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Stability and Bifurcation Analysis of a Mathematical Modeling of Measles Incorporating Vitamin A Supplement

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Abstract

Measles is transmissible disease that is common among children. The death caused by measles among children of five years and below is alarming in spite of the safe and effective vaccine. It has been discovered that Vitamin A Deficiency (VAD) in children increases their chances of measles infection. In this paper, the mathematical model of measles incorporating Vitamin A supplement as treatment was formulated and analyzed. The equilibrium points are obtained and analyzed for stability. Bifurcation and sensitivity analyses were carried out to gain further insight into the spread and control of measles. The stability analysis revealed that Disease Free Equilibrium (DFE) is stable if $R_0 < 1$. *The bifurcation analysis revealed forward bifurcation while the sensitivity analysis shows the most sensitive parameters of the model that are responsible for the spread and control of the diseases. The effect of sensitive parameters on Basic Reproduction Number,* R_0 *were presented graphically. Vaccination, recovery and Vitamin A supplement rates have been shown from the graphical presentation as the important parameter that will eradicate the measles from the population while contact and loss of immunity rates have shown that measles will persist in the population. People should be sensitized on the danger of living with infected persons. Government should do more in routine immunization and administration of Vitamin A Supplement.*

Keywords: *Stability, equilibrium, measles, trace-determinant, Lyapunov function*

1. Introduction

Measles has become a serious public health challenge to the little children globally. It is transmitted through the fluid from the nose, mouth or throat of infected persons (Atkinson, 2011). Research shows that 90% of susceptible individuals who have contact with infected person are likely to be infected (Leuridan *et al.*, 2012). High fever, a runny nose and red eyes are the signs and symptoms of measles. Blindness, severe diarrhoea and pneumonia are many at times are measles complications. In 2018 over 140,000 people died from measles (WHO, 2019). The Nigerian Centre for Disease Control (NCDC) in the beginning of 2019 recorded over

22,000 suspected cases of measles. Over 50% happen in the terrorist- affected Borno State as well as in 37 internally displaced persons (IDP) camps, this resulted to 13,753 suspected cases and 72 deaths (NCDC, 2019). Deficiency of Vitamin A has been identified as the major factor responsible for the death and other health complications resulting from measles. Almost 200 million children have been affected by its adverse health challenges, including death. Hence, WHO recommends that Vitamin A supplement be given to children from 6 months to 5 years of age (Mayo-Wilson *et al.*, 2011). They also recommend that, Vitamin A supplement be given two times in 24 hours to the children with measles (Imdad *et al.*, 2017).

Abubakar, *et al.* (2012), formulate the model of Measles dynamics using SIR model and obtained the equilibrium points. They used Belman and cook theory to carried out the stability analysis of endemic equilibrium. Abubakar, *et al.* (2013), analyzed the stability of endemic equilibrium using Hopf;s bifurcation theory. In Somma *et al.*, (2015), they modified the existing Maternally-Immune Susceptible Infected Recovered (MSIR) model by incorporating vaccination and disease-induce death rates. They obtained the equilibrium points and analyzed same for stabilities. Somma *et al.* (2017) used Lyapunov functions to carry out the stability analysis. Fred *et al.* (2014) studied mathematical modeling of measles with vaccination as control measure but did not include Maternally-derived –immunity in their model. Okyere-Siabouh and Adetunde (2013) formulated a mathematical model of measles using Susceptible-Exposed-Infected-Recovered (SEIR) epidemiological model. In their study they assumed that everybody is born into the susceptible class. Peter *et al.* (2018), Sinha *et al.* (2019) and Garba *et al.* (2020) also proposed a mathematical model of measles disease dynamics with emphasis on vaccination as means of control. Their models did not consider Vitamin A supplement.

In this paper, we obtain the Disease Free Equilibrium (DFE) and Endemic Equilibrium (EE) in terms of force of infection. The R_0 was computed and used in the stability, bifurcation and sensitivity analyses. We use trace-determinant method and Lyapunov function for the stability analyses. The sensitive parameters and R_0 are presented graphically.

2 Materials and Methods

2.1 Model Formulation

The model equation is divided into four compartments based on the epidemiological status of individuals: with $N(t)$ total population, Maternally-Derive-Immunity $M(t)$, Susceptible $S(t)$, Infected $I(t)$ and Recovered/Immune $R(t)$, where t is time . The new babies are born into M class and susceptible class at constant rate Λ. $\Lambda\beta$ is the proportion of children born with passive immunity while $\Lambda(1-\beta)$ is the proportion of children born without passive immunity. The new babies loss their immunity after some time at a rate θ and move to susceptible class. The susceptible individuals become infected with measles at a contact rate α. The susceptible class is vaccinated at a rate v and move to recovered/immuned class. The infected individuals are treated and given Vitamin A supplement at recovery rate γ and Vitamin A supplement rate τ, respectively and moved to recovered/immune class. The death rate due to disease is δ and the natural death rate of the entire population is μ. The schematic diagram and model equations for the measles transmission as discuss in this paper is presented in Fig 2.1:

