

# A Central Composite Design Method for Design of Mixture Proportion for a Laterite-cement Bricks

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## Abstract

An attempt is made in this paper to present a guideline to produce laterite cement bricks meeting a user-defined requirement. Using the Central Composite Design (CCD) of the Response Surface Methodology of experimental design, a response prediction for a three component mixture for building bricks production using water, cement and laterite with percentage sand replacement was carried out. Five blends of bricks were investigated with cement content ranging between 8-20 percent by weight of laterite and 0-20 percent sand replacements. The machine mixing, compaction using Hydraform Twin-M7 machine and curing were carried out in a controlled laboratory environment. At the specified ages of 7 and 28 days, the compressive strength of bricks was measured and responses were modeled as a second-order quadratic model. Guidelines for the development of constraint formulation for mixture proportioning and optimization formulation were carried out. An inverse relationship for response prediction for strength was obtained and compressive strength achievable ranges between 7.46 - 18.85N/mm<sup>2</sup>. Two analytical techniques, using the Genetic Algorithm (GA) stochastic search technique, and an analytical method were presented with examples and were found adaptable computationally, to obtain response prediction, satisfying the user-defined constraints of strength, cost and durability.

Keywords: user-defined, central composite design, bricks, strength

## 1. Introduction

### 1.1 Laterite and laterite-cement bricks

The need to produce laterite cement bricks satisfying user-defined requirements demands a higher complexity of the mixture design. It is quite impossible to achieve this goal without introducing a number of imposed criteria that the mixture must satisfy. To achieve this higher performance of laterite bricks, the traditional method of using trial mixes would be incapable and there is perhaps a need to employ useful numerical and optimization tools to aid the process. These performance criteria could include mechanical properties such as strength, young modulus of elasticity, creep and shrinkage.

Laterite, according to Gidigas (1976) is described as a light to dark homogeneous, vesicular, unstratified and clinker like soil material consisting mainly of oxides and hydroxides of aluminium, iron, manganese and silica which hardens on extraction and exposure. It is described as a class of pedogenics where the cementing materials are the sesquioxides content and should normally constitute not less than 50 percent of the mineralogical composition. This definition describes the material in its natural form. However, for building construction purposes, cement is usually added to improve brick properties.

Laterite brick confers technical advantage because of the primary characteristic strength requirement which is often three(3) times higher than the minimum strength requirements for the conventional commercial sandcrete building blocks in Nigeria (Alao and Jimoh, 2017). Laterite bricks have a very good thermal property, shock and earthquake resistance (Hydraform, 2014) and particularly its impact resistance.

Other research outputs (Osunade and Fajobi, 2000; Madu, 1984; Gidigas, 1976; Awoyera and Akinwumi, 2014; GIZ et al, 2013) have tried to confirm the acceptability of its properties for a series of acceptance criteria. Among these properties include strength, absorption characteristics, resistance to abrasion and reduction in the number of structural frames required in a building up to two-storey high

Various attempts have also been made to improve laterite cement material as a building material for sustainable housing construction. These include development and manufacturing of compression moulding machines for mechanical stabilization of bricks (Hydraform, 2014; Adeyemi, 1987; Cinva Ram, 1999). Stabilization of laterite soil with cement otherwise called soil-cement mixture was also investigated (Hydraform, 2014; Madu, 1984; Aguwa, 2009; Osunade and Fajobi, 2000). Stabilization with pozzolanic

material such as Corn Cob Ash (Ogunbode and Apeh, 2012). Stabilization with Locust Waste Bean Ash (Osinubi and Oyelakin, 2013), Stabilization with Coir (Aguwa, 2013). Bentonite Treatment (Amadi et al, 2011), Stabilisation with lime (Singh, 2006). Other pozzolana treatments include Rice Husk Ash, Pumice Slag Ash, Burnt Clay Brick Ash, Sugarcane bagasse Ash.

The aim of this study is to investigate an efficient optimization formulation for selection of component proportions of cement, laterite and sand for composite lateritic bricks production to meet user-defined requirements. Central Composite Design method of the Response Surface Methodology was used to model response prediction for laterites-cement bricks blended with silica sand. Compression moulding machine exerting a compactive effort of 10MN/m<sup>2</sup> was employed in this study and blending of laterite-cement mixture with river sand within grading zones 2 and 3 was considered.

## 2. Literature review

### 2.1 Statistical experimental design methodology for the laterite cement mixture

Although statistical experimental design procedure may seem rigorous, it is desired especially where user defined requirements are desired. In using this method, established experimental design procedure for selecting design points would have to be followed. It generally employs the fitting of empirical models for each of the measured responses after removing insignificant terms in the model. The resulting response equations where insignificant terms are eliminated now form the basis for optimization subject to imposed constraints using any numerical or Genetic Algorithm techniques.

