bida gravel as a partial replacement for crushed granite when the mix ratios are known and vice-versa. The predictions from the model were tested at 95% accuracy level using statistical Fisher test and regression statistic and found to be adequate with $R^2 = 0.6907$. The model was used to predict the compressive strength of all the mix ratios. The maximum compressive strength predicted by this model was 29.93N/mm^2 corresponding to mix ratio of 0.45:1:1:1.9:0.1 for water: cement: sand: granite: Bida gravel. This implies that the optimum compressive strength was achieved by replacing 5% of crushed granite with Bida gravel. The use of the model reduces time, cost and effort spent in conventional method of concrete mix design.

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