UNILORIN SCIENCE AND ENGINEERING PERIODICALS
(USEP)

1-183 ISSN 0189-8787

Journal of
Research Information in Civil Engineering
(RICE)

Published
By

Department of Civil Engineering
University of Ilorin
Ilorin, Nigeria

Vol.3. No.1, 2006

OPTIMISING THE DESIGN OF CONCRETE MIXES USING DOE METHOD FOR UNCRUSHED AGGREGATES

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Abstract

This paper discusses the numerical approach using linear programming technique for selecting proportions of various aggregates required to produce the desired concrete to meet a specification requirement. Firstly, the mix designs to produce normal grade concretes was carried out and subsequently used to obtain the proportions of constituent materials of fine and coarse aggregates. Secondly, constraints of availability, cost and specification requirements were built into the minimization procedure and the results were compared with the mix design proportions produced. The outcome was found to be reliable. The specification requirements considered here are the strength and durability requirements. The minimization procedure yields higher proportion coarse aggregate with 4.75 – 10mm grading for 20mm maximum aggregates. The application is facilitated by developing a computer solution procedure.

Keywords

Mix design, desired concrete, specification requirement, uncrushed aggregates, grading

1. Introduction

Concrete is heterogeneous in nature and the proportions of the constituent materials in a particular mix will determine the properties of the hardened concrete produced (Neville, 1993 and Tokuda et al, 1978). However, other properties of concrete in the fresh states such as workability and stability are also important as they have effects on the hardened concretes. In designing concrete mixes, strength seems to be a major criterion. However concrete strength is variable. The factors affecting concrete strength include variation in the quality of the material, variation in mix proportions and variation due to sampling and testing (Teychenne et al, 1975). Concrete strength is known to follow normal distribution such that some proportion of the results may fall below specified values and a measure of variability called standard deviation is usually applied. This is in recognition of production tolerances since there is always a deviation from the centre specification (Beal, 1981).

Various concrete mix design manuals specify their grading requirements as either homogeneous sizes or heterogeneous sizes. Some of the mix design manuals like the American Concrete Institute (ACI 211.1, 1972) method do not include uncrushed aggregates in their standards. However, the mix design method of the Department of the Environment (DOE) has accommodated mix designs using uncrushed aggregates. In this DOE mix proportioning method, the starting design estimates recommended for uncrushed aggregates are not compatible with some of the gravel deposits in this country even with the revision of the initial design, which still further produces unacceptable results (Alao, 2001). Generally, aggregate supply can be pre-graded to nominal sieve sizes or can be obtained in heterogeneous sizes consisting of various proportions of finer grades. Here, the best combination of coarse and fine aggregate contents to meet specification requirements is investigated.

2. Mathematical formulation of the problem

The task here is to produce aggregate sizes to a specification requirement given a supply of various raw aggregates. The knowledge of what percentage of the raw aggregates will pass through sieves of various aperture sizes i.e. grading requirements is required. The procedure presented here as suggested by Lewis (1983) in his technical note is based on linear programming technique for selecting proportions of various aggregates required to produce the desired concrete mix to meet the centre specification chosen. The desired mix is based on the DOE method of selecting aggregate proportions. The method involved the calculation of an optimum combination of raw aggregates of two or more nominal sizes to either meet the centre specification or to get as close as possible to it. The centre specification that is assumed here is the desired mix, therefore the problem is treated as that of trying to achieve a distribution on percentage of nominal sizes in the final mix.

The following notations were used:

Let
- A_{ij} proportion of aggregate *i* with grade size *j*
- B_j percentage of grade size *j* required in final mix
- x_i percentage of raw aggregate *i* in final mix
- P notional cost of deviation from constraint.