An advantage of this type of statistical experimental design procedure is that the responses can be characterized by an uncertainty (variability) which has an important implication for specification writing especially in site production (FHWA, 1999; Montgomery, 2001). These responses are always targeted at yielding a target or mean strength which implies that at least 95 percent of the results are expected to fall within the normal distribution curve. Or more precisely, probability  $p \leq 0.05$

### 2.2 The Central Composite Design (CCD)

This is one of statistical experimental techniques commonly used in mixture proportioning particularly to develop, improve and optimize the constituent proportions. The Central Composite Design can be used and can be run in 2-level factorial design without needing to use a complete three-level full factorial experiment (Montgomery, 2001; Lundstedt; 1998; Simon et al, 1999). These factorial designs are essentially employed for fitting response surfaces. A CCD therefore specifies  $2^n + 2n + 1$  design points for a full quadratic model with  $n$  factors. To optimize the component mixture, each response property will be optimized using the response surface method to obtain a second order quadratic model of the form (Montgomery, 2001):

$$y = \beta_0 + \sum_i^k \beta_i x_i + \sum_{i < j} \sum \beta_{ij} x_i x_j + \sum_i^k \beta_{ii} x_i^2 \quad (1)$$

where “ $y$ ” is the response representing the property of the mixture. The values  $x_i$  are the components and the parameters  $\beta_i$  and  $\beta_{ij}$  are calculated as the linear and quadratic coefficients fitting the experimental data for the linear and interactive terms respectively.

### 2.3 Design of Experiments in Box-like Domains, Coding and Normalization Procedure

Defining the region of experimental can be designed by simple lower and upper limits on the design variables which can be achieved by imposing design constraints of the type:  $x_{il} \leq x_i' \leq x_{iu}$   $i = 1, \dots, n$  where  $x_{il}$  and  $x_{iu}$  represent the lower and the upper bound limits and  $x_i'$  with the prime indicating that the variables have not been normalized. The variable  $x_i$  can now be normalized (Lundstedt et al, 1998; Simon et al, 1999) as:

$$x_i = \frac{2x_i' - x_{il} - x_{iu}}{x_{iu} - x_{il}} \quad (2)$$

and the normalized variable  $x_i$  can now be bonded within the cube as:  $-1 \leq x_i \leq 1$

Similarly, these dimensionless coded variables can also be translated to coded variables using the expression:

$$x_{actual} = x_{min} + \frac{(x_{coded} + 1)}{2} * (x_{max} - x_{min}) \quad (3)$$

where  $x_{actual}$  is the uncoded value and  $x_{min}$  and  $x_{max}$  are the uncoded lower and upper values corresponding to  $\pm 1$  coded values and  $x_{coded}$  is the coded value to be translated.

### 3. Methodology

#### 3.1 Materials and method

The laterite sample was obtained within Ilorin environs, Kwara State, Nigeria (KW-31, Elevation 317, and Coordinates 663093, 935109). The laterite sample was obtained from an existing burrow pit. The method of disturbed sampling was used at a depth 0.5m – 1.5m depth for the collection. Two grading zones of sand, namely zones 2 and 3 sand otherwise called coarse (C) and fine (F) sands were used. The sand samples for Coarse and Fine sands were collected from the Stream beds of Egbejila and Tepatan respectively all within Ilorin environs, both tributaries of Asa River. Mixing water was obtained from public mains and Ordinary Portland Cement conforming to BS 12 was obtained from a cement depot.

The physical properties and geotechnical tests of the component mixes of sand and laterite using laboratory investigation were carried out to confirm the suitability of the deposits in accordance with BS 1377(1990). Additional tests also include the mineralogical tests using the “Energy Dispersive X-Ray Fluorescence Method”. This is shown in Table 1. Soil classification test was carried out in accordance with American Association of State Highways and Transport Officials, AASHTO.

Table 1: Properties of the laterite Sample measured

Physical and Geotechnical Properties		Value
i)	Liquid limit (%)	49
ii)	Plastic limit (%)	30.6
iii)	Plasticity Index (%)	18.4
iv)	Specific gravity	2.64
v)	Linear Shrinkage (mm)	10.1
vi)	Maximum Dry Density (kg/m <sup>3</sup> )	1821
vii)	Optimum Moisture Content (%)	14.1
viii)	Colour	Reddish Brown
ix)	Condition of Sample	Air Dry
x)	Soil Classification	A-2-7
<b>Mineralogical Properties</b>		
i)	Iron Oxide Content (Fe <sub>2</sub> O <sub>3</sub> ) (%)	18.01
ii)	Sesquioxide Content (%)	42.21

Specimen preparation was carried out by batching, mixing and casting of specimen samples. Using 0% laterite-cement mixture as a control, two percentage sand replacements with proportion (0%, 10% and 20%) silica sand was carried out. Initially, a starting set of mixtures was designed using the absolute volume method within a domain of 8-percent and 20-percent cement content at a water cement ratio of 0.5 and this starting mixing water was later revised to produce a mix that would produce one cubic meter of the maximum dry density mixture. Machine mixing was used and compaction using hydraulically compressed M7-Twin Hydraform brick moulding machine. Once the batch constituent proportions are mixed in the mixer, the moulding was immediately carried out without delay (Adedokun and Dandela, 2011) before hydration of cement commences. ASTM C 170-90 test plan was used. The prepared specimen samples were cured and tested at 7 and 28 days to obtain compressive strengths and other mechanical properties using a Testometric Universal Testing Machine Model FS300CT.