2.2 Model Equations

$$M' = \Lambda\beta - (\theta + \mu)M \tag{2.1}$$

$$S' = \Lambda(1-\beta) - \frac{\alpha SI}{N} + \theta M - (\mu + v)S \tag{2.2}$$

$$I' = \frac{\alpha SI}{N} - (\gamma + \tau + \mu + \delta)I \tag{2.3}$$

$$R' = (\gamma + \tau)I - \mu R + vS \tag{2.4}$$

Where,

$$N = M + S + I + R \tag{2.5}$$

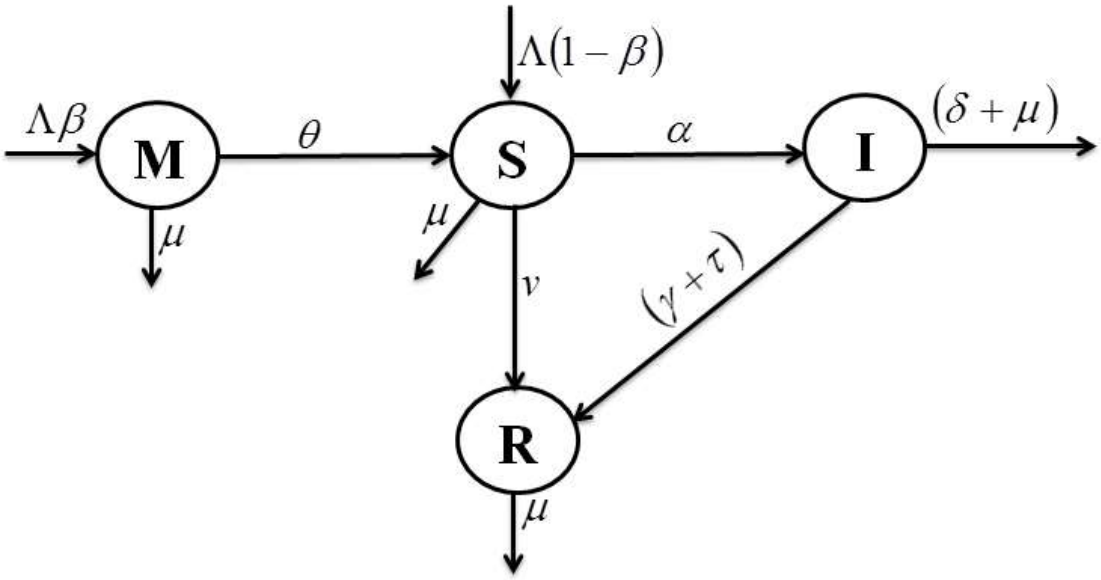

Figure 2.1: Schematic Diagram of the Model

2.3 Existence of Equilibrium Points

At equilibrium $M' = S' = I' = R' = 0$ (2.6)
Let

$$E^* = (M^*, S^*, I^*, R^*) = (M, S, I, R) \tag{2.7}$$

$$\Lambda\beta - A_1 M^* = 0 \tag{2.8}$$

$$\Lambda(1-\beta) - \frac{\alpha S^* I^*}{N^*} + \theta M^* - A_2 S^* = 0 \tag{2.9}$$

$$\frac{\alpha S^* I^*}{N^*} - A_3 I^* = 0 \tag{2.10}$$

$$A_4 I^* + vS^* - \mu R^* = 0 \tag{2.11}$$

Where, $A_1 = (\theta + \mu)$, $A_2 = (\mu + v)$, $A_3 = (\gamma + \tau + \mu + \delta)$, $A_4 = (\gamma + \tau)$

Table 1: Description of Variables and Parameters of the Model

Variables/Parameter	Description
N	Total Population
M	Maternally-Derived –Immunity
S	Susceptible
I	Infected
R	Recovered/Immune
Λ	Recruitment rate
β	Birth with Temporary Passive immunity
θ	Loss of Immunity Rate
α	Contact Rate
δ	Death Rate due to Disease
γ	Recovery Rate
τ	Vitamin A supplement Rate
μ	Natural Death Rate
v	Vaccination Rate

From (2.10)

$$I^* = 0 \tag{2.12}$$

or

$$\left(\frac{\alpha S^*}{N^*} - A_3\right) = 0 \tag{2.13}$$

It is shown from (2.12) and (2.13) that there exist two equilibria; (2.12) is the Disease- Free Equilibrium (DFE) while (2.13) is the Endemic Equilibrium (EE).

Let,

$$E^0 = (M^0,\ S^0,\ I^0,\ R^0) = (M,\ S,\ I,\ R) \tag{2.14}$$

be the DFE points

Substituting (2.12) into (2.8) to (2.11) and solve simultaneously gives the DFE:

$$\left(M^0, S^0, I^0, R^0\right) = \left(\frac{\Lambda\beta}{A_1},\ \frac{A_6}{A_1 A_2},\ 0,\ \frac{vA_6}{A_1 A_2 \mu}\right) \tag{2.15}$$

Where, $A_5 = \Lambda(1-\beta)$, $A_6 = A_1 A_5 + \Lambda\beta\theta$

At DFE

$$\left.\begin{aligned} N^0 &= M^0 + S^0 + I^0 + R^0 \\ N^0 &= \frac{A_7}{A_1\mu} \end{aligned}\right\} \tag{2.16}$$