If there are m aggregates and n grades, then the problem can be formulated by the following $n+1$ equations:

$$\sum_{i=1}^{m} x_i A_{ij} = B_j \qquad j = 1,2,3 \ldots n \qquad (1)$$

and

$$\sum_{i=1}^{m} x_i = 100 \qquad (2)$$

with the provision that

$$x_i \geq 0 \qquad (3)$$

Equation 1 ensures that there are correct proportions of each grade in the final mix. Equation 2 ensures that the percentages add up to 100 and constraint 3 prevents negative quantities. Equations 1 – 3, if not modified will yield a trivial solution and in a practical case, there is no solution because equation 1 insists that the centre specification must be met (because of the equality sign). In reality, a small deviation is expected from the centre specification and in this situation, a re-formulation of the problem that will lead to a solution can be written as:

$$\sum_{i=1}^{m} x_i A_{ij} \approx B_j \qquad j = 1,2,3 \ldots n \qquad (4)$$

while Equation 2 and constraint 3 still hold and the solution procedure can proceed to find the 'best' one.

2.1 Solution to the Problem Formulation

The modified problem formulation given in Equations 2 – 4 has the characteristics of the constraint of a linear programming problem (Lewis, 1983). By choosing a suitable objective function of the type.

$$\min U = \sum_{i=1}^{m} x_i \qquad (5)$$

the simplex algorithm can be used to obtain the optimum solution and that is to minimize Equation 5 subject to Equations 2 – 4. To re-write the constraints in a standard linear programming problem, two dummy variables need to be added, which are the surplus, and the artificial variables respectively to the constraint equations. This addition of the dummy variables should be carried out per equation in order to convert the problem to a standard linear programming problem. Thus, equation 4 will be modified to become:

$$\sum_{i=1}^{m} x_i A_{ij} - y_{1j} + y_{2j} = Bj \quad j = 1,2,..n \tag{6}$$

with $y_{1j}, y_{2j} \geq 0$

where y_{1j} and y2j are surplus and artificial variables respectively.

Also, to discourage the dummy variables from appearing in the final solution, they can be written into the objective function with a small notional cost of deviation as:

$$\min V = \sum_{i=1}^{m} x_i + P \sum_{k=1}^{2} \sum_{j=1}^{n} y_{kj} \tag{7}$$

This revised expression of the objective function is the Alternate Simplex Method of solution procedure or otherwise called the Big – M method, It uses a multiplier P, which is problem dependent. A starting approximation of P can be 10. The objective function after re-formulation can now be re-written to minimize Equation 7 subject to Equations 2 and 6 and

$$x_i \ , \quad y_{lj} \ , \quad y_{2j} \geq 0 \tag{8}$$

2.2 Further Refinements

If C_i is the cost per unit quantity of aggregate *i*, then Equation 7 can be adjusted as:

$$\min \ V = \sum_{i=1}^{m} C_i x_i + P \sum_{k=1}^{2} \sum_{j=1}^{n} y_{kj} \tag{9}$$

Also, extra restrictions in the specification can be handled by the addition of extra constraint equations. For example, the specification requirement that the proportion of one particular raw aggregate should not fall below or should fall below a certain percentage can be added respectively as:

$$x_p \geq B \tag{10a}$$

and

$$x_q \leq B \tag{10b}$$

Equations 10 (a) and (b) should be converted to a standard linear programming problem by the addition of a 'slack' or 'surplus' variable. In this case, it's dummy variable will not appear in the objective function (Arora, 1989).

Also if practical considerations dictate that some of the raw aggregate materials can only be mixed in simple proportions, then constraints of the following form can be added e.g.:

$$x_p - x_q = 0 \tag{11a}$$

$$x_p - 2x_q = 0 \tag{11b}$$

$$2x_p - 3x_q = 0 \tag{11c}$$

Equation 11(a) implies that two particular aggregate sizes should be in the same proportion. Equation 11(b) insists that two particular aggregates sizes must be in ratio 2:1 and Equation 11(c) insists that two particular aggregates sizes must be in ratio 3:2. Alternatively, if the above problem formulation is to be solved using the 'two-phase' simplex method, then the algorithm has to be re-defined. This can be done by defining an auxiliary function to be known as the artificial cost function. This artificial cost function is the sum of all the artificial variables 'w' defined as:

$$w = \sum_{j=1}^{n} y_{2j} \tag{12}$$

and the problem can be re-written as:

$$w = \sum_{j=1}^{n} Bj - \sum_{i=1}^{m} \sum_{j=1}^{n} x_i A_{ij} \tag{13}$$

which is the cost function written in terms of non basic variables. This can be re-written in a standard form as:

$$\min w - \sum_{j=1}^{n} Bj = -\sum_{i=1}^{m}\sum_{j=1}^{n} x_i A_{ij} \qquad (14)$$