**3.2 Mix proportioning process**

The development and proper formulation of requirements and imposed conditions can yield a more rational approach to mix proportioning process. It is however improper to search for an optimal solution without a clear declaration of a feasible region within which a solution can be found. Constraint equations are used to construct these ranges of this feasible region.

**3.3 Methodology for estimation of constituent proportions within the design domain**

The expression of the absolute volume of the mixture is expressed (Neville, 1990; Aguwa, 2009) as:

$$\frac{water}{G_{s_{water}} \times 1000} + \frac{cement}{G_{s_{cement}} \times 1000} + \frac{laterite}{G_{s_{laterite}} \times 1000} = 1 \tag{4}$$

where  $G_s$  = specific gravity and subsequently, each of the proportions of the variables between 8 and 20 percent of cement by weight of laterite within the domain can be estimated. For the lower limit of cement content of 8% of the dry weight of laterite, the mix ratio can be expressed as 1:12.5. Here, a starting water/cement ratio adopted was 0.5, which represents an assumed starting mixing water to produce  $1m^3$  of maximum dry weight/density of the laterite cement mixture.

- i) The ratio 1:12.5 represents one part of cement and twelve and a half parts of laterite and water represents 0.5 by weight of cement. This represents a water:cement:laterite ratio 0.5:1:12.5 water:cement:laterite. The laterite content can then be expressed as Laterite,  $L=12.5*C$ . Subsequently, the water content based on the adopted starting water/cement ratio can similarly be expressed as Water,  $W=0.5*C$
- ii) The equation which satisfies the constraint condition of equation (4) can therefore be re-written as:

$$\frac{0.5C}{1.0 \times 1000} + \frac{C}{3.15 \times 1000} + \frac{12.5C}{2.64 \times 1000} = 1$$

where = specific gravity of 1.0, 3.15 and 2.64 for water, cement and laterite respectively

Collecting the like term and solving for the unknown Cement C, the solution is obtained as: Cement,  $C=180.11kg/m^3$ , Water,  $W=0.5*C= 90.05kg/m^3$  and Laterite,  $L=12.5*C= 2251.32kg/m^3$ . Similarly, for the upper limit of 20 percent cement content, representing a ratio 0.5:1:5; Cement,  $C=368.81kg/m^3$ , Water,  $W=0.5*C= 184.41kg/m^3$  and Laterite,  $L=5*C= 1844.07kg/m^3$ . The Scheffe's augmented [3,2]

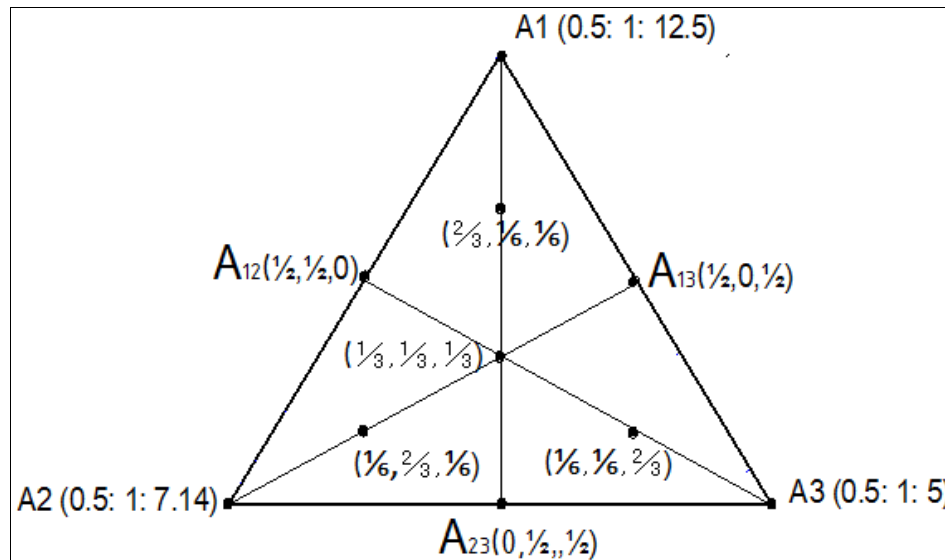


Figure 1. An augmented [3, 2] Simplex lattice points

Simplex lattice design was used to obtain a design matrix (Anya and Osadebe, 2015) whose vertices were 8%, 14% and 20% cement contents representing ratios 1:12.5; 1:7.14 and 1:5 fitted in a manner as to yield an optimum within the design domain selected. This is represented in Figure 1. The pseudo component

variables of all other binary, interior and centre points were transformed into their actual factor variables by the method described Scheffe (Anya and Osadebe, 2015).