Where, $A_7 = \Lambda\beta\mu + A_6$

2.4 The Basic Reproduction Number R_0.

Basic Reproduction Number, is the largest eigenvalue or spectral radius of FV^{-1} is the basic reproduction number of the model. The approach of (Driessche and Watmough, 2002) will be used to compute the R_0.

$$F = \frac{\alpha\mu A_6}{A_2 A_7} \tag{2.17}$$

$$V = A_3 \tag{2.18}$$

$$V^{-1} = \frac{1}{A_3} \tag{2.19}$$

$$FV^{-1} = \frac{\alpha\mu A_6}{A_2 A_3 A_7} \tag{2.20}$$

Hence,

$$R_0 = \frac{\alpha\mu A_6}{A_2 A_3 A_7} \tag{2.21}$$

2.5 Stability Analysis of the Model

Theorem 2. 1: The DFE point E^0 of the model is Locally Stable if $R_0 < 1$.

Proof: The trace-determinant approach of will be implored to prove the above theorem. The E^0 is locally stable if trace of the Jacobian Matrix is negative and determinant of the same matrix is positive.
Therefore the Jacobian of the model at E^0 is given as

$$J(E^0) = \begin{bmatrix} -A_1 & 0 & 0 & 0 \\ \theta & -A_2 & -A_8 & 0 \\ 0 & 0 & A_8 - A_3 & 0 \\ 0 & v & A_4 & -\mu \end{bmatrix} \tag{2.22}$$

Where, $A_8 = \frac{\alpha\mu A_6}{A_2 A_7}$

$$trJ(E^0) = (-A_1) + (-A_2) + (A_8 - A_3) + (-\mu) \tag{2.23}$$

It is shown from (2.23) that the trace of $J(E^0) < 0$ if $(A_8 - A_3) < 0$
This implies that

$$(A_8 - A_3) < 0 \tag{2.24}$$

Further simplification of (2.24) gives

$$\frac{\alpha\mu A_6}{A_2 A_3 A_7} < 1 \tag{2.25}$$

The Left Hand Side (LHS) of (2.25) is equivalent to (2.21). Hence,

$$R_0 < 1 \tag{2.26}$$

$$\det J(E^0) = \mu(A_1 A_2 A_3 - A_1 A_2 A_8) \tag{2.27}$$

$\det J(E^0) > 0$ implies

$$A_3 > A_8 \tag{2.28}$$

Equation (2.2) also implies

$$A_8 < A_3 \tag{2.29}$$

Further simplification of (2.29) gives

$$R_0 < 1 \tag{2.30}$$

Hence, equation (2.26) and (2.30) proves the Theorem 2.1 that is the DFE is Locally Stable.

Theorem 2: The DFE point E^0 of the model is Globally Stable if $R_0 \leq 1$.

Proof:

Consider the Lyapunov candidate

$$V = A_3 I \tag{2.31}$$

Taking the time derivative of (2.31) we have

$$\frac{dV}{dt} = A_3\left[\frac{\alpha S}{N} - A_3\right]I \tag{2.32}$$

Since $S \leq S^0$ and $N \leq N^0$

$$\frac{dV}{dt} \leq A_3\left[\frac{\alpha S^0}{N^0} - A_3\right]I \tag{2.33}$$

Further simplification of (2.33) gives

$$\frac{dV}{dt} \leq A_3{}^2\left(R_0 - 1\right)I \tag{2.34}$$

Hence, from equation (2.34) $\frac{dV}{dt} \leq 0$ if, $R_0 \leq 1$. We conclude that $V(M, S, I, R)$ is negative definite and this proves that the DFE, E^0 is Global Stability.

2.6 Endemic Equilibrium (EE) Point in Terms of Force of Infection

Let,

$$E^{**} = \left(M^{**}, S^{**}, I^{**}, R^{**}\right) = \left(M, S, I, R\right) \tag{2.35}$$

be the Endemic Equilibrium point

Therefore, equation (2.8) to (2.11) becomes,

$$\left.\begin{aligned} &\Lambda\beta - A_1 M^{**} = 0 \\ &\Lambda(1-\beta) - \lambda^{**} S^{**} + \theta M^{**} - A_2 S^{**} = 0 \\ &\lambda^{**} S^{**} - A_3 I^{**} = 0 \\ &A_4 I^{**} + \nu S^{**} - \mu R^{**} = 0 \end{aligned}\right\} \tag{2.36}$$

Where,

$$\lambda^{**} = \frac{\alpha I^{**}}{N^{**}} \tag{2.37}$$

Solving (2.36) simultaneously gives the Endemic Equilibrium points Infection in terms of force of infection

$$\begin{pmatrix} M^{**} \\ S^{**} \\ I^{**} \\ R^{**} \end{pmatrix} = \begin{pmatrix} \frac{\Lambda\beta}{A_1} \\ \frac{A_6}{A_1\left(\lambda^{**}+A_2\right)} \\ \frac{\lambda^{**}A_6}{A_1A_3\left(\lambda^{**}+A_2\right)} \\ \frac{\lambda^{**}A_4A_6+\nu A_3A_6}{\mu A_1A_3\left(\lambda^{**}+A_2\right)} \end{pmatrix} \quad (2.38)$$

