Local and Global Stability Analysis of a Mathematical Model of Measles Incorporating Maternally-Derived-Immunity

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ABSTRACT

In this paper, the local stabilities of both the Disease Free Equilibrium (DFE) and Endemic Equilibrium (EE) were analyzed using the Jacobian matrix stability technique. The global stabilities were analyzed using Lyapunov function. The analysis shows that the DFE is locally and globally stable if the basic reproduction number $R_0 < 1$ and $R_0 \leq 1$ respectively. The EE is also locally and globally stable if $R_0 > 1$. Vaccination and recovery rates have been shown from the graphical presentation as the important parameter that will eradicate measles from the population.

Keywords: Stability; equilibrium; measles; Lyapunov function

1. Introduction

Measles is a disease which transmits through the coughs and sneezes of an infected person. Contact with saliva or nasal discharges is also another way it spread (WHO, 2016). The infection defects the mucous layers, at that point spreads all through the body. Measles is an illness that infects only human beings and is not known to infect other mammals. It is one of the main sources of death among little children despite the fact that a safe and practical immunization is accessible (Atkinson, 2011). There were 134 200 measles deaths in 2015, all over the world in which around 367 deaths occur every day or 15 deaths every day. The death due to measles have decrease 79% from 2000 and 2015 worldwide as a result of vaccination. About 20.3 million deaths between 2000-2015 have been prevented by measles vaccination, making the vaccine a standout amongst other purchases in public health (WHO, 2016).

Lyapunov functions are needed apparatus in the stability analysis of dynamical systems, both in theory and applications (Korobeinikov, 2004). The general problem of creating a Lyapunov function is a very hard problem. There have been several efforts and methods in the literature of how to calculate Lyapunov functions for many kinds of systems. Some of them use a mental understanding into the system to have a good perception about a candidate for a Lyapunov function; others use more logical means, including numerical algorithms. These techniques have emanate from diverse groups in Engineering, Mathematics, and Informatics (Giesl and Hafstei, 2015).

Abubakar, *et al.* (2012), formulate the model of Measles dynamics using SIR model and obtained the equilibrium points. They carried out the stability analysis of endemic equilibrium using Belman and cook theory. Abubakar, *et al.* (2013), used Hopf;s bifurcation theory to analyzed the stability of endemic equilibrium. In Somma *et al.*, (2015), they modified the existing Maternally-Immune Susceptible Infected

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Recovered (MSIR) model by incorporating vaccination rate and death rate due to the disease. They obtained the Disease Free Equilibrium (DFE) and calculate the Basic Reproduction Number R_0.

In this paper, we obtain the Endemic Equilibrium (EE) and analyzed the Local and Global stability of both DFE and EE of the by (Somma *et al.*, 2015). The Jacobian Matrix technique was used to analyze the local stabilities and Lyapunov function to analyze the global stability. Carryout the stability analysis of the model and to also simulate the model graphically.

2. Material and Methods

2.1 Model Formulation

The model considered the total population $N(t)$ and divided into four compartment based on the epidemiological status of individuals: Maternally-Derive-Immunity $M(t)$, Susceptible $S(t)$, Infected $I(t)$ and Recovered/Immune $R(t)$, where t is time. In this model it is assume that the new babies are born into M class with temporary immunity from their mothers at constant rate Λ. The new babies loss their immunity after some time at a rate θ and move to susceptible class. The susceptible individuals become infected with measles at a contact rate α. The susceptible class is vaccinated at a rate v and thereby move to recovered/immune class. The treated infected individuals recover at a rate γ and move to recovered/immune class. The death rate due to measles δ while the natural death rate of the entire population is μ. The schematic diagram and model equations for the measles transmission as discuss in this paper are presented below:

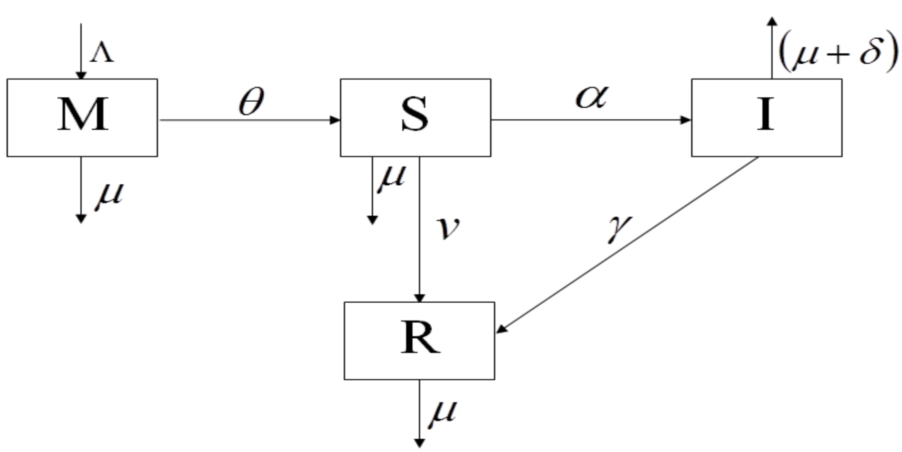

Figure 2.1: Flow Diagram of the Model

2.2 Model Equations

$$\frac{dM}{dt} = \Lambda - \theta M - \mu M \quad (2.1)$$

$$\frac{dS}{dt} = \theta M - \alpha SI - (\mu + v)S \quad (2.2)$$

$$\frac{dI}{dt} = \alpha SI - (\gamma + \mu + \delta)I \quad (2.3)$$

$$\frac{dR}{dt} = \gamma I - \mu R + vS \quad (2.4)$$

Table 2.1: Definition Variables and Parameters of the Model

Variables/Parameter	Description
N	Total Population
M	Maternally-Derived –Immunity
S	Susceptible
I	Infected
R	Recovered/Immune
Λ	Recruitment rate
θ	Loss of Immunity Rate
α	Contact Rate
δ	Death Rate due to Disease
γ	Recovery Rate
μ	Natural Death Rate
v	Vaccination Rate

2.3 Existence of Equilibrium Points

At equilibrium $\frac{dM}{dt} = \frac{dS}{dt} = \frac{dI}{dt} = \frac{dR}{dt} = 0$ (2.5)

Let $E^* = (M^*, S^*, I^*, R^*)$ be the arbitrary equilibrium points of the model system

$$\Lambda - A_1 M^* = 0 \quad (2.6)$$

$$\theta M^* - \alpha S^* I^* - A_2 S^* = 0 \quad (2.7)$$

$$\alpha S^* I^* - A_3 I^* = 0 \quad (2.8)$$

$$\gamma I^* - \mu R^* + vS^* = 0 \quad (2.9)$$

Where

$$A_1 = (\theta + \mu),\ A_2 = (\mu + v),\ A_3 = (\gamma + \mu + \delta)$$

From (2.8)

$$I^* = 0 \tag{2.10}$$

or

$$\left(\alpha S^* - A_3\right) = 0 \tag{2.11}$$

It is shown from (2.10) and (2.11) that there exist two equilibria; (2.10) is the Disease Free Equilibrium (DFE) while (2.11) is the Endemic Equilibrium (EE).

2.4 Disease Free Equilibrium (DFE), E^0

In the absence of the disease, this implies that $\left(I^* = 0\right)$,

Let $E^0 = (M^0, S^0, I^0, R^0)$ be the DFE points

Substituting (2.10) into (2.6) to (2.9) and solve simultaneously gives the DFE:

$$E^0 = \left(M^0, S^0, I^0, R^0\right) = \left(\frac{\Lambda}{A_1}, \frac{\Lambda\theta}{A_1 A_2}, 0, \frac{v\Lambda\theta}{A_1 A_2 \mu}\right) \tag{2.12}$$

2.5 The Basic Reproduction Number R_0.

The R_0, was calculated using the approach of (Driessche and Watmough, 2002). The detail of the R_0 is in (Somma *et al.*, 2015).