### 3.4 Determination of dry density/moisture content relationship and estimation of revised mixing water

The optimum moisture content corresponding to the maximum dry density was used to determine the quantity of mixing water to produce maximum dry compacted soil per cubic meter of the soil-cement mixture. The 4.5kg rammer method was used in accordance with the procedure described in BS 1377 (1990). The heavy compaction is used because; the machine compactive effort is 10MN/m<sup>2</sup> (Hydraform, 2014).

### 3.5 Revised mixing water estimation

The optimum moisture required which corresponds to the maximum dry density was used to replace the starting mixing water. A linear mathematical relationship connecting water requirement to the cement:laterite ratio was obtained based on the initial water requirement estimated to yield maximum dry density, with a probability  $p < 0.05$ . The expression relating water requirement was obtained and the constituent proportions that will now satisfy equation (4) was now re-calculated.

An example of re-estimation of the constituent mixture illustrated in section 3.3 is repeated thus. Using the same cement content of 8% of the dry weight of laterite, the mix ratio which was expressed as 1:12.5, can now be re-calculated. Here, the new water/cement ratio for 8% cement content is 1.83, which represents the measured optimum moisture content to produce a maximum dry density.

- i) The ratio 1:12.5 is still maintained and represents one part of cement and twelve and a half parts of laterite and water represents 1.83 by weight of cement. This represents a water:cement:laterite ratio 1.83:1:12.5. The laterite content can similarly be expressed as Laterite,  $L=12.5*C$ . Subsequently, the water content based on the adopted starting water/cement ratio can similarly be expressed as Water,  $W=1.83*C$ .
- ii) The equation which satisfies the constraint condition of equation (4) can therefore be re-written again as:  $\frac{1.83C}{1000} + \frac{C}{3.15 \times 1000} + \frac{12.5C}{2.64 \times 1000} = 1$ . Collecting the like term and solving for the unknown Cement C, the solution is obtained as Cement,  $C = 145.33\text{kg/m}^3$ , Water,  $W = 1.83*C = 265.75\text{kg/m}^3$  and Laterite,  $L=12.5*C= 1816.63\text{kg/m}^3$

This when divided by their respective absolute volumes of 1000 kg/m<sup>3</sup>, 3150 kg/m<sup>3</sup> and 2640 kg/m<sup>3</sup> for water, cement and laterite respectively gives Water,  $W=0.266$ , Cement,  $C=0.046$  and Laterite,  $L=0.688$  cubic metres. Similarly, the upper limit of 20 percent cement representing ration 1:5 cement to laterite ratio with mixing water representing water cement ratio of 0.78. Using similar method, Cement,  $C = 334.06\text{kg/m}^3$ , Water,  $W = 0.78*C = 261.26\text{kg/m}^3$  and Laterite,  $L = 1670.30\text{kg/m}^3$ . Re-representing these quantities in absolute volumes gives: Water,  $W=0.261$ , Cement,  $C=0.106$  and Laterite,  $L=0.633$ . The methodology for the revised mixing water determination can be summarized thus:

- i) Start with a starting water cement ratio of 0.5 to estimate quantity of all mixture proportions;
- ii) Replace this starting quantity of mixing water with measured water requirement from Optimum Moisture Content;
- iii) Adopting a statistical significance with probability  $p \leq 0.05$ , carry out a response prediction for water requirement (the response) against the cement:laterite ratio (the variable) to obtain a linear relationship for water requirement. This is to correct the variability. Recalculate the actual mixing water required and use it to replace the water requirement in (ii). This resulting mixture proportions is now in excess of one cubic metre of laterite-cement mixture, although still at maximum dry density.
- iv) revise the mixture proportions to reflect the summation of all the absolute volumes equal to unity using the same procedure for estimation of constituent proportions. The revised water:cement ratio to be used is obtained by dividing the numerical value of water obtained in column (9) by the numerical value of cement in column (10) of Table 2.

Using a statistical significance with probability  $p \leq 0.05$ , re-calculate the expression for mixing water requirement based on the revised table of mixture proportions to yield maximum dry density per m<sup>3</sup> of the

laterite cement mixes. The re-calculated limits/domains for the five blends are as shown in equations 5(a) – 5(e).