At Endemic Equilibrium

$$\left.\begin{aligned} N^{**} &= M^{**}+S^{**}+I^{**}+R^{**} \\ N^{**} &= \frac{A_9\lambda^{**}+A_2A_3A_7}{\mu A_1A_3\left(\lambda^{**}+A_2\right)} \end{aligned}\right\} \quad (2.39)$$

Where, $A_9 = \left(\Lambda\beta\mu A_3+\mu A_6+A_4A_6\right)$

Substituting I^{**} and N^{**} into (2.37) and simplify gives

$$\lambda^{**} = \frac{A_2A_3A_7}{A_9}\left(R_0-1\right) \quad (2.40)$$

Equation (2.40) is significant in this work because that is what we are going to use to carry out the bifurcation analysis.

3. Results and Discussion

Table 3.1 is the table of estimated values of variables and parameters of the model that are used to carry out the bifurcation analysis, sensitivity analysis and the graphical presentation of sensitive parameters against R_0. The details of the estimation of values are in Appendices.

Table 3.1: Estimated Values of Variables and Parameters of the Model

Variables	Values per year	Source
$M(0)$	4,445,373	B9
$S(0)$	191,497,210	B10
$I(0)$	61,931	B3
$R(0)$	4,959,085	B4
N	200,963,599	B1
Λ	7,534,530	B2
β	0.75	B13
α	0.9	B12
δ	0.005	B6
γ	0.99	B5
τ	0.83	B14
μ	0.12	B7
θ	0.39	B11
ν	0.65	B8

3.1 Bifurcation Analysis

The stability of DFE at $R_0 < 1$ is not sufficient to conclude that the measles can be eradicated of in the population. However, we carried out the bifurcation analysis using equation (2.40) to see whether the model exhibit Backward or Forward bifurcation.

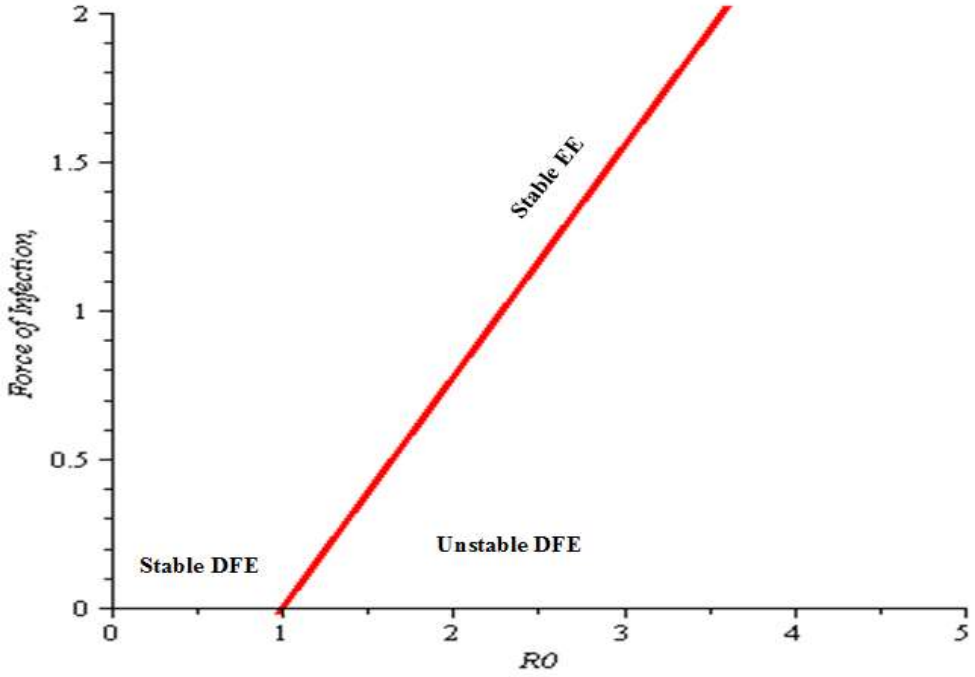

Figure 3.1: Forward Bifurcation Diagram for the Model

In figure 3.1, it was revealed that the stabilities of DFE and EE points depending on the value of R_0. A forward bifurcation in the equilibrium points occur at $R_0 = 1$. The DFE is stable if, $R_0 < 1$ and unstable if $R_0 > 1$. However, $R_0 > 1$, implies the stable EE. Thus, the model exhibit forward bifurcation because the stable endemic equilibrium does not exist when $R_0 < 1$. The force of infection, λ^{**} is an increasing function of R_0.

3.2 Sensitivity Analysis of the R_0 with Some Parameter of the Model

The approach of Abdulrahman *et al.* (2013) will be followed to carry out the sensitivity analysis. The normalized forward sensitivity indices with respect to a parameter values, Q is defined as

$$S_Q^{R_0} = \frac{\partial R_0}{\partial Q} \times \frac{Q}{R_0} \tag{4.1}$$

Where,

$$Q = \{\alpha,\ \gamma, \nu, \theta, \tau\} \tag{4.2}$$

Maple 13 software is used to calculate the sensitivity indices using the values in Table 3.1.

