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Application of System of Linear Equations to A 3-Arm Roundabout Network Flows

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Abstract

A mathematical model was presented and used to determine turning movements at roundabouts based on field data. Assumptions were made in order to simplify the model; such as no U-turns from and to the same arm of a roundabout, total traffic into the roundabout is equal to total traffic out of the roundabout and traffic is homogenous (i.e. mainly consisting of vehicles). Using Gaussian elimination, turning movements could be estimated for 3-arm roundabouts for the indeterminate traffic steam movements when inflows and outflows for each arm of the roundabout is known together with a flow stream on one internal circulating (weaving) section between any two arms of the roundabout. The model has practical use in reducing the number of detectors or counters (whether automatic, videoing techniques or manual methods are in use) which are needed in collecting data to determine the estimated flows from and to the different parts of a roundabout. The reduction in the number of detectors (or traffic counts) could be due to site limitations caused by faulty or limited number of counters used, inaccessible sections for obtaining video images for later analysis (e.g. presence of sharp bends buildings or large trees obscuring vision). The benefits of saving costs could be significant in terms of time and man-power needed on site and this could depend on the amount of traffic flow through the roundabout.

Keywords: Roundabouts, traffic, detector, inflows, indeterminate and outflows.

1.0 Introduction

A linear equation is an algebraic equation in which each term is either a constant or the product of a constant and it is the first power of a single variable. A network on the other hand is the connection of points to each other and it is continuous. A wide variety of engineering and management problems are involved in the optimization of network flows; that is, how objects move through a network. Practical examples include coordination of trucks in a transportation system, routing of packets in a communication network and sequencing of legs for air travel. Such problems sometimes involve few indivisible objects, and this leads to a finite set of feasible solutions example is the problem of finding a minimal cost sequence of legs for air travel from Kaduna to Lagos. People took the transportation problem up early in the Second World War. It was used to determine how to move troops to the battlegrounds in Europe and Asia.

Several researchers had attempted to estimate turning movements at road junctions and a number of methods were developed over the years making use of traffic flows entering and leaving a junction [1-5]. Also the work in [6] used detailed information on individual vehicle's trajectories through the roundabout and proposed estimates of turning movements. The accuracy of turning flow estimates at road junctions from traffic counts was also examined in [7] who accepted a relative difference of about 22.5% for traffic flows in the region of about 1575 vehicles per hour and an absolute difference of 74 vehicles per hour for traffic flows of 454 vehicles per hour. In addition, Jadaan[8] studied the accuracy of turning flows and accepted a relative difference between actual and estimated flow of 13.1% for traffic flows of 168 vehicles per hour. Other efforts went into developing complex mathematical models for estimating turning flows at intersections and comparing results obtained from each model [9]. Other studies, such as [10], used dynamic methods where traffic flow through a facility is considered as a dynamic process and these have shown high accuracy results. Besides, Marshall [11] proposed labour-saving methods for counting traffic movements at 3-arm and 4-arm junctions and showed that two observers together with two automatic counters are sufficient for 3-arm junctions.

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However, this task is rather difficult when traffic flow is high and where observers need to trace the path of vehicles individually. Furthermore, [12] studied 4-arm intersections and reported on the potential of cost saving from replacing direct observation of turning flows with estimates based on counts taken from entry and exits flows. The use of algorithms in manual traffic counts of turning flows at road junctions to save on labour costs was proposed in [13]. Multi-objective programming formulations for estimating ordinary differential (O - D) matrices was presented in [14], while Lam and Lo [15] used direct traffic counts to estimate an O – D matrix and Nitron and Davis [16] used a single set of traffic data (based on input/output counts) to estimate a static ordinary differential matrix rather than using time-series of counts to track a dynamic O -D matrix. The used of automatic vehicle identification in traffic data counts for real-time O - D estimation was proposed in [17].