$$R_0 = \frac{\Lambda\alpha\theta}{A_1 A_2 A_3} \tag{2.13}$$

2.6 Endemic Equilibrium (EE) Point

Equation (2.11) give the existence of EE (i.e., $I^* \neq 0$)

Let $E_1 = (M_1, S_1, I_1, R_1)$ be the EE points

Therefore equation (2.6) to (2.9) become

$$\left.\begin{aligned} &\Lambda - A_1 M_1 = 0 \\ &\theta M_1 - \alpha S_1 I_1 - A_2 S_1 = 0 \\ &\alpha S_1 I_1 - A_3 I_1 = 0 \\ &\gamma I_1 - \mu R_1 + v S_1 = 0 \end{aligned}\right\} \tag{2.14}$$

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Solving (2.14) simultaneously gives the Endemic Equilibrium points with respect to R_0,

$$(M_1, S_1, I_1, R_1) = \left(\frac{\Lambda}{A_1}, \ \frac{A_3}{\alpha}, \frac{A_2(R_0 - 1)}{\alpha}, \frac{\gamma A_2(R_0 - 1) + vA_3}{\alpha\mu} \right) \quad (2.15)$$

3. Result and Discussion

3.1 Local stability of Disease Free Equilibrium (DFE)

Theorem 3.1: The DFE Equilibrium point E^0 of the model is Locally Asymptotically Stable (LAS) if $R_0 < 1$.

Proof:

Therefore, the Jacobian of the model at E^0 is given as

$$J(E^0) = \begin{bmatrix} -A_1 & 0 & 0 & 0 \\ \theta & -A_2 & A_4 & 0 \\ 0 & 0 & A_4 - A_3 & 0 \\ 0 & v & \gamma & -\mu \end{bmatrix} \quad (3.1)$$

Where $A_4 = \dfrac{\Lambda \alpha \theta}{A_1 A_2}$

Using elementary row operation to reduce (3.1) to upper triangular matrix gives

$$J(E^0) = \begin{bmatrix} -A_1 & 0 & 0 & 0 \\ 0 & -A_2 & A_4 & 0 \\ 0 & 0 & A_4 - A_3 & 0 \\ 0 & 0 & 0 & -\mu \end{bmatrix} \quad (3.2)$$

$$\left| J(E^0) - \lambda I \right| = 0 \quad (3.3)$$

$$\begin{vmatrix} -A_1 - \lambda & 0 & 0 & 0 \\ 0 & -A_2 - \lambda & A_4 & 0 \\ 0 & 0 & A_4 - A_3 - \lambda & 0 \\ 0 & 0 & 0 & -\mu - \lambda \end{vmatrix} = 0 \quad (3.4)$$

The characteristic equation of (3.4) is given as

$$(-A_1 - \lambda)(-A_2 - \lambda)(A_4 - A_3 - \lambda)(-\mu - \lambda) = 0 \quad (3.5)$$

From (3.5)

$$\lambda_1 = -A_1, \ \lambda_2 = -A_2, \ \lambda_3 = A_4 - A_3 \text{ and } \lambda_4 = -\mu \quad (3.6)$$

From (3.6)

$\lambda_1 < 0, \lambda_2 < 0,$ and $\lambda_4 < 0$, for the condition of stable stability λ_3 must be less than zero (i.e. $\lambda_3 < 0$)

Hence,

$$\left.\begin{array}{l} \lambda_3 < 0 \\ \Rightarrow A_4 < A_3 \end{array}\right\} \quad (3.7)$$

Recall $A_4 = \frac{\Lambda \alpha \theta}{A_1 A_2}$ therefore, (3.7) becomes

$$\frac{\Lambda \alpha \theta}{A_1 A_2 A_3} < 0 \quad (3.8)$$

Thus, (3.8) becomes

$$R_0 < 0 \quad (3.9)$$

Equation (3.9) proved the theorem 3.1, the model is (LAS) at DEF, E_0.