Table 3.2: Sensitivity Indices of the Parameters of the Model

Parameter	Low transmission Sensitivity Index	High transmission Sensitivity Index
α	1.000000000	1.00000000
γ	-0.0861244019	-0.5089974293
v	-0.6756756756	-0.8441558441
θ	0.2507331378	0.1638655463
τ	-0.171003717	-0.4267352185

It is shown in Table 3.2 that the parameters have either positive or negative effects on the R_0. The positive parameters will increase the R_0 while the negative parameters will decrease the R_0. The contact rate α has the highest sensitivity index follow by vaccination rate, v and other parameters follow.

3.3 Graphical Presentation of R_0 and Some Parameters of the Model

Figures 3.2 to 3.6 are the graphical presentations of sensitive parameters and R_0

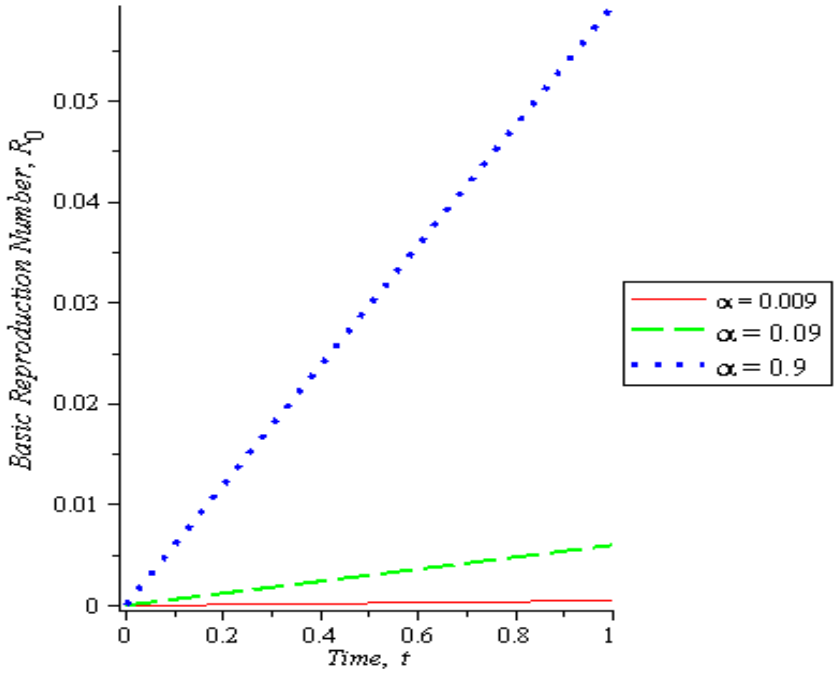

Figure 3.2: The Effect of Contacts Rate Graph on R_0

Figure 3.2 shows that as contact rate increases with time the R_0 increases. This implies that vulnerable people should avoid contact with infected persons. It is observed from figure 3.3 that as recovery rate increases with time the R_0, decreases to almost zero. This shows the vital role the treatment of the

symptoms of measles can play in eradicating it from the population. The more people are treated the less the R_0 and it will result to total eradication from the population.