Moreover, several researchers attempted the use of O-D matrices to describe the turning movements from and to the various arms at road junctions [18-20]. The process of obtaining accurate measurements of turning movements could be particularly complex when dealing with more than 3-arm junctions whether they are roundabouts or any other type of intersections. Any attempts to reduce the need to obtain counts from each and every entry or exit sections as well as other circulating sections from a roundabout will become very attractive indeed especially when trafficflows are high. This may lead to the fact that the number of equations used in forming the matrices (i.e. independent algebraic equations) is less than the number of unknowns representing the different turning movements in a roundabout.

To solve this problem, an analogy has been made for estimating turning flows at road junctions with that used in solving "statically indeterminate" structures in Civil and Structural Engineering where the number of unknowns exceeds the number of independent algebraic equations [21]. The Gaussian elimination procedure [22] is applied for this purpose and the following terms, as defined in [23] and [4] are used: - "traffically determinate" road junction: which is one that provides a number of linearly independent algebraic equations equal to the number of traffic streams in that junction, and - "traffically indeterminate" road junction: where the number of linearly independentalgebraic equations is less than the number of traffic streams in that junction.

2.0 Materials and Methods

Several methods exist as solution to network flow problem but the use of linear equation has so far been the be to tackle network flow problems in the sense that flow problems affects everyman in the society. The one conversant to everybody is the traffic flow problem. So the easiest way to solve these problems is through the system of linear equation by Gaussian elimination method.

3.0 Solution to Network Flow Using Gaussian Elimination

A system of linear equations is one which may be written in the form

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \qquad (1_a)$$
$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \qquad (1_b)$$
$$\vdots \qquad\qquad \vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m \qquad (1_m)$$

Here, all of the coefficientsa_{ij} and all of the right hand sides b_j are assumed to be known constants. All of the x_i's are assumed to be unknowns, that we are to solve for. Imagine that we are in the midst of applying Gaussian elimination, as in the above flow chart, and that we have finished dealing with rows $1, \dots, e-1$. These rows will not change during the rest of the elimination process. Denote by M_e the matrix consisting of those rows of the current coefficient matrix having index at least e. If the augmented matrix now looks like

$$\begin{bmatrix} * & * & ** \\ 0 & 0 & ** \\ 0 & * & ** \end{bmatrix}$$

and $e = 2$ (in other words, we are about to start work on row 2) then

$$M_e = \begin{bmatrix} 0 & 0 & * \\ 0 & * & * \end{bmatrix}$$

4.0 Analysis of Traffic Network Flow Problem to Estimate Turning Movements at Roundabouts Using Field Data

Systems of linear equations arise when we investigate the flow of some quantity through a network. Such networks arise in science, engineering and economics. Two such examples are the pattern of traffic flow through a city and distribution of products from manufacturers to consumers through a network of wholesalers and retailers. A network consists of a set of points, called the nodes, and directed lines connecting some or all of the nodes. The flow is indicated by a number or a variable. We observe the following basic assumptions:

i. The total flow into a node is equal to the total flow out of a node.
ii. The total flow into the network is equal to the total flow out of the network.

For any roundabout, each equation corresponds to flow counts at a certain cross section. These flow counts could be obtained easily by unskilled laborers, automatic counters or video cameras. However, certain assumptions should be made in arriving at a logical and simple model for use in accurately estimating turning movements at roundabouts. These assumptions are as follows:

a. Traffic flow is continuous through the junction (i.e. no stopping or parking within the roundabout). Accordingly,

$$\sum E_i = \sum L_i \qquad (1)$$

Where

E_i and L_i are traffic entering and leaving arm i, respectively.

b. No U-turns were made (i.e. from and to the same arm). Accordingly,

$$T_{ii} = 0 \text{ (for all i values)} \qquad (2)$$

Where, T_{ii} is traffic from and to arm i.

c. Traffic is homogenous (i.e. traffic composition is the same for all links).