The solution procedure can proceed as minimizing Equation 14 subject to constraints of equations (6), (5) and (2). However, the Alternate Simplex solution technique converges faster than the two-phase simplex algorithm (Constantinides, 1983).

3.0 Materials and Methods

The uncrushed coarse aggregate used has been found to satisfy specification requirements based on the physical properties obtained (Alao, 2001). The mix design for normal grade 20, 25 and 30 concrete mixes with slumps 10 – 30mm and 30 – 60mm were carried out. Maximum coarse aggregate size of 20 was used. The mix design method presented by DOE was used to obtain constituent proportions of water, cement, fine and coarse aggregate materials. Several grading requirements considered include; continuous grading, single size grading, gap grading (continuous) and gap grading (single) size. This obviously, is to reveal coarse aggregate proportions that would yield specification requirements. The percentage combinations of aggregates within the grading limits suggested by (Neville, 1993) and (Teychenne et al, 1975) were used.

4. Results and Discussion

The summary of the mix design is presented in Table 1. The results of the compressive strength tests at 7 and 28 days are also presented in Table 2. The compressive strength test results recorded are averages of three test results.

The summary of concrete cube strength results recorded before applying minimization procedure has revealed that strength depends on the percentage proportions of coarse aggregate materials. Also, all grading requirements have failed to show consistency in strength gains above grade 20 concrete.

Table 1 : Summary of the Mix Design for 20mm Maximum Aggregate Size.

Parameters	Grade 20		Grade 25		Grade 30	
Characteristic Strength at 28-Days $f_{cu,28}$ (N/mm²)	20		25		30	
Target Mean at 7-Days $f_{t,7}$ (N/mm²)	20		24		29	
Target Mean Strength at 28-Day $f_{t,28}$ (N/mm²)	33		38		43	
Expected Slump (mm)	10-30	30-60	10-30	30-60	10-30	30-60
Water –Cement Ratio	0.55	0.55	0.53	0.53	0.48	0.48
Cement (kg)	291	327	301	340	333	427
Water (Litres)	160	180	160	180	160	180
Fine Aggregates (kg)	576	590	613	587	533	558
Coarse Aggregates (kg)	1410	1313	1363	1306	1404	1300

Table 2: Cube strength in N/mm² for different grading types using 20mm maximum aggregate size.

Grading Type	Concrete Grade					
	20		25		30	
	7 day	28 day	7 day	28 day	7 day	28 day
Continous, CG	17.80	22.67	18.22	24.45	20.00	26.96
Single Size, SS	18.22	20.89	18.67	23.85	18.67	25.93
Natural Deposit, ND	22.37	15.11	23.55	15.78	26.22	19.11
Gap Continous, CG	20.44	21.78	21.78	21.78	29.78	29.78
Gap Single Size, GS	23.56	19.56	23.56	20.44	25.78	25.33

The underlined quantities and the values not underlined are strength development at 7 and 28 days respectively. CG, SS, ND, GC and GS = Continuous grading, Single size grading, Natural Deposit grading, Gap grading (continuous) and Gap grading (single size) respectively.