The implication of equation (3.9) is that the measles can be eradicated from the population.

3.2 Local stability of Endemic Equilibrium (EE)

Theorem 3.2: The EE point E_1 of the model is (LAS) if $R_0 > 1$.

Proof:

Therefore the Jacobian of the model at EE, E_1 is given as

$$J(E_1) = \begin{bmatrix} -A_1 & 0 & 0 & 0 \\ \theta & -A_2 R_0 & -A_3 & 0 \\ 0 & A_2(R_0 - 1) & 0 & 0 \\ 0 & v & \gamma & -\mu \end{bmatrix} \quad (3.10)$$

Using elementary row operation to reduce (3.10) to upper triangular matrix gives

$$J(E_1) = \begin{bmatrix} -A_1 & 0 & 0 & 0 \\ 0 & -A_2 R_0 & -A_3 & 0 \\ 0 & 0 & \frac{A_3 - A_3 R_0}{R_0} & 0 \\ 0 & 0 & 0 & -\mu \end{bmatrix} \quad (3.11)$$

$$\left| J(E_1) - \lambda I \right| = 0 \quad (3.12)$$

$$\begin{vmatrix} -A_1-\lambda & 0 & 0 & 0 \\ 0 & -A_2R_0-\lambda & -A_3 & 0 \\ 0 & 0 & \dfrac{A_3-A_3R_0}{R_0}-\lambda & 0 \\ 0 & 0 & 0 & -\mu-\lambda \end{vmatrix}=0 \quad (3.13)$$

The characteristic equation of (3.4) is given as

$$\left(-A_1-\lambda\right)\left(-A_2R_0-\lambda\right)\left(\frac{A_3-A_3R_0}{R_0}-\lambda\right)\left(-\mu-\lambda\right)=0 \quad (3.14)$$

From (3.14)

$$\lambda_1=-A_1,\ \lambda_2=-A_2R_0,\ \lambda_3=\frac{A_3-A_3R_0}{R_0} \text{ and } \lambda_4=-\mu \quad (3.15)$$

From (3.15)

$\lambda_1<0,\ \lambda_2<0,$ and $\lambda_4<0$, for the condition of stable stability λ_3 must be less than zero (i.e. $\lambda_3<0$)

Hence,

$$\left.\begin{aligned} &\lambda_3<0 \\ &\Rightarrow 1-R_0<0 \end{aligned}\right\} \quad (3.16)$$

Thus,

$$R_0>1 \quad (3.17)$$

Equation (3.17) proves theorem 3.2, the model is (LAS) at EE, E_1. The consequence of equation (3.17) is that the measles will continue in the population.

3.3 Global Stability Analysis of Disease Free Equilibrium (DFE), E^0

Theorem 3.3: If $R_0\leq 1$, the DFE, E^0 is Globally Asymptotically Stable (GAS).

Proof:

Define the following Lyapunov-Lasalle function

$$V=A_3I \quad (3.18)$$

Taking the time derivative of (3.18) we have

$$\frac{dV}{dt}=A_3\left[\alpha S-A_3\right]I \quad (3.19)$$

Since $S \le S^0$

$$\frac{dV}{dt} \le A_3\left[\alpha S^0 - A_3\right]I$$

$$\frac{dV}{dt} \le A_3\left[\frac{\Lambda\alpha\theta - A_1A_2A_3}{A_1A_2}\right]I \qquad (3.20)$$

Divide equation (3.20) through by $A_3^{\ 2}$ gives

$$\frac{dV}{dt} \le A_3^{\ 2}\left(R_0 - 1\right)I \qquad (3.21)$$

Hence, from equation (3.21), $R_0 \le 1$ implies that $\frac{dV}{dt} \le 0$. We conclude that $V(M, S, I, R)$ is negative definite and this proves that the model is (GAS) of the DFE, E^0.

3.4 Global Stability Analysis of Endemic Equilibrium (EE), E_1

Theorem 3.4: If $R_0 > 1$, the EE is (GAS).