In order to be able to solve an O-D matrix, one needs a "traffically determinate" junction. However, when dealing with a "traffically indeterminate" junction (as defined above) and because of the missing data from and to certain arms or unavailability of data (which could be due to shortage in the number of data collectors or counters on site), extra data is required. Such data could be obtained by considering extra information on flows for a selected weaving section within the circulating traffic between roundabout arms (and possibly the use of information from other "redundant" traffic streams which may easily be obtained from observations).

Table 1 gives the number of movements within a chosen weaving section for different numbers of roundabout arms, say W_{12} as shown in Figure 1. Considering a particular weaving section within a circulating traffic and assuming no U-turns, it is clear from Table 2 that as the number of roundabout arms increases, the number of movements within the weaving section increases sharply. This shows that the complexity of monitoring turning movements at roundabout increases.

Table 1: Number of Movements within a Selected Weaving Section as a Function of the Number of Roundabout Arms

Number of Roundabout Arms	**Number of Movements within a Weaving Section (including those from adjacent arms)**	**Details of Traffic Movements**
3	2	$W_{12}=E_1+T_{32}$
4	4	$W_{12}=E_1+T_{32}+T_{42}+T_{43}$
5	7	$W_{12} = E_1 + T_{32} + T_{42} + T_{43} + T_{52}+T_{53}+T_{54}$
6	11	$W_{12} = E_1 + T_{32} + T_{42} + T_{43} + T_{52}+T_{53}+T_{54}+T_{62}+T_{63}+T_{64}+T_{65}$

5.0 Matrices Formation For "Traffically Indeterminate" Roundabouts

In this research work, we focused on a 3-arm roundabout located along Mobil in Minna, Niger state Nigeria. A simple example is chosen for a 3-arm roundabout to describe the proposed mathematical model to form the matrices used in solving "traffically indeterminate" roundabouts. Here the assumptions described in previous sections (i.e. homogenous traffic, $E_i = L_i$ and $T_{ii} = 0$) were used together with the assumption that some data on traffic flow movements were missing due to absence of counters.

The following matrix describes the algebraic equations representing turning flows if the weaving section W_{12} is chosen for the 3-arm roundabout shown in Equation (3):

$W_{12}=E_1+T_{32}$

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} T_{13} \\ T_{12} \\ T_{31} \\ T_{32} \\ T_{21} \\ T_{23} \end{bmatrix} = \begin{bmatrix} E_1 \\ E_3 \\ E_2 \\ L_1 \\ L_3 \\ W_{12} \end{bmatrix} \qquad (3)$$

Note that W_{12} is equal to $T_{12} + T_{13} + T_{32}$ while E_1 is equal to $T_{12}+T_{13}$. Here the required counts are E_1, E_2, E_3, L_1, L_3 and W_{12} (assuming that the missing flow counts for this case is L_2).

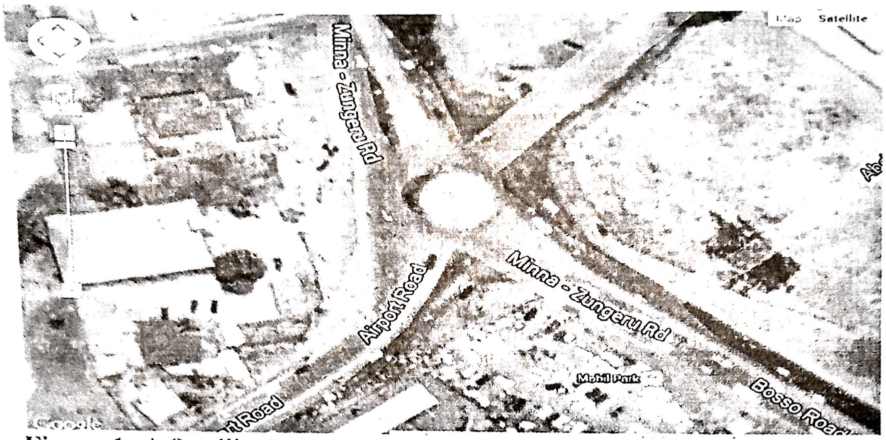

Figure 1: A Satellite Image of Mobil Roundabout along Zungeru Road

6.0 Application of Systems of Linear Equations to Traffic Flows

In recent years, the concept and tools of network analysis have been found to be useful in transportation system. The level urbanization in the developing world indicates that more people live in the cities than before. The population of Niger state grew from 2,421,581(1991) to 3,950,249 based on the 2006 Census. This implies an increase in vehicles from the year 2004 to 2007 of about 5,676 to 8,063 vehicles. The following analysis of traffic flow through a road network during the peak period illustrates how systems of linear equations can arise in practice.