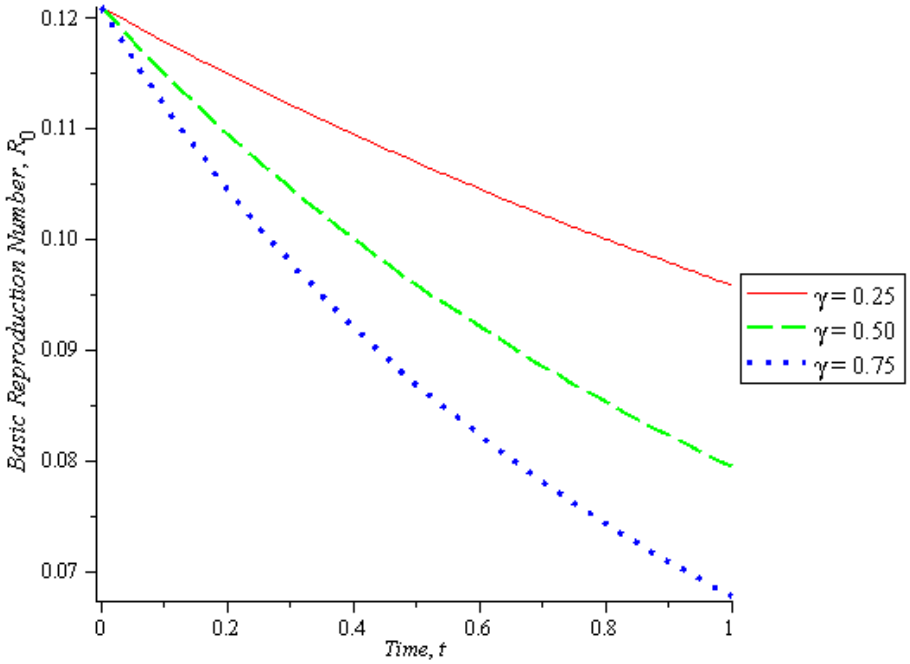

Figure 3.3: The Effect of Recovery Rates on R_0

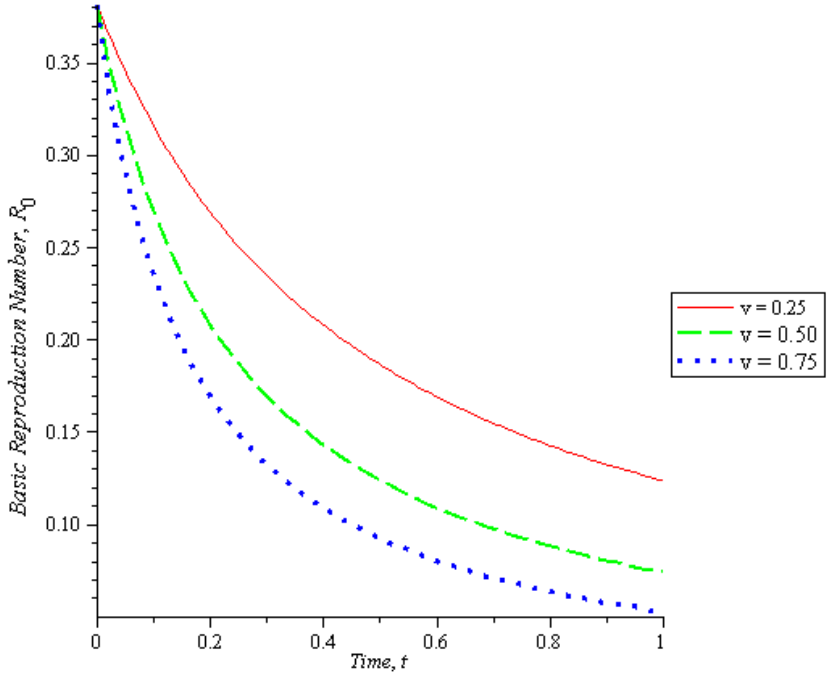

Figure 3.4: The Effect of Vaccination Rates on R_0

In Figure 3.4 it is revealed that as vaccination rate increases with time the R_0 decreases. Vaccination of children between nine months to one year of age is crucial to eradicate measles. It is shown from Figure

3.5 that as loss of immunity increase with time the R_0, increases. Strong immunity is needed in the fight to eradicate measles. In Figure 3.6 as the Vitamin A supplement rate increases with time the basic reproduction number R_0, decreases. Vitamin A supplement is recommended to boost the immune system of the children infected with measles.

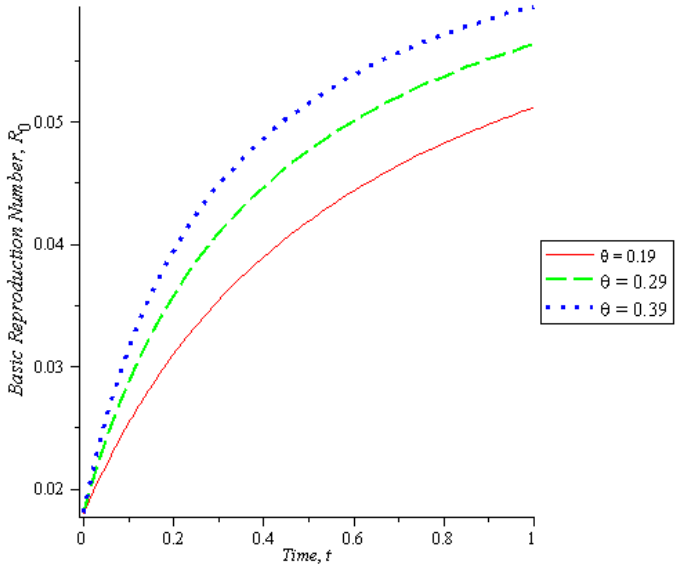

Figure 3.5: The Effect of Loss of Immunity Rate on R_0

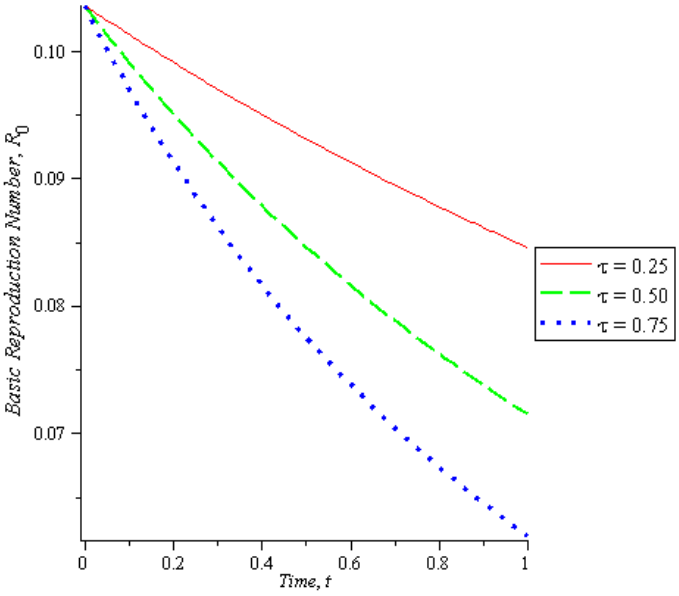

Figure 3.6: The Effect of Vitamin A Supplement Rates on R_0

4. Conclusion

In this paper, we obtained the DFE and EE in terms of force of infection and analyzed the Local and Global stabilities of DFE. Trace -determinant method was used to analyze the local stability and

