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A LAGRANGE REVENUE MODEL FOR ETHANOL PRODUCTION FROM BIOMASS

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Abstract

In this work, we modified a revenue model in order to handle the costing of Ethanol Production from Banana Biomass through a heuristic approach. The methodology of Lagrange multiplier that involves budgetary constraints with local search was employed to formulate Lagrange cost model as multi-item, multi-level capacitated revenue generation. The Lagrange multiplier are updated by using surrogate subgradient method that ensures the convergence of the approximate solution. A feasible solution of the original problem is constructed from the solution of the Lagrange multiplier problem at each iteration which is later improved by local search that changes the values of one or more of the variables at each time. The result obtained from the numerical experiment shows that by using the formulated model the producers of Ethanol stand to have optimal revenue which is 13.25% above the current market price.

Keywords: Heuristic Optimization, Lagrange multiplier, Ethanol, Biomass, Convergence, Revenue.

1.0 Introduction

Historically, fermentation products were mainly food products, but in recent years an increased interest has been observed in the production of bulk chemicals (ethanol and other solvents), specialty chemicals (Pharmaceuticals, industrial enzymes), biofuels and food additives (flavor modifiers); Fermentation processes are also used in agriculture.

Mathematical optimization can be used as a computational engine to arrive at the best solution for a given problem in a systematic and efficient way. In the context of biochemical systems, coupling optimization with suitable simulation modules open a whole new avenue of possibilities. Two types of important applications as in [1] have been highlighted:

i. Design problems: How to rationally design improved metabolic pathways to maximize the flux of interesting products and minimize the production of undesired by-products (metabolic engineering and biochemical evolution studies);
ii. Parameter estimation: Given a set of experimental data, calibrate the model so as to reproduce the experimental results in the best possible way.

Banana trunk biomass is a renewable polymer abundant in nature particularly in Nigeria; as Nigeria is ranked among the highest producers of banana in West Africa. The biomass is often wasted after harvesting. Currently there are trends in hydrolyzing banana trunk polymers, using enzyme processes to produce fermentable sugars and the fermentable sugar is further converted into ethanol. This is a cheaper way of producing ethanol and it can be used as renewable fuel.

The mathematical model for the optimization of the various parameters leading to the production of ethanol from this abundant and renewable polymer has not been achieved.

2.0 Problem Statement

The market price for Ethanol currently stands at N4,000.00 per 2.5 litres container. The production of Ethanol involves the hydrolysis of biomass into residual and glucose through fermentation in the presence of enzymes. This is not without human labor. Modeling this process in order to achieve optimal revenue for a producer is believed to be a researchable venture.

3.0 Aim and Objectives

This work aims at formulating a mathematical model for optimal revenue of ethanol production from banana trunk biomass and solving the resulting equations using local search.

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4.0 Literature Review

Bioethanol is currently produced from sugars and starch materials. Ethanol made from sugar cane biomass (bagasse & straw) as well as other lignocelluloses materials provide unique environmentally sustainable energy sources, economic strategic benefits and can be considered as a safe and cleanest liquid fuel alternative to fossil fuels. So lignocellulose biomass can act as a cheap substrate with constant supply as a substrate for bioconversion to fuel ethanol[2]. Ethanol is currently produced from sugars, starches and cellulosic materials. However due to the concomitant growth in demand for human feed such as starches and sugars there is an urgent need for potentially less competitive and perhaps less expensive feedstock such as lignocellulose materials as main resources for ethanol production in the near future [3]. The alternative fuels are expected to satisfy several requirements which include most importantly substantial reduction of greenhouse gas emission, worldwide availability of raw materials, and capability of being produced from renewable feedstock[4]. Therefore this quest of turning lignocellulosic materials such as biomasses into useful product via chemical and enzymatic processes has become of keen interest [5].

5.0 Methodology

Let *X, Y* and *Z* be Banach space, and let $y_1 \leq_k$ be the linear partial order in *Y* induced by a closed, nonempty, convex cone *K* in *Y*: in $y_1 \leq_k y_2$ *iff* $y_2 - y_1 \in K$. We denote the polar cone of *K* by $K^{+} := \{y^{*} \epsilon Y^{*} : \langle y^{*}, y\rangle \geq 0, \forall\, y \in K\}$. Consider the following class of constrained optimization problems, for $(y; z) \in Y \times Z$,

$P(y; z)$: min $f(x)$ subject to: $g(x) \leq K\ y$, $h(x) = z$; $x \in C$;

where C is a closed subset of X, $f : X \rightarrow IR$ is lower semi-continuous, $g : X \rightarrow Y$ is lower semi-continuous with respect to $\leq K$ and $h : X \rightarrow Z$ is continuous [6, 7, 8].