Consider the typical road network of Figures 2 and 3;it represents an area of Bosso, Minna. The roads are all one-way with the arrows indicating the directions of traffic flow.

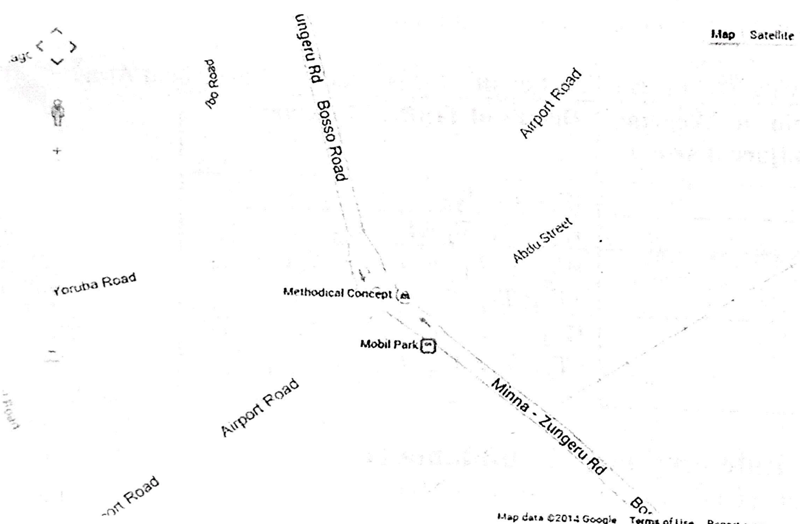

Figure 2: A Map of Minna Showing t

The flow of traffic in and out of the network is measured in terms of vehicles per hour based on 15 minutes counts. The figures given in Table 3 are based on a mid week peak traffic hour 5pm to 6pm. A mathematical model was used to analyze this network in Figure 2 as shown below:

Table 3: Average flow per hour for a 3-arm Roundabout (Based on 15 minutes counts at evening peak)

Time of the day	Entry Arm 1 (Bosso Zungeru from the North West) to Arm		Entry Arm 2 (Airport Road from the North East) to Arm		Entry Arm 3 (Tunga Zungeru Road from the South) to Arm		
	2	3	3	1	1	2	Total
5:15	205	270	110	98	315	156	1154
5:30	217	303	175	52	300	131	1178
5:45	136	164	74	38	127	88	627
6:00	112	218	107	70	228	104	839
6:15	279	296	185	89	243	213	1305
Total	949	1251	651	347	1213	692	

Table 3 shows the required traffic counts and those assumed missing. The assumed missing (in this case T_{13},T_{12},T_{31},T_{32},T_{21}, and T_{23}) counts was calculated using the Gaussian elimination procedure. Where $W_{12} = T_{13} + T_{12} + T_{32}$ with the help of Math Lab