$$\left. \begin{matrix} 0.261 \leq x_1 \leq 0.266 \\ 0.046 \leq x_2 \leq 0.106 \\ 0.633 \leq x_3 \leq 0.688 \end{matrix} \right\} CCD - 0; \quad (5a)$$

$$\left. \begin{matrix} 0.262 \leq x_1 \leq 0.267 \\ 0.046 \leq x_2 \leq 0.105 \\ 0.628 \leq x_3 \leq 0.691 \end{matrix} \right\} CCD - C1; \quad (5b)$$

$$\left. \begin{matrix} 0.259 \leq x_1 \leq 0.262 \\ 0.046 \leq x_2 \leq 0.106 \\ 0.633 \leq x_3 \leq 0.694 \end{matrix} \right\} CCD - F1; \quad (5c)$$

$$\left. \begin{matrix} 0.263 \leq x_1 \leq 0.266 \\ 0.046 \leq x_2 \leq 0.106 \\ 0.631 \leq x_3 \leq 0.688 \end{matrix} \right\} CCD - C2; \quad (5d)$$

$$\left. \begin{matrix} 0.256 \leq x_1 \leq 0.268 \\ 0.046 \leq x_2 \leq 0.107 \\ 0.637 \leq x_3 \leq 0.687 \end{matrix} \right\} CCD - F2; \quad (5e)$$

The Letters C1, F1 and C2, F2 immediately after the hyphen represents Coarse (C) and Fine (F) sand. The figures 0, 1 and 2 represents zero (0), ten (10) and twenty (20) percent sand replacement respectively.

Table 2: Design matrix at Optimum Moisture Content using an augmented [3, 2] Simplex lattice design

S/no.	Coordinate Points	Pseudo component ratios			Actual components ratios			Actual component mixes, kg/m <sup>3</sup> (0% sand replacement)			
		x1=water, x2=cement, x3=laterite			x1	x2	x3				
		X1	X2	X3	water	Cement	Laterite	water	cement	sand	laterite
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1	A1	1	0	0	1.83	1.00	12.50	265.75	145.33		1816.63
2	PURE A2	0	1	0	1.09	1.00	7.14	264.69	243.32		1737.29
3	A3	0	0	1	0.78	1.00	5.00	261.26	334.06		1670.30
4	A12	½	½	0	1.46	1.00	9.82	265.66	181.90		1786.22
5	BINARY A13	½	0	½	1.31	1.00	8.75	265.45	202.25		1769.70
6	A23	0	½	½	0.94	1.00	6.07	263.55	281.44		1708.35
7	C1	⅓	⅔	⅓	1.16	1.00	7.68	265.03	227.79		1749.40
8	CONTROL C2	⅔	⅓	⅓	1.53	1.00	10.36	265.71	173.11		1793.44
9	C3	⅓	⅓	⅔	1.01	1.00	6.61	264.22	260.80		1723.88
10	CENTRE O	⅓	⅓	⅓	1.24	1.00	8.21	265.28	214.37		1760.00

\*The highlighted are the upper and the lower limits on the domains of constituent proportions

\*The domains of other blends are constructed in like manner

\*The quantities in columns 9,10,12 can be divided by the respective unit weights of 1000, 3150 and 2640kg/m<sup>3</sup> for water, cement and laterites respectively to obtain the absolute volumes

### 3.6 Mathematical relationship between mixture proportions

The Scheffe’s augmented [3,2] Simplex lattice design was used to obtain a linear mathematical relationship connecting water requirement to the cement: laterite ratio based on the revised water requirement estimated to yield 1m<sup>3</sup> of maximum dry density. Using a probability  $p < 0.05$  statistical significance, the expression relating the new water requirement (Y) was obtained as shown in equation (6).

$$CCD - 0; \quad Y = 269.5 - 36.93 * \left( \frac{Cement}{Laterite} \right) \quad (6a)$$

$$CCD - C1; \quad Y = 259.8 - 39.40 * \left( \frac{Cement}{Laterite} \right) \quad (6b)$$

$$CCD - F1; \quad Y = 258.5 - 18.42 * \left( \frac{Cement}{Laterite} \right) \quad (6c)$$

$$CCD - C2; \quad Y = 269.4 - 29.77 * \left( \frac{Cement}{Laterite} \right) \quad (6d)$$

$$CCD - F2; \quad Y = 276.6 - 99.04 * \left( \frac{Cement}{Laterite} \right) \quad (6e)$$

Once the ratio of cement to laterite has been selected, the mixing water requirement can be estimated. Similarly, the equations relating laterite quantity based on cement quantity selected is shown in equation (7). The letters C and F immediately following the hyphen represents coarse and fine sands respectively. The figures 0, 1 and 2 represents zero (0), ten (10) and twenty (20) percent sand replacement respectively.

$$CCD - 0; \quad L = 1927 - 0.7767 * Cement \quad (7a)$$

$$CCD - C1; \quad L = 1956 - 0.9058 * Cement \quad (7b)$$

$$CCD - F1; \quad L = 1959 - 0.8697 * Cement \quad (7c)$$

$$CCD - C2; \quad L = 1928 - 0.7886 * Cement \quad (7d)$$

$$CCD - F2; \quad L = 1907 - 0.6749 * Cement \quad (7e)$$

### 3.7 Construction of the CCD design matrix

The expression for coding and decoding of equation 1 is used to construct the matrix for the CCD design in Table 3(a) – (e). The bounds represent the proportions of the constituent mixtures for low and high levels on cement of 8 and 20 percent respectively as shown in Table 2. The construction of the design matrix is also implementable using the Design Expert statistical software (Design Expert, 2000). The inclusion of the axial points, alpha ( $\alpha$ ) in the design is to account for any missing linear expression. The choice of alpha is also to make the design rotatable and the inclusion and augmentation of the centre points are for building confidence intervals, Montgomery (2001).