5.1 Steps to Optimize the Multivariate Function

To maximize or minimize a multivariable function f(x, y,…) subject to the constraint that another multivariable function equals a constant, g(x,y,…) = C. The following steps are applicable:

i. Introduce a new variable λ, and define a new function L as follows:

$$L(x,y,\ldots) = f(x,y,\ldots) - \lambda(g(x,y,\ldots) - C).$$

The function L is called the "Lagrangian", and the new variable λ, is referred to as "Lagrangian multiplier".

ii. Set the gradient of L equal to the zero vector

$$\nabla L(x,y,\ldots\ \lambda) = 0$$

In other words, find the critical points of L.

i. Consider each solution, which will look like $(X_0, Y_0, \ldots, \lambda_0)$. Plug each one into f, whichever one gives the greatest or smallest value is the maximum or minimum point we are seeking. According to [9], production function is given by

$Q = AK^{a}L^{b}$ where '*a*' and '*b*' are positive fractions. $Q = AK^{a}L^{(1-a)}$

We modified a revenue model in order to handle the costing with a heuristic approach for the production of ethanol from biomass. The methodology of heuristic combined Lagrange multiplier that involves budgetary constraints with local search was employed to formulate Lagrange cost model as multi-item, multi-level capacitated revenue generation. We updated the Lagrange multiplier are by using surrogate sub gradient method that ensures the convergence of the approximate solution. A feasible solution of the original problem is constructed from the solution of the Lagrange multiplier problem at each iteration which is later improved by local search that changes the values of one or more of the variables at each time.

6.0 Model Formulation

6.1 Budgetary Constraints

For running a factory producing ethanol that requires Biomass as a raw material; our cost is predominantly human labour which is =N= 120 per hour for workers, initial Biomass weight is 500kg, 40% of residue and 70% of the glucose is produced from the biomass per 2500ml of ethanol. We want to maximize the function R,

$R(h, b, r, g) = Kh^{m}b^{n}r^{p}g^{q}$; m, n, p, q = 0-1; K = 200

Subject to,

$$120h + 500b + 40r + 70g = 4000.$$

Where,

K = constant,

h = hour of human labour,

b = mass of Biomass,

r = quantity of residue used and

g = quantity of glucose used.

6.2 Maximizing a Function

We need to maximize a function,

$$\text{Max}\quad R(h,b,r,g) = Kh^{m}b^{n}r^{p}g^{q}$$

subject to:
$120h + 500b + 40r + 70g = 4000.$
We begin by writing the Lagrange function for the formulation above.

Let
$m = \frac{3}{4}$, $n = \frac{1}{2}$, $p = \frac{1}{6}$, $q = \frac{1}{12}$.
The constrained optimization problem above becomes an unconstrained problem,
$L(h,b,r,g) = 200h^{3/4}b^{1/2}r^{1/6}g^{1/12} - \lambda(120h + 500b + 40r + 70g - 4000).$
This is our formulated model.

7.0 Model Solution

We set the gradient ∇L equal to 0. This is the same as setting each partial derivative equal to 0 as follows:

$$\frac{\partial L}{\partial h} = 0 \quad (1)$$

$$\frac{\partial L}{\partial b} = 0 \quad (2)$$

$$\frac{\partial L}{\partial r} = 0 \quad (3)$$

$$\frac{\partial L}{\partial g} = 0 \quad (4)$$

$$\frac{\partial L}{\partial \lambda} = 0 \quad (5)$$

Differentiating with respect to h, we have:

$$\frac{\partial}{\partial h}\left(200h^{3/4}b^{1/2}r^{1/6}g^{1/12} - \lambda[120h + 500b + 40r + 70g - 4000]\right) = 0$$

We obtained,

$$\left(\frac{600}{4}\right)h^{(-1/4)}b^{1/2}r^{1/6}g^{1/12} - \lambda 120 = 0 \quad (6)$$

Differentiating with respect to b, we have:

$$\frac{\partial}{\partial b}\left(200h^{3/4}b^{1/2}r^{1/6}g^{1/12} - \lambda[120h + 500b + 40r + 70g - 4000]\right) = 0$$

We obtained,

$$\left(\frac{200}{2}\right)h^{3/4}b^{(-1/2)}r^{1/6}g^{1/12} - 500\lambda = 0 \quad (7)$$

Differentiating with respect to r, we have:

$$\frac{\partial}{\partial r}\left(200h^{3/4}b^{1/2}r^{1/6}g^{1/12} - \lambda[120h + 500b + 40r + 70g - 4000]\right) = 0$$

We obtained,

$$\left(\frac{200}{6}\right)h^{3/4}b^{1/2}r^{(-5/6)}g^{1/12} - 40\lambda = 0 \quad (8)$$

Differentiating with respect to g, we have:

$$\frac{\partial}{\partial g}\left(200h^{3/4}b^{1/2}r^{1/6}g^{1/12} - \lambda[120h + 500b + 40r + 70g - 4000]\right) = 0$$

We obtained,

$$\left(\frac{200}{12}\right)h^{3/4}b^{1/2}r^{1/6}g^{(-11/12)} - 70\lambda = 0 \quad (9)$$