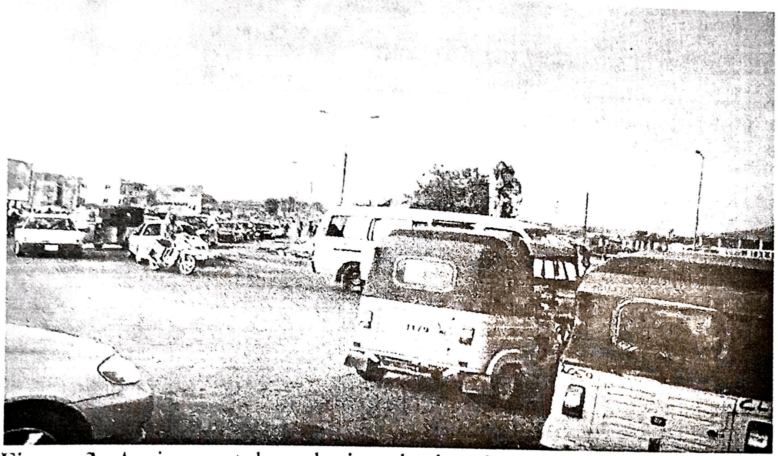

Figure 3: An image taken during the head counts of vehicles

Table 3: Traffic Count Matrix for the 3-Arm Roundabout

	To			
From	1	2	3	Total
1	0	------	------	2200
2	------	0	-------	990
3	------	------	0	1905
Total	1560	-------	1902	
W_{12}=2892				

The above values in Table 3 give the total number of vehicles entering arm 1, 2, 3 and exit of arm 1, 3 and W_{12} with the assumed missing turns.

$$\begin{aligned} T_{13} + T_{12} &= 2200 \\ T_{31} + T_{32} &= 1905 \\ T_{21} + T_{23} &= 990 \\ T_{31} + T_{21} &= 1560 \\ T_{13} + T_{23} &= 1902 \\ T_{13} + T_{12} + T_{32} &= 2892 \end{aligned} \quad (4)$$

Note: The above system of linear equation can be formed into matrix by substituting the above gives

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} T_{13} \\ T_{12} \\ T_{31} \\ T_{32} \\ T_{21} \\ T_{23} \end{bmatrix} = \begin{bmatrix} 2200 \\ 1905 \\ 990 \\ 1560 \\ 1902 \\ 2892 \end{bmatrix} \quad (5)$$

The solution to Equation (5) using Gaussian elimination with the help of Math Lab the result gives:

$$\begin{bmatrix} T_{13} \\ T_{12} \\ T_{31} \\ T_{32} \\ T_{21} \\ T_{23} \end{bmatrix} = \begin{bmatrix} 1251 \\ 949 \\ 1213 \\ 692 \\ 347 \\ 651 \end{bmatrix} \quad (6)$$