Proof:

Consider the Lyapunov function

$$L(M_1, S_1, I_1, R_1) = \left(M - M_1 \ln M\right) + \left(S - S_1 \ln S\right) + \left(I - I_1 \ln I\right) + \left(R - R_1 \ln R\right) \qquad (3.22)$$

Taking the time derivative of (3.18) gives

$$\frac{dL}{dt} = \left(\frac{M - M_1}{M}\right)\left(\Lambda - A_1M\right) + \left(\frac{S - S_1}{S}\right)\left(\theta M - \alpha SI - A_2S\right) + \left(\frac{I - I_1}{I}\right)\left(\alpha SI - A_3I\right) + \left(\frac{R - R_1}{R}\right)\left(\gamma I - \mu R + vS\right) \qquad (3.23)$$

at the endemic equilibrium E_1 we have

$$\frac{dL}{dt} = -\frac{A_1\left(M - M_1\right)^2}{M} + \frac{\theta\left(S - S_1\right)\left(M - M_1\right)}{S_1} + \frac{v\left(S - S_1\right)\left(R - R_1\right)}{R_1} + \frac{\gamma\left(I - I_1\right)\left(R - R_1\right)}{R_1} \qquad (3.24)$$

$$\frac{dL}{dt} = -\frac{A_1\left(M - M_1\right)^2}{M} + P(M, S, I, R) \qquad (3.25)$$

Where

$$P(M, S, I, R) = \frac{\theta\left(S - S_1\right)\left(M - M_1\right)}{S_1} + \frac{v\left(S - S_1\right)\left(R - R_1\right)}{R_1} + \frac{\gamma\left(I - I_1\right)\left(R - R_1\right)}{R_1}$$

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Following the approach of Korobeinikov, (2004). , P is non-negative for $M, S, I, R > 0$.

Therefore, $\frac{dL}{dt} = 0$ if $M = M_1,\ S = S_1,\ I = I_1,\ R = R_1$

And

$\frac{dL}{dt} < 0$ if

$$P(M, S, I, R) < \frac{A_1(M - M_1)^2}{M} \tag{3.26}$$

for $M, S, I, R > 0$.

Thus, if $R_0 > 1$ then, Model system (2.1) - (2.4) has a unique EE point E_1 which is (GAS).

3.5 Graphical Presentation of Basic Reproduction Number, R_0 and Some Parameters of the Model

Table 3.1 is the table of values used for graphical presentation basic reproduction number and some parameters of the model.

Table 3.1: Values for Parameters used for the Graphical Presentation

Variables	Values per year	Source
$M(0)$	82,010,000	B9
$S(0)$	7,099,464,364	B10
$I(0)$	254,918	B3
$R(0)$	118,270,718	B4
N	7,300,000,000	B1
Λ	139,000,000	B2
α	0.9	B12
δ	0.53	B6
γ	0.47	B5
μ	0.008	B7
θ	0.39	B11
v	0.85	B8

See appendix B, for the estimation of variables and parameter values used in graphical presentation as shown on Table 4.1 above.

