Lapai Journal of Applied and Natural Sciences
LAJANS Vol 1(1): 84 - 88

RESEARCH PAPER

A Mathematical Model for Bank Asset and Liability Portfolio System

[1]Jiya M., [2]Bawa M., [1]Shehu M.D., [1]Adamu G. and [3]Doko U.C.

[1]Department of Mathematics, Federal University of Technology, Minna, Nigeria
[2]Department of Mathematics & Computer Science, Ibrahim Badamasi Babangida University, Lapai, Nigeria
[3]Department of Mathematics, Niger State College of Education, Minna, Nigeria

ABSTRACT

A mathematical model for Asset and Liability Portfolio System (ALPS) was developed and analyzed on banking system using partial differential equation technique. The dynamic nature of decision making support for asset and liability management was presented so as to choose a realistic result. The model was tested with the use of maple17 software which shows that commercial banks can manage their assets and liabilities through cash flow of deposits and loans. Setting the bank's initial position, and different deposit flow situations, the result revealed that people are discouraged to save their money when the interest rate of deposit is low. To the contrary, people are encouraged to take loan when the interest rate is so low. Banks should effectively oversee the asset and liability portfolio process for proper accountability of capitals. The result of stress-testing shows the kind of dynamic processes taking position in commercial banks. In this case, it is possible for managers to adjust bank liabilities to earn assets as much as possible.

Keywords: Partial Differential Equation, Asset, Liability, Deposit, Loan

INTRODUCTION

A banking firm is a complex structure surrounded by the state of managing crisis. It is motivated by a substantial amount of financial flows and its capital, having a variety of starting point and overturns by dynamic and probabilistic uniqueness and at the same time structure the incorporated pattern (Selyutin & Rundenko, 2013). The basic model for Asset and Liability Management (ALM) has been investigated upon by many researchers. This work will be an extension of the following existing models (Voloshyh, 2014; Selyutin & Rudenko, 2013), their work assume no investment on trading securities. Alekseev & selyutin (2014), formulated a mathematical model using a software tool designed for modelling financial flows in commercial banks. The program is capable of calculating the dynamics of the main components of ALM and financial stability of the bank under the influence of the control parameters at various scenarios (stress-testing) in an interactive mode. The model was designed to minimize the risks (interest rate, liquidity), with a certain balance sheet structure. Rajaeyan *et al.* (2014) formulated a mathematical model for control and optimization of assets in banks by using the stochastic method. The model was tested using data taken from Bank of Industry and Mine of Iran. Naderi (2013), formulated an asset-liability management model to solve multi-period investment problems. The model aims to maximize the overall revenue and deal with uncertainties as well as risks. The results show that the non-linear function outperform the piecewise linear function and generates better asset allocation. They demonstrate how ALM model can be applied for asset allocation in financial institutions.

Mastoureh and Babak (2015) formulated a mathematical model for the optimal management of asset and liability using goal programming in Eghtesad-e-Novin Bank. The purpose of this model is to create a balance among the conflict objectives of profitability, liquidity and risk in banks. The authors used financial statements information relevant to the fiscal year of 2005-2014, the goal programming model was designed according to the structural restrictions, goal restrictions and the objective function for the optimal allocation of resources using LINGO software. Their results suggest

Received 25 October, 2016
Accepted 25 November, 2016
Address Correspondence to:
ahlul_baity@yahoo.com

that the design quantitative model for the optimal management of assets and liabilities in bank is possible, using the GP model as a decision making tool in ALM, while risk is under control, return can be increased. Using the Analytic Hierarchy Process (AHP), the ALM can be quantitatively measured. Naderi *et al.* (2013), design a mathematical model called goal programming in order to manage the optimal assets and liabilities. In the work, the result confirmed that it was likely to propose a model of optimum supervision of assets and liabilities to verify appropriate formation for items of balance sheet and increase it to overseeing of asset items. It is possible to extend it to other commercial banks. The model determines the appropriate balance sheet structure and the management of assets' elements in such a way that it reduced the liquidity risk, which is the model's advantage.

METHODOLOGY

The Model Formulation

Consider

$A = L + C$ and

$L^* = D + E$

where A = asset, L^* = liability, L = loans, C = reserves, D = deposits and, E = equity.