Figure (4) represents the solution to the problem using the Math Lab software

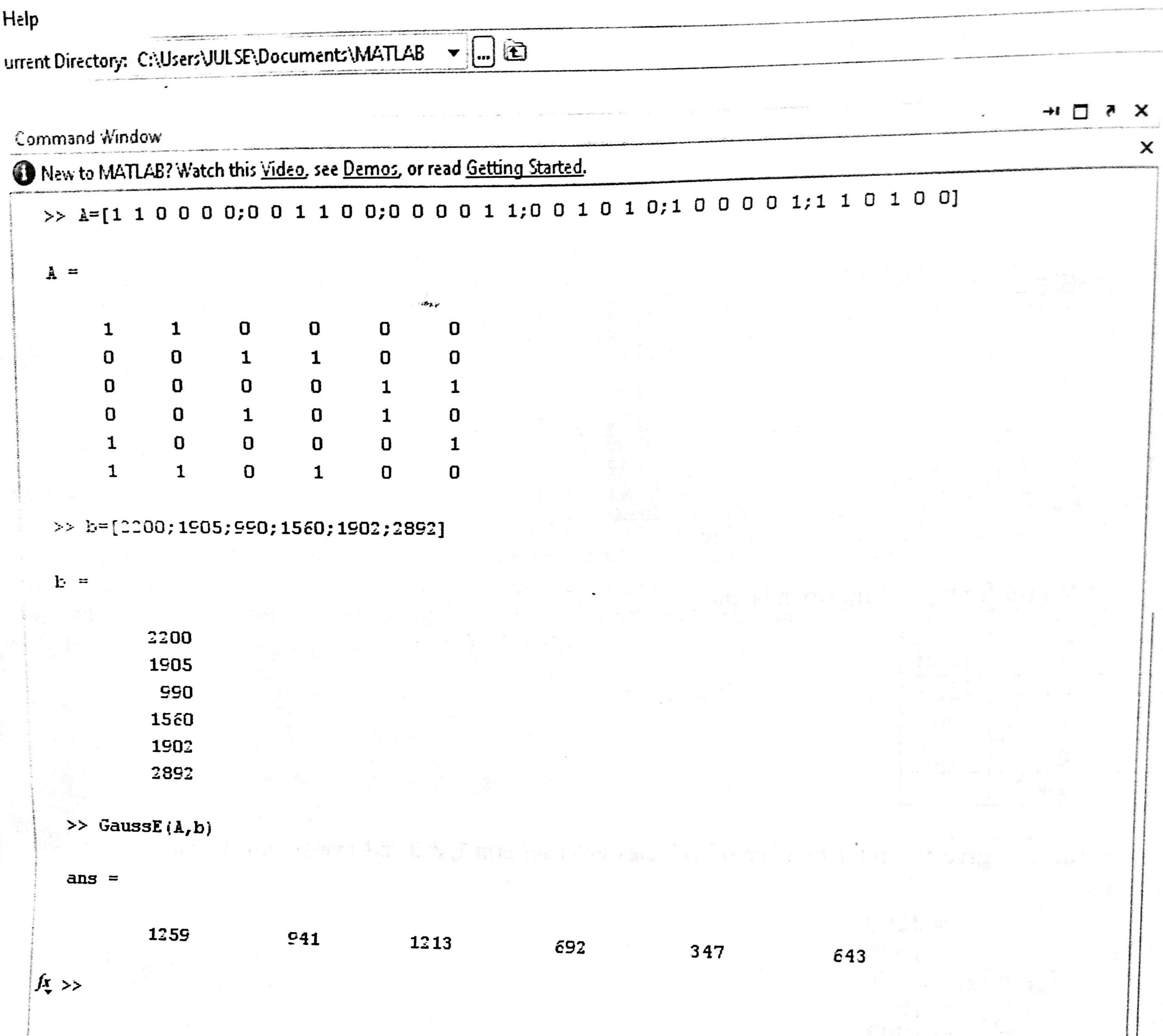

Figure 4: Solution to the Problem using the Math Lab software

7.0 Results and Discussion

When comparing values of flow counts from Equation with those of Table 3 the results show complete agreement. There is more traffic flow in T_{13}arm (1251). This implies that there is more traffic flow in Bosso Zungaru towards Tunga road. If traffic light is to be mounted at this round about more preference is to be given to Bosso Zungaru - Tunga road in order to eliminate unnecessary hold up. This indicates that this mathematical method used for solving"traffically determinate" 3-arm roundabout results is useful and significant saves time and labour cost.

This research work has solved problems network flow. Problems like transport should not look difficult to the industrialists again. In some cases manual traffic counts are used and on others more sophisticated devices, such as automatic traffic counters, video cameras are in use to gather the necessary information on traffic such as traffic flow, composition of traffic and turning movements at junctions. . The solution to transport problem helps to minimize cost, example in distribution of goods and in the usage of traffic counters, to maximize profit for businessmen and businesswomen, safes time, safes life for example reduces emission of carbon into the atmosphere when traffic is controlled and safes the environment from global warming.

This research work has come at right time to assist government especially transport management for proper planning and management of traffic in a particular place.

8.0 References

[1] Jefferys, M. and Norman, M. (1977) On finding realistic turning flows at road junctions, *Traffic Engineering and Control*, January, Vol. 18, No. 1, 19-21, 25.

[2] Mekky, A. (1979) On estimating turning flows at road junctions, *Traffic Engineering and Control*, October, Vol. 20, No. 10, 486-487.

[3] Van Zylen, H.J. (1979), The estimation of turning flows on a junction, *Traffic Engineering and Control*, November, Vol. 20, No. 11, 539-541.