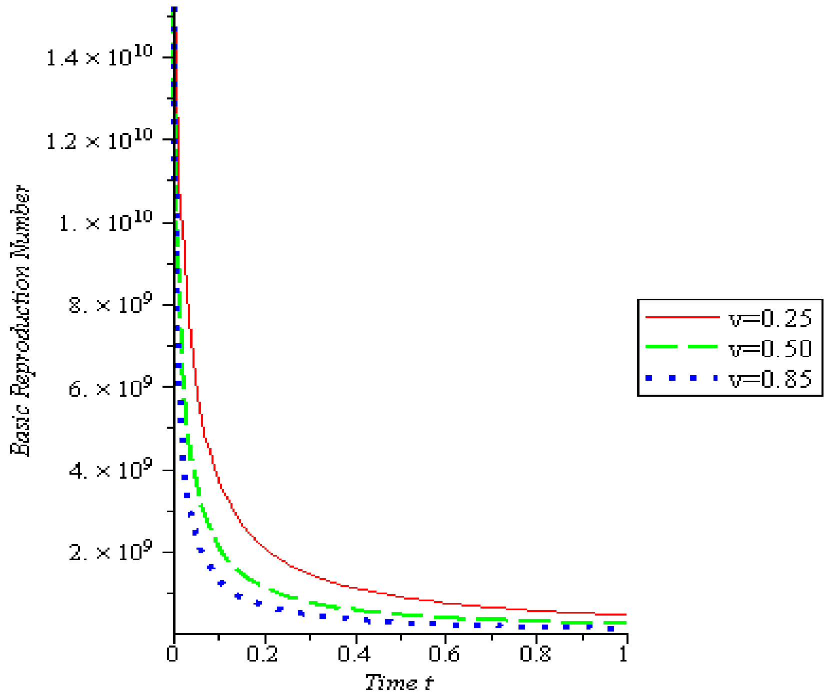

Figure 3.1: The Graph of R_0 against different values of Vaccination Rate

Figure 3.1 shows that as vaccination rate increases with time the R_0 decreases. It is observe that, with increase in vaccination rate, the basic reproduction number decrease to almost zero. This shows that, immunizing new babies will eradicate the measles from the population with time.

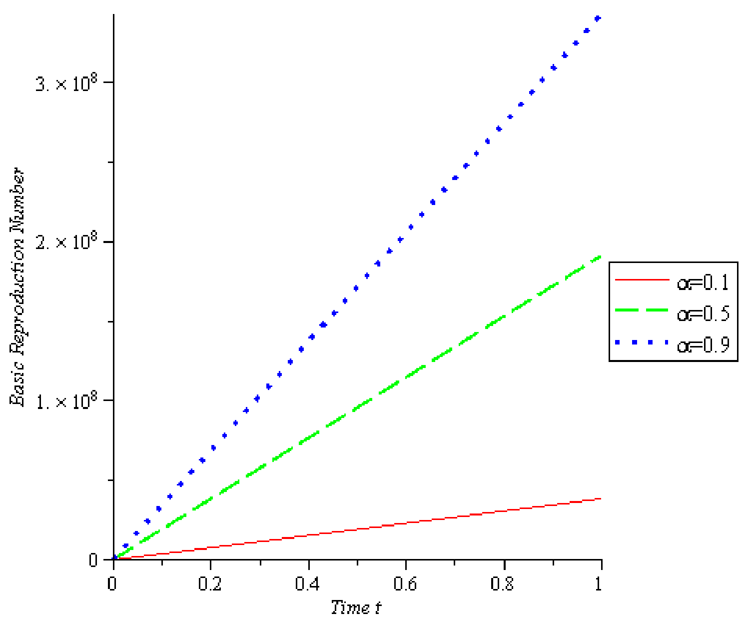

Figure 3.2: The Graph of R_0 against different values of Contact Rate

Figure 3.2 shows that as contact rate increases with time the R_0 increases. It also shows that low contact rate gives low basic reproduction number. The children infected with measles should be separated from those that are not infected.