The following partial differential equation representations (Voloshyh, 2014; Selyutin & Rudenko, 2013) and the dynamics system of total loans issued and total amount of deposits can be described by partial differential equation of the first order of the form:

$$\frac{\partial l}{\partial t} + \frac{\partial l}{\partial \tau} = u(t,\tau) \quad (2.1)$$

$$\frac{\partial d}{\partial t} + \frac{\partial d}{\partial \tau} = v(t,\tau) \quad (2.2)$$

With $l =$ Total amount of loans issued, $t =$ Current time $0 \le t < \infty$

$\tau =$ Current age on loans

$u(t,\tau)$ = Temporary outflow with respect to current time and age of maturity

$v(t,\tau)$ = Temporary inflow with respect to current time and age of maturity (2.3)

Let; $u(\tau, l) =$ Value of amortization of loans

$u(t) =$ Temporary outflows with respect to time

$$u(\tau, l) = -R_t l \quad (2.4)$$

where; $R_t =$ Repayment of loans issued

Following the Macaulay duration for an asset as in Selyutin & Rudenko (2013),

$$l(t,\tau) = u(t-\tau)e^{-R_t l} \quad (2.5)$$

For stability initial and boundary condition at $t < \tau \le T$, it is essential to set $u(t)$, $t \in [-T, 0)$

It follows that

$$l(0,\tau) = \phi(\tau) = u(-\tau)e^{-R_t \tau} \quad (2.6)$$

Substituting τ for $-t$, gives

$$u(t) = \phi(-t)e^{R_t t} \text{ under } T \le t < 0 \quad (2.7)$$

The total input of loans on the current age is given by,

$$L(t) = \int_0^T l(t,\tau)d\tau \quad (2.8)$$

Substituting (2.5) into (2.8) we have

$$L(t) = \int_0^T u(t-\tau)e^{-R_t l}d\tau \quad (2.9)$$

Integrating gives the ordinary differential equation:

$$\frac{dL}{dt} = u(t) - R_t L - l(t,T) = u(t) - R_t L - u(t-T)e^{-R_t T} \quad (2.10)$$

Assets with various terms of repayment are in portfolio of assets or liabilities, it is likely to replace variable $L(t)$ in equation (2.10) with vector (r), where vector's components are financial tools with various terms of repayments, which give

$$\frac{dL_r}{dt} = u_r(t) - R_r L_r - l_r(t,T_r) = u_r(t) - R_r L_r - u_r(t-T_r)e^{-R_r T_r} \quad (2.11)$$

Suppose $T_r = r$ where r is expressed in months.

Then the preceding equation can be offered as:

$$\frac{dL_r}{dt} = u_r(t) - R_r L_r - l_r(t,r) = u_r(t) - R_r L_r - u_r(t-r)e^{-R_r r} \quad (2.12)$$

where;

$R_t =$ Repayment of loans,

$u_r(t) =$ Cash outflows with respect to time.

According to Selyutin & Rudenko (2013), the dynamics of reserves C_r can be described by:

$$\frac{dC_r}{dt} = \mu C_r + \sigma C_r + \lambda(t) \tag{2.13}$$

where;

$C_r =$ Cash reserves as tools in assets,
$\mu =$ Securities portfolio return,
$\sigma =$ Volatility of security portfolio,
$\lambda =$ Operation cost of bank activities.

Equally from equation (2.2), consider the dynamics of deposits given by partial differential equation of the first order:

$$\frac{\partial d}{\partial t} + \frac{\partial d}{\partial \tau} = v(t, \tau) \tag{2.14}$$

where;

d = Total amount of deposits,
$t =$ Current time deposits
$\tau =$ Current age deposits

Consider the initial and boundary condition $v(0) = \varphi(0)$, t is the current time $0 \le t < \infty$, τ is the current age on deposits $0 < \tau \le T$, $v(t) =$ Temporary inflow with respect to time, $\omega =$ Inflow on cash deposits.

Then, the equation follows as:

$$\frac{dD_r}{dt} = v_r(t) - \omega_r D_r - d_r(t, r) = v_r(t) - \omega_r D_r - v_r(t - r)e^{-\omega_r r} \tag{2.15}$$

The equation of dynamics of equity can be obtained by differentiation and making corresponding substitution of equations (2.12) to (2.15) to get

$$\frac{dE}{dt} = \rho_r L_r - \eta_r D_r + \frac{dC_r}{dt} + \gamma(t) - \lambda(t) \tag{2.16}$$

For the dynamics of capital $\frac{dC_r}{dt} = 0$ hence,

$$\frac{dE}{dt} = \rho_r L_r - \eta_r D_r + \gamma(t) - \lambda(t) \tag{2.17}$$

where, ρ_r = interest on loans, η_r = interest on deposits, γ = investment on security portfolio return on trade

$\lambda =$ operation cost on bank activities

From equation (2.17),

let;

$$\alpha_r = \frac{L_r}{L}, \beta_r = \frac{D_r}{D}$$

Substituting the value of L_r and D_r in equation (2.17), gives

$$\frac{dE}{dt} = L\rho_r \alpha_r - D\eta_r \beta_r + \gamma(t) - \lambda(t) \tag{2.18}$$

where; $\alpha_r =$ Loan structure, $\beta_r =$ Deposit structure and $\gamma(t)$ = purchase (+) or sale (-)