## 4. Results and discussion

### 4.1 Results

The modeling of response predictions for brick strength at 7 and 28 days and cost was carried out here. The compressive strength of bricks still remains the primary response prediction for describing all other properties and hence its choice for model prediction. For example the bricks with higher strength yield high Young's Modulus of elasticity and lower strain. Similarly, the brick with higher strength corresponds with higher cost. Tables 3(a) – (e) show the summary of the design matrix for each of the design points and their responses respectively.

### 4.2 Description of the selected model using the Central Composite Design method

The models that adequately explain the fitted data are shown in Tables 4(a) - (c). The CCD model contains a constant term by default which describes the responses from input data. The runs are often randomized so as to avoid extraneous variables in the experiment (Design Expert, 2000; Simon et al, 1999; Montgomery, 2001) and the Design Expert does this randomization by default. Replicate mixes are also added to provide an estimate of repeatability or for estimating statistical significance of the fitted coefficients.

The statistical significance with low probability value of  $p \leq 0.05$  calculated shows that a model, coefficient or intercept is significant and should be included in the model. Similarly, other inferences and residuals are calculated, to validate the fitted model prediction. Contour plots can also be used to identify the conditions that give the extremum visually in one dimensional view. Contour plots therefore show only two (2) components at a time by default. Here, the interaction terms are eliminated; which shows that they are not significant in the model. The general form of the second order quadratic model is of the form:  $ax^2 + bx + c$ .

Table 3(a): Mixture proportions in coded and actual components and their responses (100% laterite)

CCD-0										
The design matrix		x <sub>1</sub> =water; x <sub>2</sub> =cement x <sub>3</sub> =100% laterite			Y1=f <sub>c7</sub> Y2=f <sub>c28</sub> Y3=cost					
Experiment no.	Point	Variables						Response		
		coded			actual (kg)			N/mm <sup>2</sup>	N/mm <sup>2</sup>	Naira
		x <sub>1</sub>	x <sub>2</sub>	x <sub>3</sub>	x1	x2	x3	Y1	Y2	Y3
1	Factorial	-1	-1	-1	261.26	145.33	1670.30	6.46	10.47	25.52
2	Factorial	1	-1	-1	265.75	145.33	1670.30	6.51	9.11	25.40
3	Factorial	-1	1	-1	261.26	334.06	1670.30	12.51	16.65	38.00
4	Factorial	1	1	-1	265.75	334.06	1670.30	11.20	17.37	37.83
5	Factorial	-1	-1	1	261.26	145.33	1816.63	6.60	10.53	25.26
6	Factorial	1	-1	1	265.75	145.33	1816.63	7.63	8.40	25.15
7	Factorial	-1	1	1	261.26	334.06	1816.63	12.17	16.15	37.10
8	Factorial	1	1	1	265.75	334.06	1816.63	12.40	17.28	36.94
9	Axial	-1.682	0	0	259.73	239.69	1743.46	9.54	12.02	31.69
10	Axial	1.682	0	0	267.28	239.69	1743.46	8.03	11.71	31.45
11	Axial	0	-1.682	0	263.50	80.97	1743.46	4.37	5.89	20.84
12	Axial	0	1.682	0	263.50	398.42	1743.46	14.71	19.90	41.27
13	Axial	0	0	-1.682	263.50	239.69	1620.40	7.76	12.84	32.10
14	Axial	0	0	1.682	263.50	239.69	1866.52	8.61	13.93	31.10
15	Centre	0	0	0	263.50	239.69	1743.46	9.08	13.12	31.57
16	Centre	0	0	0	263.50	239.69	1743.46	9.05	13.59	31.57
17	Centre	0	0	0	263.50	239.69	1743.46	9.23	13.10	31.57
18	Centre	0	0	0	263.50	239.69	1743.46	9.07	12.88	31.57
19	Centre	0	0	0	263.50	239.69	1743.46	8.99	12.76	31.57
20	Centre	0	0	0	263.50	239.69	1743.46	9.71	13.78	31.57

Table 3(b): Mixture proportions in coded and actual components and responses (10% C-Sand replacement)