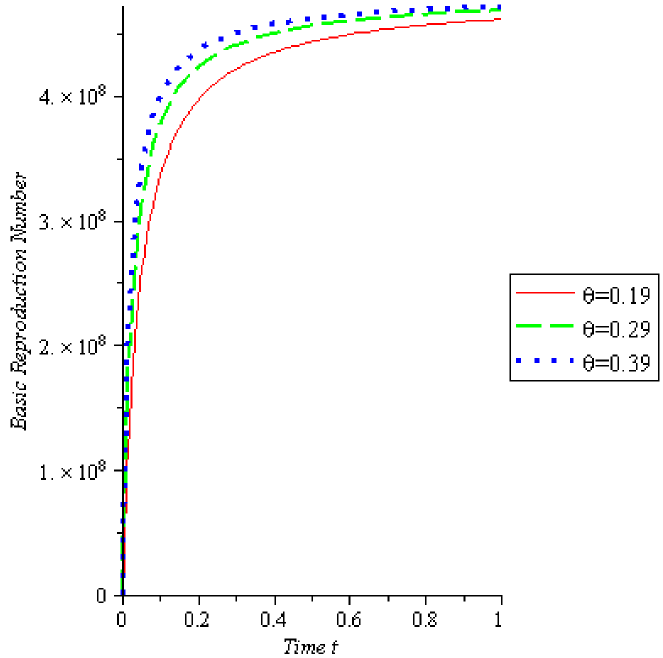

Figure 3.3: The Graph of R_0 against different values of loss of Immunity Rate

Figure 3.3 shows that as loss of immunity rate increases with time the R_0 increases. The immunity depends on vaccination and treatment.

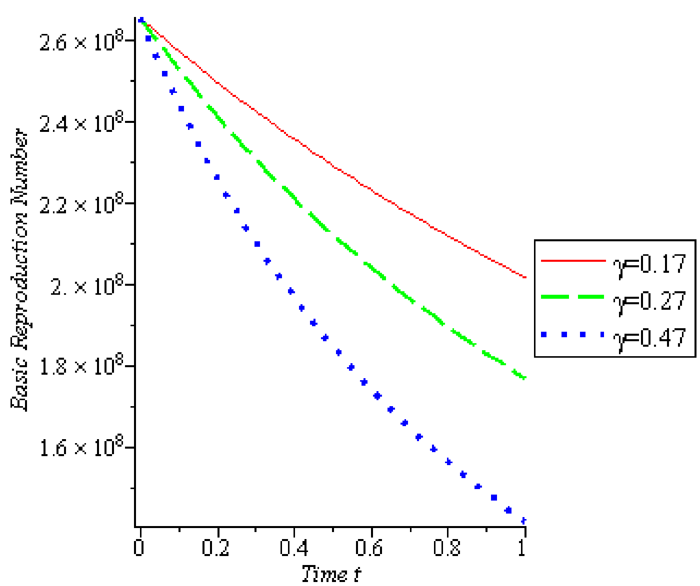

Figure 3.4: The Graph R_0 against different values of Recovery Rate

Figure 3.4 shows that as recovery rate increases with time the R_0 decreases. It is observe that, with increase in recovery rate, the basic reproduction number decrease to almost zero.

4. Conclusion

In this paper, we obtained the Endemic Equilibrium (EE) and analyzed the Local and Global stabilities of both DFE and EE. We used the Jacobian matrix stability technique to analyze the local stabilities and Lyapunov function to analyzed the global stabilities. The DFE and EE were locally and globally asymptotically stable. Measles will be eliminated from the population if $R_0 \leq 1$ or persist in the population if $R_0 > 1$. Graphical presentation shows that, vaccination rate and recovery rate are important parameters in eradicating the measles from the population.

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Appendices

Appendix A: Reported Measles Cases by WHO Region 2015 and 2016, as of November 2016

Table A1: Reported Measles Cases by WHO Region 2015, as of November 2016

WHO Region	Member States Reported (Expected)	Total Suspected	Total Measles	Clinical Confirmed	Epidemiological Link	Laboratory Confirmed	Data Received
Africa	41(47)	86984	55263	21111	26163	7989	Nov-16
America	34(35)	18670	210	0	0	210	Nov-16
Eastern Mediterranean	21(21)	34654	14053	639	4559	8855	Nov-16
Europe	50(53)	28025	26776	19835	1014	5926	Nov-16
South-East Asia	11(11)	114726	90860	64484	22353	4023	Nov-16
Western Pacific	27(27)	143289	67756	22337	611	44808	Nov-16
Total	**184(194)**	**426348**	**254918**	**128406**	**54700**	**71811**	

Source: WHO (2016)