This gives clear account of the dynamics of capital to changes of major parameters of assets and liabilities. The major goal of shareholders and bank executives is the raise in capital. Then the dynamics of bank asset and liability portfolio can be described by the following ordinary differential equations for numerical simulations.

$$\frac{dL_r}{dt} = u_r(t) - R_r L_r - u_r(t - r)e^{-R_r r}$$
$$\frac{dC_r}{dt} = \mu C_r + \sigma C_r + \lambda(t)$$
$$\frac{dD_r}{dt} = v_r(t) - \omega_r D_r - v_r(t - r)e^{-\omega_r r}$$
$$\frac{dE}{dt} = L\rho_r \alpha_r - D\eta_r \beta_r + \gamma(t) - \lambda(t)$$

Numerical Simulations

The stress-testing were carried out using the following variables and parameters for initial conditions, computations were run in maple17 software for analysis. We assume some variables and parameters due to restrictions on banking industry: $L =$ 1000, $D =$ 1500, $u_r(t) =$ 0.20, $v_r(t) =$ 0.10, $R_t =$ 0.25, $e =$ 2.7, $r =$ 0.5, $C =$ 300, $E =$ 500, $\alpha =$ 0.45, $\beta =$ 0.20, $\gamma(t) =$ 0.01, $\lambda(t) =$ 0.5, $\omega =$ 0.81, $\rho_r =$ 0.5, $\eta_r =$ 0.02, $\sigma =$ 0.8, $\mu =$ 75

RESULTS

The results of numerical simulations are presented on Figures 1-4 below.

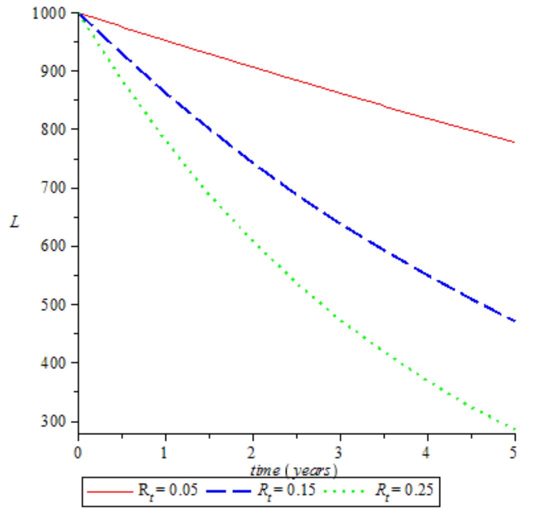

Fig: 1. Dynamics of interest rate (loans portfolio) on repayment rate (R_t) for R_0 = 5%, R_1 = 15%, R_2 = 25% over time.

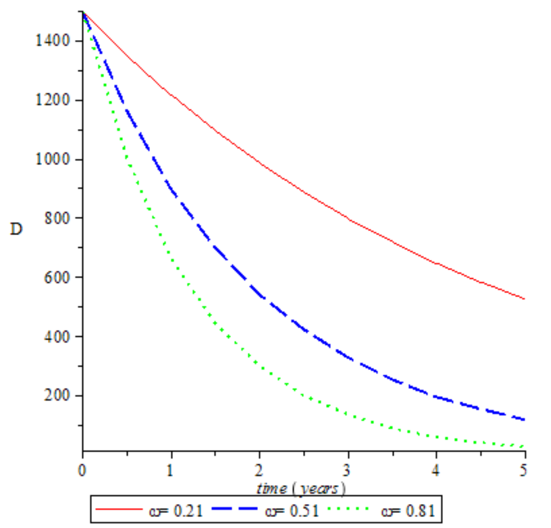

Fig: 2. Dynamics of deposits against time with different rate of cash inflows over time with parameters as indicated.

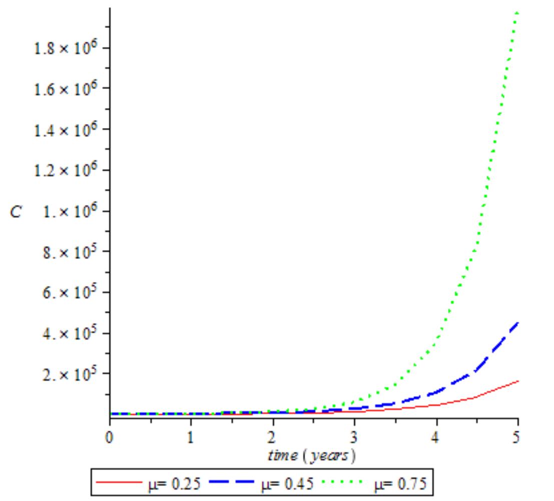

Fig: 3. Cash reserve against time with different rate of average security portfolio return on investment with parameters as indicated.