Lyapunov function to analyzed the global stability. The DFE is locally and globally stable. Measles will be put under control if $R_0 < 1$. Bifurcation analysis reveals that the model exhibit forward bifurcation (i.e. measles can be eradicated when $R_0 < 1$). It is evident from the sensitivity analysis that the contact rate α, is the most sensitive parameter to increase the R_0 and vaccination rate v is the most sensitive parameter to reduce the R_0. Graphical presentation shows that, vaccination rate and recovery rate are important parameters in eradicating the measles from the population. It was also discovered from Figure 3.2 that increase in contact with people infected with measles increases the basic reproduction number. Hence, the infected people should be separated from the susceptible individuals in order to reduce the spread of epidemics. It is also recommended that government should do more in routine immunization of measles and administration of Vitamin A supplement to the infected people.

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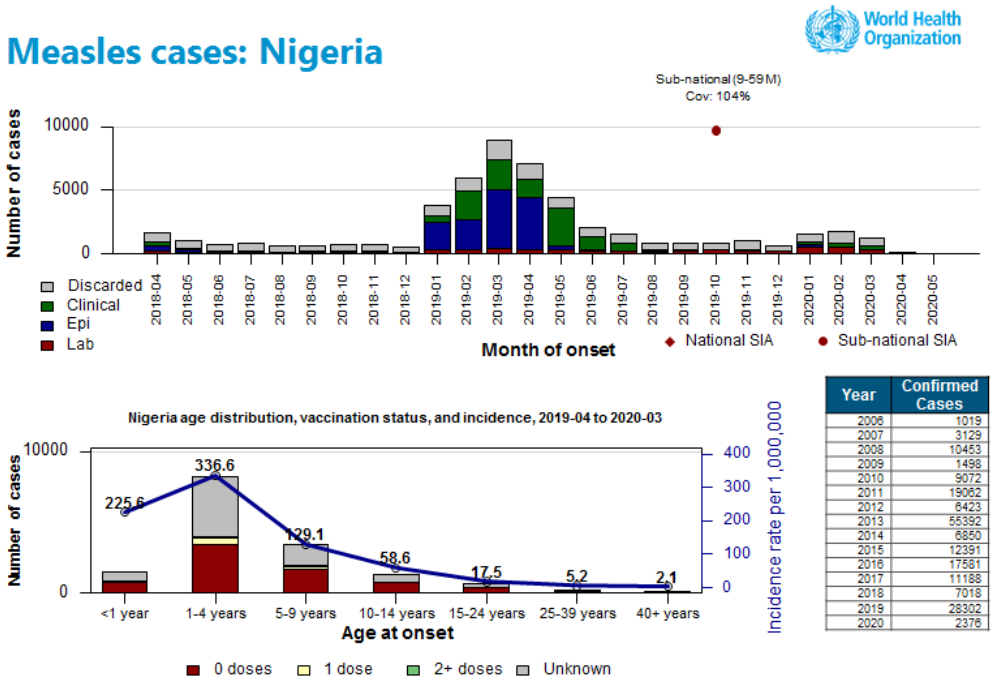
APPENDICES

Appendix A: Reported Measles Cases in Nigeria by WHO Region 2006 to 2020 as of April, 2020

Table A1a: Reported Measles Cases in Nigeria by WHO Region 2006 to 2020 as of April, 2020



Year	Confirmed Cases
2006	1019
2007	3129
2008	10453
2009	1498
2010	9072
2011	19062
2012	6423
2013	55392
2014	6850
2015	12391
2016	17581
2017	11188
2018	7018
2019	28302
2020	2376

Table A1b: Reported Measles Cases in Nigeria by WHO Region 2006 to 2020 as of April, 2020

Year	Confirmed Cases
2006	1019
2007	3129
2008	10453
2009	1498
2010	9072
2011	19062
2012	6423
2013	55392
2014	6850
2015	12391
2016	17581
2017	11188
2018	7018
2019	28302
2020	2376

Source: WHO (2020)

Table A2: Distribution of Measles Cases in Nigeria, Year and Month

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	TOTAL
2011	4514	3970	4499	2518	1054	610	431	434	232	250	341	209	19062
2012	374	845	1186	796	355	220	137	190	229	189	807	1095	6423
2013	6604	12228	17785	9417	4385	1502	966	685	543	551	507	219	55392
2014	1557	1881	1285	685	449	287	68	80	55	115	243	145	6850
2015	641	1558	1927	1686	1390	932	691	599	664	969	924	410	12391
2016	2638	3947	3052	1823	1547	972	699	976	781	581	316	249	17581
2017	1142	1142	1913	1548	1231	1041	696	805	696	578	242	154	11188
2018	1223	1457	1699	906	425	238	228	155	183	190	166	148	7018
2019	3004	4935	7344	5808	3570	1325	825	318	305	340	292	236	28302
2020	927	779	607	63	0								2376