CCD-C1										
The design matrix		x <sub>1</sub> =water; x <sub>2</sub> =cement x <sub>3</sub> =10% C-sand+90% laterite			Y1=f <sub>c7</sub> Y2=f <sub>c28</sub> Y3=cost					
Experiment no.	Point	Variables						Response		
		coded			actual (kg)			N/mm <sup>2</sup>	N/mm <sup>2</sup>	Naira
		x <sub>1</sub>	x <sub>2</sub>	x <sub>3</sub>	x1	x2	x3	Y1	Y2	Y3
1	Factorial	-1	-1	-1	262.36	146.00	1657.21	5.55	7.59	25.56
2	Factorial	1	-1	-1	267.05	146.00	1657.21	4.70	7.19	25.43
3	Factorial	-1	1	-1	262.36	331.44	1657.21	9.86	13.61	37.88
4	Factorial	1	1	-1	267.05	331.44	1657.21	13.45	17.75	37.70
5	Factorial	-1	-1	1	262.36	146.00	1825.01	6.60	9.73	25.26
6	Factorial	1	-1	1	267.05	146.00	1825.01	5.42	7.98	25.15
7	Factorial	-1	1	1	262.36	331.44	1825.01	8.58	13.46	36.86
8	Factorial	1	1	1	267.05	331.44	1825.01	10.29	13.56	36.69
9	Axial	-1.682	0	0	260.76	238.72	1741.11	8.10	11.50	31.61
10	Axial	1.682	0	0	268.65	238.72	1741.11	7.10	10.58	31.36
11	Axial	0	-1.682	0	264.70	82.77	1741.11	3.13	4.72	20.94
12	Axial	0	1.682	0	264.70	394.68	1741.11	9.82	13.43	41.02
13	Axial	0	0	-1.682	264.70	238.72	1599.99	7.61	10.77	32.08
14	Axial	0	0	1.682	264.70	238.72	1882.23	7.86	10.99	30.94
15	Centre	0	0	0	264.70	238.72	1741.11	9.31	14.14	31.48
16	Centre	0	0	0	264.70	238.72	1741.11	6.65	11.16	31.48
17	Centre	0	0	0	264.70	238.72	1741.11	9.06	13.10	31.48
18	Centre	0	0	0	264.70	238.72	1741.11	8.91	12.88	31.48
19	Centre	0	0	0	264.70	238.72	1741.11	9.14	13.21	31.48
20	Centre	0	0	0	264.70	238.72	1741.11	9.53	13.78	31.48



Table 3(c): Mixture proportions in coded and actual components and responses (10% F-Sand replacement)

CCD-F1										
The design matrix		x <sub>1</sub> =water; x <sub>2</sub> =cement			x <sub>3</sub> =10% F-sand+90% laterite			Y1=f <sub>c7</sub>	Y2=f <sub>c28</sub>	Y3=cost
Experiment no.	Point	Variables						Response		
		coded			actual (kg)			N/mm <sup>2</sup>	N/mm <sup>2</sup>	Naira
		x <sub>1</sub>	x <sub>2</sub>	x <sub>3</sub>	x1	x2	x3	Y1	Y2	Y3
1	Factorial	-1	-1	-1	259.29	146.61	1669.85	7.04	9.49	25.66
2	Factorial	1	-1	-1	261.46	146.61	1669.85	5.50	9.12	25.61
3	Factorial	-1	1	-1	259.29	333.97	1669.85	13.65	17.07	38.07
4	Factorial	1	1	-1	261.46	333.97	1669.85	11.14	12.89	37.99
5	Factorial	-1	-1	1	259.29	146.61	1832.60	6.88	8.25	25.37
6	Factorial	1	-1	1	261.46	146.61	1832.60	6.94	9.95	25.31
7	Factorial	-1	1	1	259.29	333.97	1832.60	13.10	19.21	37.07
8	Factorial	1	1	1	261.46	333.97	1832.60	11.66	17.19	36.99
9	Axial	-1.682	0	0	258.55	240.29	1751.22	9.68	15.40	31.74
10	Axial	1.682	0	0	262.20	240.29	1751.22	10.00	14.17	31.62
11	Axial	0	-1.682	0	260.38	82.72	1751.22	4.81	6.00	21.04
12	Axial	0	1.682	0	260.38	397.86	1751.22	15.66	19.79	41.31
13	Axial	0	0	-1.682	260.38	240.29	1614.35	8.66	12.71	32.27
14	Axial	0	0	1.682	260.38	240.29	1888.10	10.79	15.09	31.15
15	Centre	0	0	0	260.38	240.29	1751.22	10.94	14.94	31.68
16	Centre	0	0	0	260.38	240.29	1751.22	10.54	14.56	31.68
17	Centre	0	0	0	260.38	240.29	1751.22	9.56	13.10	31.68
18	Centre	0	0	0	260.38	240.29	1751.22	9.40	12.88	31.68
19	Centre	0	0	0	260.38	240.29	1751.22	9.65	13.21	31.68
20	Centre	0	0	0	260.38	240.29	1751.22	10.06	13.78	31.68

Table 3(d): Mixture proportions in coded and actual components and responses (20% C-Sand replacement)