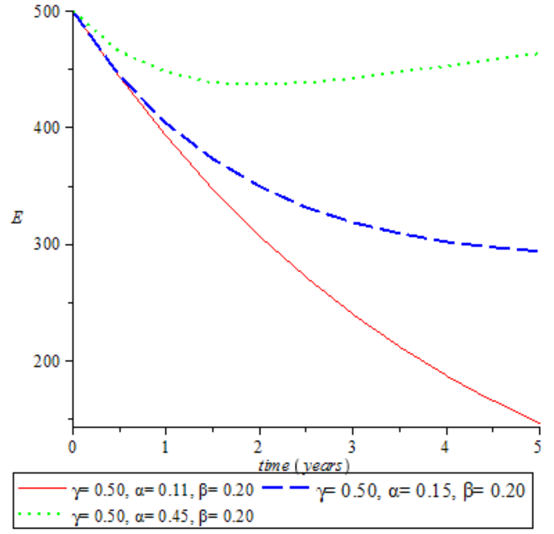

Fig: 4. Dynamics of equity against time with different rates of purchase (+) or sale (-), loan and deposit structures with parameters as indicated.

DISCUSSION

The dynamics of asset and liability portfolio system on bank is described by only four equations for total loans issued and deposits on cash flows of interest rate, cash reserve and equity. The model is an internal activity of a bank. Fig.1, with $R_t =$ 25%, 15% and 5% shows that the bank expects low interest income on loans as the repayment rate gradually decreases with time. In this case the duration of loans issued over time end long in maturity age, whereby the bank will experience liquidity risk. Fig. 2, shows dynamic cash inflow on deposits against time with different rate. It shows from the graph that people are discouraged to save their money in the bank when the interest rate of deposit is low. In this case it increases the rate of liquidity risk. Here the bank assets must be well checked in order to manage the liabilities.

Fig. 3, shows the graph of cash reserve against time with different rates of security portfolio return. This shows that the banks can obtain much profit on their capital by lending out cash to borrowers instead of holding it in their vaults or depositing it in similar institutions. Fig. 4, indicated that when there is no investment on trade, the owner's interest decreases as that of purchase or sale, loans and deposits decreases over time down to zero.

Conclusion

A mathematical model for asset and liability portfolio system was developed using partial differential equation and variation method for numerical simulation. The model was tested with the use of maple17 software, which show that banks can manage their asset and liability portfolio, taking into account only financial flow of deposits and loans. The graph of cash flow shows desired behavior when compared with the graph of Voloshyh (2014), Jiya *et al.* (2016) and Selyutin & Rudenko (2013).

Recommendations

i. It is necessary to develop a modelling approach as part of decision support system for asset and liability management in commercial banks. However, availability of data to researcher in studying financial sectors is necessary and parameters of some special cases need no assumptions.
ii. The research can be extended by incorporating the fixed asset component of assets and stability of liquidity

REFERENCES

Alekseev, I. V., & Selyutin, V. V. (2014). Interactive Computer Models of the Dynamics of Assets and Liabilities: *Terra Economicus in Russian* 9(4), part 2, 42 – 47.

Jiya, M., Adamu, G., & Doko, U. C. (2016). Mathematical Model for Controlling Asset and Liability Management in a Commercial Bank: 53rd Annual National Conference of MAN on Mathematics as an Instrument for the Desired Change, Zaria, Nigeria. 297 - 314.

Mastoureh, M., & Babak, J. (2015). Design on Mathematical Model of Asset and Liability Management using Goal programming: *GMP Review* 18(3), Kermanshah, Iran.

Naderi, S., Mehrzad, M., & Hadi, P. G. (2013). Liability Optimal Management Mathematical Modeling for Bank: *A journal of Basic and Applied Scientific Research*, 3(1) 484 – 493.

Naderi, B., & Mehmet, C. (2013). Asset-Liability Management Models in Decision making: *A journal of Business and Finance.* 1(1), 11 – 16.

Rajaeya, A., Ahmadi, P., Sarlak, M., & Raee, R. (2014). A Mathematical Model for Managing Bank Asset and Liability using a Stochastic Method: *1st Conference on Monetary and Banking Management,Tehran, Iran.*

Selyutin, V., & Rudenko, M. (2013). Mathematical Model of Banking Firm as Tool for Analysis, Management and Learning: *Research Institute of Mechanics and Mathematics, Southern Federal University, Russia.* Retrieved from http://ceur-ws.org/Vol1000/ICTERI-2013-p-401-408-ITER.pdf.

Voloshyn, I. (2014). A Continuous- Time Dynamics of Interest Cash flows from a Bank's loan Portfolio: *SSRN Electronic Journal* DOI: 10.2139/ssrn.2461150. Retrieved from https://www.researchgate.net/publication/263546472.