Source: WHO (2020)

Table 3: Summary of Reported Suspected Measles Cases by State in 2019

States	December	Deaths in December	Week 01 – 52, 2019	Total Deaths week 1-52 (CFR%)
Abia	17	0	809	0
Adamawa	79	0	1,864	46 (2.5%)
Akwa Ibom	8	0	429	0
Anambra	48	0	861	2 (0.2%)
Bauchi	24	0	1,244	0
Bayelsa	7	0	368	0
Benue	0	0	325	0
Borno	118	0	22,234	118 (0.5%)
Cross River	23	0	432	0
Delta	29	0	682	0
Ebonyi	8	0	346	0
Edo	4	0	409	0
Ekiti	28	0	911	0
Enugu	22	0	546	2 (0.4%)
FCT	0	0	216	0
Gombe	17	0	493	4 (0.8%)
Imo	11	0	712	0
Jigawa	0	0	1,160	0
Kaduna	75	0	1,829	14 (0.8%)
Kano	3	0	4,141	27 (0.7%)
Katsina	252	0	9,353	53 (0.6%)
Kebbi	64	3	801	4 (0.5%)
Kogi	12	0	277	0
Kwara	2	0	228	1 (0.5%)
Lagos	0	0	763	0
Nasarawa	5	0	279	0
Niger	5	0	223	0
Ogun	22	0	736	0
Ondo	9	0	467	0
Osun	37	0	866	0
Oyo	5	0	1,062	0
Plateau	3	0	481	2 (0.4%)
Rivers	20	0	436	0
Sokoto	91	0	1,503	2 (0.1%)
Taraba	11	0	143	0
Yobe	47	0	3,897	16 (0.4%)
Zamfara	3	0	405	0
Total Cases	**1,109**	**3**	**61,931**	**291 (0.5%)**

Source: (NCDC)

Appendix B: Estimation of Variables and Parameter Values

The values of variables and parameters were estimated based on the available data from the World Health Organization (WHO), Population Reference Bureau and reliable related literature.

B1: The Total Population, N

According to Worldometer, the Nigeria total population for 2019, is 200,963,599.

$$N = 200{,}963{,}599$$

B2: Recruitment Number, Λ

According to Knoema the number of new birth in Nigeria in 2019 is 7,534,530.

Therefore,

$$\Lambda = 7{,}534{,}530$$

B3: Number of Infected, I

The Nigeria Centre for Disease Control (NCDC) estimate that, there are 61,931 reported cases of measles in Nigeria in 2019, resulting in 291 deaths. (See Table A2)

$$I = 61{,}931$$

B4: Number of Recovered/Immune, R

R = recovered + immune
From B3 the number of cases is 61,931 and number of death is 291.
Recovered= 61,931 - 291 = 61,640
According to UNICEF Data: Monitoring the situation of children and women

the number of infants in Nigeria in 2019 is 7,534,530 and the percentage of vaccination is 65%. Therefore,
Vaccinated = 65% of 7,534,530 = 4,897,445.
Hence,
Recovered/Immune Human population, $R = 61{,}640 + 4{,}897{,}445$

$$R = 4{,}959{,}085$$

B5: Recovery Rate, γ

From B3 and B4

$$\gamma = \frac{\text{Recovered}}{\text{Number of cases}}$$

$$\gamma = \frac{61{,}640}{61{,}931} = 0.99$$

B6: Disease Induce death rate, δ

From B3 the number of cases of measles is 61,931 and the number of death from measles is 291

$$\delta = \frac{\text{Number of Death from measles}}{\text{Number of cases}}$$

$$\delta = \frac{291}{61{,}931} = 0.005$$

B7: Natural Death Rate, μ

According to UNICEF, the death rate is 120 deaths per 1,000 live births. Therefore,

$$\mu = \frac{120}{1000} = 0.12$$

B8: Vaccination rate, v

According to, WHO in 2019, about 65% of the Nigeria's children received one dose of measles vaccine. Therefore,

$$v = 0.65$$

B9: Maternally-Derived-Immunity, M

According to Millennium Development Goal (MDG4), close to 41% of deaths among newborn infants, occurred in their first 28 days of life or the neonatal period.

$M = 59\%$ of $7{,}534{,}530$

$M = 4{,}445{,}373$

B10: Number of Susceptible, S

Recall $N = M + S + I + R$ therefore,

$$S = N - (M + I + R)$$

$$S = 200,963,599 - (4,445,373 + 61,931 + 4,959,085)$$

$$S = 191,497,210$$

B11: Loss of immunity, θ

According to WHO Immunization coverage fact sheet, national immunization schedule reported that, only 65% of children received 1 doses of measles. Therefore,

$$\theta = 35\% = 0.35$$

B12: Contact Rate, α

$$\alpha = \frac{9}{10} = 0.9$$

B13: Birth with Temporary Passive immunity

$$\beta = 0.75$$

B14: Vitamin A, Supplementation Coverage Rate:
According to Trading Economics the Vitamin A, supplementation coverage in 2018 is 83%
$$\tau = 0.83$$