Table A2: Reported Measles Cases by WHO Region 2016, as of November 2016

WHO Region	Member States Reported (Expected)	Total Suspected	Total Measles	Clinical Confirmed	Epidemiological Link	Laboratory Confirmed	Data Received
Africa	42(47)	46474	28126	12459	11085	4582	Nov-16
America	34(35)	9564	65	0	0	65	Nov-16
Eastern Mediterranean	20(21)	19763	4518	153	947	3418	Nov-16
Europe	50(53)	3849	2537	241	385	1910	Nov-16
South-East Asia	11(11)	86302	63169	51015	11004	1150	Nov-16
Western Pacific	27(27)	100517	55620	27594	638	27388	Nov-16
Total	**184(194)**	**266469**	**154035**	**91462**	**24059**	**38513**	

Source: WHO (2016)

Appendix B: Estimation of Variables and Parameter Values

It is difficult to get a reliable data, we estimated the parameter values based on the available data from the World Health Organization (WHO), Population Reference Bureau and reliable related literature. The estimates are clearly explained in the following sub-sections.

B1: The Total Population, N

According to Population Reference Bureau, the world total population at 2015, is 7.3 billion.

$$N = 7{,}300{,}000{,}000$$

B2: Recruitment Number, Λ

According to Population Reference Bureau the birth rate per year is $\frac{19}{1{,}000}$

The number of new birth in 2015 is 139, 000, 000.
Therefore,

$$\Lambda = 139{,}000{,}000$$

B3: Number of Infected, I

The WHO estimate that, there are 254, 918 cases of measles worldwide each year, resulting in 134,200 deaths. (See Table A1)

$$I = 254{,}918$$

B4: Number of Recovered/Immune, R

Recovered/Immune Human population, R = recovered + immune
From B3 the number of cases is 254, 918 and number of death is 134,200.
Recovered= 254, 918 -134,200 = 120,718 the number of surviving infants in 2015 is 139,000,000 and the percentage of vaccinated is 85%. Therefore,
Vaccinated = 85% of 139,000,000 =118,150,000.
Hence,

Recovered/Immune Human population, $R = 120{,}718 + 118{,}150{,}000$

$R = 118{,}270{,}718$

B5: Recovery Rate, γ

From B3 and B4

$$\gamma = \frac{\text{Recovered}}{\text{Number of cases}}$$

$$\gamma = \frac{120{,}718}{254{,}918} = 0.47$$

B6: Disease Induce death rate, δ

From B3 the number of cases of measles is 254,918 and the number of death from measles is 134,200

$$\delta = \frac{\text{Number of Death from measles}}{\text{Number of cases}}$$

$$\delta = \frac{134{,}200}{254{,}918} = 0.53$$

B7: Natural Death Rate, μ

According to WHO, the death rate is 8 deaths per 1,000. Therefore,

$$\mu = \frac{8}{1000} = 0.008$$

B8: Vaccination rate, v

According to, WHO in 2015, about 85% of the world's children received one dose of measles vaccine. Therefore,

$$v = 0.85$$

B9: Maternally-Derived-Immunity, M

According to Millennium Development Goal (MDG4), every year nearly 41% of all under-five child deaths are among newborn infants, babies in their first 28 days of life or the neonatal period.

$$M = 59\% \text{ of } 139{,}000{,}000$$

$$M = 82{,}010{,}000$$

B10: Number of Susceptible, S

Recall $N = M + S + I + R$ therefore,

$$S = N - (M + I + R)$$

$$S = 7{,}300{,}000{,}000 - (82{,}010{,}000 + 254{,}918 + 118{,}270{,}718)$$

$$S = 7{,}300{,}000{,}000 - 200{,}535{,}636$$

$$S = 7{,}099{,}464{,}364$$

B11: Loss of immunity, θ

According to WHO Immunization coverage fact sheet, national immunization schedule reported that, only 61% of children received 2 doses of measles. Therefore,

$$\theta = 39\% = 0.39$$

B12: Contact Rate, α

Nine out of ten people who are not immune and share living space with an infected person will catch it, (Atkinson, (2011),[8]). Therefore

$$\alpha = \frac{9}{10} = 0.9$$

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