[4] Razouki, S.S. and Jadaan, K.S. (1997) Estimation of turning movement at three-arm rotaries, *Dirasat, an International refereed research Journal, Natural and Engineering Sciences*, Vol. 24, No. 2, 305-312, Amman, Jordan.

[5] Razouki, S.S. (2000) A mathematical model for determining the origin-destination matrix for five-arm rotaries, *Journal of Engineering and Development,* Vol. 4, No. 3, 86-99, College of Engineering, Al-Mustansiriya University, Baghdad, Iraq.

[6] Eisenman, S.M. and List, G. (2005) A technique for data collection and estimation of turning movements at roundabouts, *Proceedings of the 84th Annual Conference of the Transportation Research Board,* 2005, Washington D.C.

[7] Bell, M.G.H. (1984) The estimation of junction turning volumes from traffic counts: the role of prior information, *Traffic Engineering and Control,* May, Vol. 25, No. 5, 279-283.

[8] Jadaan, K. (1989) Accuracy of turning flow estimates at road junctions, *Journal of Transportation Engineering, ASCE,* July, Vol. 115, No. 4, 438-449.

[9] Maher, M.J. (1984) Estimating the turning flows at a junction: a comparison of three models, *Traffic Engineering and Control,* Vol. 25, No. 1, 19-22.

[10] Cremer, M. and Kellerg, H. (1987) A new class dynamic methods for the identification of origin-destination flows, *Transportation Research B,* Vol. 21B, No. 2, 117-132.

[11] Marshall, M.L. (1979) Labour-saving methods for counting traffic movements at three- and four-arm junctions, *Traffic Engineering and Control,* April, Vol. 20, No. 4, 159-162.

[12] Mountain, L.J., Maher, M. and Maher, S. (1986) The estimation of turning flows from traffic counts at four-arm intersections, *Traffic Engineering and Control,* Vol. 27, No. 10, 501-507.

[13] Adebisi, O. (1987)Improving manual counts of turning traffic volumes at road junctions, *Journal of Transportation Engineering,* ASCE, 113(3), 256-267.

[14] Goethe, M, Jornsten, K. and Lundgren, J. (1989) Estimation of origin-destination matrices from traffic counts using multiobjective programming formulations, *Transportation Research B,* Vol. 23B, No. 4, 257-269.

[15] Lam, W.H.K. and Lo, H.P. (1990) Accuracy of O-D estimates from traffic counts, *Traffic Engineering and Control,* Vol. 31, No. 6, 358-367.

[16] Nihan, N.L. and Davis, G.A. (1987) Recursive estimation of origin-destination matrices from input/output counts, *Transportation Research B*, Vol. 21B, No.2, pp.149-163.

[17] Dixon, M.P. and Rilett, L.R. (2002) Real-time OD Estimation using automatic vehicle identification and traffic count data, *Computer-aided Civil and Infrastructure Engineering*, Vol. 17, 7:21.

[18] Fisk, C.S. and Boyce, D.E. (1983) A note on trip matrix estimation from link traffic count data, *Transportation Research B*, Vol. 17B, No. 3, 245-250.

[19] Mountain, L. and Steele, D. (1983) Prior information and the accuracy of turning flow estimates, *Traffic Engineering and Control*, Vol. 24, No. 12, 382-388.

[20] Mountain, L.J. and Westwell, P.M. (1983) The accuracy of estimation of turning flows from automatic counts, *Traffic Engineering and Control*, Vol. 24, No. 1, 3-7.

[21] Bhatt, P. and Nelson, H.M. (1990) *Structures / Marshall and Nelson*, 3rd Edition, Longman Scientific & Technical, John Wiley & Sons, Inc, New York.

[22] Kreyszig, E. (2006). *Advanced engineering mathematics*, 9th Ed., John Wiley & Sons, Singapore.

[23] Razouki, S.S. (1997) Modelling turning movements at traffically determinate four-arm road junction, *Journal of Engineering and Development,* Vol. 1, No. 1, 62-71, College of Engineering, Al-Mustansiriya University, Baghdad, Iraq