Only natural deposit grading and continuous grading having higher proportions of coarse aggregates within 4.75mm – 10mm grading limits has shown a better improvement in strength. The failure mode exhibited by single size and gap grading which produced low strengths are not visible in natural and continuous grading. The foregoing facts have enabled the building – in of

In constructing the constraint equations, it is more convenient to relate the proportion of an aggregate i with grade size j to the aggregate i with the maximum grade size j. Thus for example 1, using natural deposit grading:

$$1.295\ A_{11} = A_{13} \quad (15)$$
$$1.124\ A_{12} = A_{13} \quad (16)$$

and Bj, the percentage of grade size j required in the final mix is:

$$1.295\ (x_1\%) + 1.124\ (x_2\%) + x_3\% = 112.71\% \quad (17)$$

The constraint equations for obtaining percentage proportions of aggregate i from Table 1 can be written out as:

$$1.295x_1 + 1.124x_2 + x_3 - y_1 + y_2 = 112.71 \quad (18)$$

$$1.640x_1 + 2.030x_2 + x_3 = 142.71 \quad (19)$$

$$x_1 - y_5 + y_6 = 26 \quad (20)$$

$$x_1 + x_2 + x_3 + y_7 = 100 \quad (21)$$

and the objective function is:

$$-50.359x_1 - 42.543x_2 - 31x_3 + 10y_1 + 10y_5 = f - 3814.29 \quad (22)$$

The additional constraint of Equation 10 is on minimum percentage of fine aggregate in order to avoid a harsh mix. The solution yields:

x_1 = 26%
x_2 = 40.57%
x_3 = 33.43%

This shows that 26% of the total aggregate content is to be used as sand while 40.57% and 33.43% of the total aggregate are to be used as coarse aggregate within 4.75 – 10mm and 11 – 20 mm respectively. The problem converges in five iterations.

5.2 Example 2

If only one single size or single grading is to be used, then the solution becomes trivial. The constraint is written out as:

$$2.448x_1 + x_2 - y_1 + y_2 = 142 \quad (23)$$
$$x_1 + x_2 + y_3 = 100 \quad (24)$$

and the objective function is

$$-35.48x_1 - 21x_2 + 10y_1 = f - 2420 \quad (25)$$

the solution is:

(fine) $x_1 = 29\%$

(coarse) $x_2 = 71\%$

The best combination is the problem, hence, a trivial solution. The problem converges in only two iterations.

5.3 Example 3

Also, a decision to vary proportions of fine aggregates within the limits permitted in the concrete design manual will generally indicate the use of upper limits of fines. This means that the mortar matrix will additionally be used to carry the compressive stresses and in this case of the aggregates materials under consideration, it may be desirable because of aggregate breakages exhibited.

The constraints can be written out as:

$$\left.\begin{aligned} 2.849x_1 + x_2 - y_1 + y_2 &= 148 \\ 2.705x_1 + x_2 - y_3 + y_4 &= 146 \\ 2.572x_1 + x_2 - y_5 + y_6 &= 144 \\ 2.448x_1 + x_2 - y_7 + y_8 &= 142 \\ 2.332x_1 + x_2 - y_9 + y_{10} &= 140 \\ 2.224x_1 + x_2 - y_{11} + y_{12} &= 138 \\ x_1 + x_2 + y_{13} &= 100 \end{aligned}\right\} \quad \text{(xii)}$$

and the objective function is

$$-162.3x_1 - 71x_2 + 10y_1 + 10y_3 + 10y_5 + 10y_7 + 10y_9 + 10y_{11} = f - 9580 \quad \text{(xiii)}$$

The solution is:

(fine) $x_1 = 31\%$

(coarse) x_2 = 69%
f = 100%

the problem converges in ten iterations.

6. Conclusion

From the foregoing results, the following conclusions can be made:

(i) The minimization procedure yields higher proportion coarse aggregate with 4.75 – 10mm aggregate grading for 20mm maximum aggregates.

(ii) The method can conveniently handle constraints of costs, availability and strengths.

(iii) The proposition that when single size or gap graded aggregates are used, that they bear upon one another thus producing higher strength concrete do not hold here. This is because this uncrushed aggregates are weak. The minimization procedure has produced the use of upper limits on sand which would obviously lead to increase in strength.

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