Differentiating with respect to λ, we have:

$$\frac{\partial}{\partial \lambda}\left(200h^{3/4}b^{1/2}r^{1/6}g^{1/12} - \lambda[120h + 500b + 40r + 70g - 4000]\right) = 0$$

We obtained,

$$-120h - 500b - 40r - 70g - 4000 = 0 \quad (10)$$

The maximum revenue can be obtained, using the relationship below,

$$R(h) = \left(200h^{3/4}b^{1/2}r^{1/6}g^{1/12}\right) \quad (11)$$

Inputting equations (6) to (11) into MAPLE10 Software, we have

$(\frac{600}{4}.h^{\frac{-1}{4}}.b^{\frac{1}{2}}.r^{\frac{1}{6}}.g^{\frac{1}{12}} - 120.\lambda = 0, \frac{200}{2}.h^{\frac{3}{4}}.b^{\frac{-1}{2}}.r^{\frac{1}{6}}.g^{\frac{1}{12}} - 500.\lambda = 0, \frac{200}{6}.h^{\frac{3}{4}}.b^{\frac{1}{2}}.r^{\frac{-5}{6}}.g^{\frac{1}{12}} - 422.\lambda = 0, \frac{200}{12}.h^{\frac{3}{4}}.b^{\frac{1}{2}}.r^{\frac{1}{6}}.g^{\frac{-11}{12}} - 383.\lambda = 0, -120.h - 500.b - 422.r - 383.g + 4000 = 0)$ (12)

$\frac{150\sqrt{b}\, r^{\frac{1}{6}} g^{\frac{1}{12}}}{h^{\frac{1}{4}}} - 120\lambda = 0,\ \frac{100 h^{\frac{3}{4}} r^{\frac{1}{6}} g^{\frac{1}{12}}}{\sqrt{b}} - 500\lambda = 0,\ \frac{100}{3}\frac{h^{\frac{3}{4}}\sqrt{b}\, g^{\frac{1}{12}}}{r^{\frac{5}{6}}} - 422\lambda = 0, \frac{50}{3}\frac{h^{\frac{3}{4}}\sqrt{b}\, r^{\frac{1}{6}}}{g^{\frac{11}{12}}} - 383\lambda = 0, -120h - 500b - 422r - 383g + 4000 = 0$ (13)

Solving for h, b, r, g andλ;we have

$h = \frac{50}{3} = 16.67;\quad b = \frac{8}{3} = 2.67;\quad r = \frac{2000}{1899} = 1.05;\quad g = \frac{2000}{3447} = 0.58$

$\lambda = \frac{1}{392751180} 200^{\frac{1}{4}} 1899^{\frac{5}{6}}\, 3447^{\frac{11}{12}} 1350000^{\frac{1}{4}} = 0.87$ (14)

By applying the Revenue Function Formulae using the results in (14), we have

$R_0 = \frac{200}{58912677} 50^{\frac{3}{4}} 3^{\frac{3}{4}} \sqrt{8}\, 2000^{\frac{1}{4}} 1899^{\frac{5}{6}} 3447^{\frac{11}{12}} = 2396$ (15)

8.0 Discussion of Result

The results in equation 14 which were used to give equation 15 suggest that the producer of Ethanol should employ 16.67 hours of human Labour, use 2.67kg of banana trunk biomass,1.05ml of residue and 0.58ml of glucose to obtain revenue of 2,396.00 Naira per 2.5 litres. This result is not optimal. By varying the parameters, MAPLE10 Software has shown that, 16.67 hours of human labour, 2.67kg of banana trunk biomass, 11.11ml of residue and 3.37ml of glucose will yield an optimal revenue of N4,530.00using similar computation as in equation 15. See the variations in Table 1.

Table 1: Result of the Variation of the Parameter using MAPLE10 Software.

S/No	K	m	n	p	q	Hour (h)	Biomass (b)	Residue (r)	Glucose (g)	Lagrange Function (λ)	Revenue (R)
1	200	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{12}$	16.67	2.67	1.053	0.58	0.87	2396
2.	200	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{12}$	17.78	2.13	13.33	3.81	1.0	3119
3	200	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{12}$	16.67	2.67	11.11	3.17	1.6	4530
4	200	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{1}{6}$	$\frac{1}{12}$	14.035	3.37	10.53	3.0075	1.6	4043
5	200	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{1}{6}$	$\frac{1}{8}$	13.68	3.28	10.26	4.40	1.8	4280
6	200	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	30.90	0.27	2.58	0.74	4.0	1310
7	200	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{4}$	30.90	0.27	2.58	0.74	4.0	1646

9.0 Conclusion

The result obtained from the numerical experiment shows that by using the formulated model the producers of Ethanol stand to have optimal revenue of N4,530 per 2.5 litres. This amount is higher by 13.25% than the current market price. By this, the producers of Ethanol will be able to supply to the retailers at a cost that enables retailers to make more gain and even re-sell at a lower price than the current N4000 per 2.5litres.

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