CCD-C2										
The design matrix		x <sub>1</sub> =water; x <sub>2</sub> =cement			x <sub>3</sub> =20% C-sand+80% laterite			Y1=f <sub>c7</sub>	Y2=f <sub>c28</sub>	Y3=cost
Experiment no.	Point	Variables						Response		
		coded			actual (kg)			N/mm <sup>2</sup>	N/mm <sup>2</sup>	Naira
		x <sub>1</sub>	x <sub>2</sub>	x <sub>3</sub>	x1	x2	x3	Y1	Y2	Y3
1	Factorial	-1	-1	-1	262.65	145.23	1667.16	4.76	7.14	25.48
2	Factorial	1	-1	-1	266.28	145.23	1667.16	4.32	6.59	25.38
3	Factorial	-1	1	-1	262.65	333.43	1667.16	11.21	12.90	37.93
4	Factorial	1	1	-1	266.28	333.43	1667.16	12.39	15.63	37.79
5	Factorial	-1	-1	1	262.65	145.23	1815.32	5.68	8.50	25.22
6	Factorial	1	-1	1	266.28	145.23	1815.32	6.40	8.95	25.13
7	Factorial	-1	1	1	262.65	333.43	1815.32	13.25	17.43	37.02
8	Factorial	1	1	1	266.28	333.43	1815.32	13.83	16.60	36.89
9	Axial	-1.682	0	0	261.41	239.33	1741.24	9.35	12.86	31.62
10	Axial	1.682	0	0	267.51	239.33	1741.24	7.86	10.89	31.43
11	Axial	0	-1.682	0	264.46	81.05	1741.24	3.82	4.72	20.83
12	Axial	0	1.682	0	264.46	397.61	1741.24	15.85	20.17	41.20
13	Axial	0	0	-1.682	264.46	239.33	1616.63	8.86	13.01	32.06
14	Axial	0	0	1.682	264.46	239.33	1865.84	9.40	12.89	31.05
15	Centre	0	0	0	264.46	239.33	1741.24	9.03	12.62	31.53
16	Centre	0	0	0	264.46	239.33	1741.24	9.24	11.80	31.53
17	Centre	0	0	0	264.46	239.33	1741.24	9.89	13.29	31.53
18	Centre	0	0	0	264.46	239.33	1741.24	9.72	13.06	31.53
19	Centre	0	0	0	264.46	239.33	1741.24	10.15	13.63	31.53
20	Centre	0	0	0	264.46	239.33	1741.24	10.40	13.97	31.53

Table 3(e): Mixture proportions in coded and actual components and responses  
(20% F-Sand replacement)

CCD-F2										
The design matrix		x <sub>1</sub> =water; x <sub>2</sub> =cement x <sub>3</sub> =20% F-sand+80% laterite			Y1=f <sub>c7</sub> Y2=f <sub>c28</sub> Y3=cost					
Experiment no.	Point	Variables						Response		
		coded			actual (kg)			N/mm <sup>2</sup>	N/mm <sup>2</sup>	Naira
		x <sub>1</sub>	x <sub>2</sub>	x <sub>3</sub>	x1	x2	x3	Y1	Y2	Y3
1	Factorial	-1	-1	-1	255.81	144.93	1682.61	5.59	9.12	25.62
2	Factorial	1	-1	-1	267.77	144.93	1682.61	4.76	8.02	25.29
3	Factorial	-1	1	-1	255.81	336.52	1682.61	14.25	19.00	38.28
4	Factorial	1	1	-1	267.77	336.52	1682.61	9.68	18.21	37.83
5	Factorial	-1	-1	1	255.81	144.93	1811.62	5.51	9.50	25.38
6	Factorial	1	-1	1	267.77	144.93	1811.62	6.64	9.37	25.08
7	Factorial	-1	1	1	255.81	336.52	1811.62	12.81	17.53	37.47
8	Factorial	1	1	1	267.77	336.52	1811.62	12.11	19.10	37.04
9	Axial	-1.682	0	0	251.74	240.73	1747.11	9.04	14.33	32.00
10	Axial	1.682	0	0	271.85	240.73	1747.11	8.47	13.70	31.36
11	Axial	0	-1.682	0	261.79	79.60	1747.11	3.73	5.17	20.78
12	Axial	0	1.682	0	261.79	401.86	1747.11	13.81	20.42	41.51
13	Axial	0	0	-1.682	261.79	240.73	1638.62	8.58	13.01	32.14
14	Axial	0	0	1.682	261.79	240.73	1855.61	7.71	13.69	31.25
15	Centre	0	0	0	261.79	240.73	1747.11	8.77	14.99	31.68
16	Centre	0	0	0	261.79	240.73	1747.11	7.76	12.62	31.68
17	Centre	0	0	0	261.79	240.73	1747.11	8.48	13.29	31.68
18	Centre	0	0	0	261.79	240.73	1747.11	8.33	13.06	31.68
19	Centre	0	0	0	261.79	240.73	1747.11	8.55	13.40	31.68
20	Centre	0	0	0	261.79	240.73	1747.11	8.92	13.97	31.68

Table 4(a) Response prediction for 28-day strength: CCD method

CCD-0;	$1/(f_{c,28}) = 0.21363 - 0.000856443 * \text{Cement} + 0.00000116686 * \text{Cement}^2$
CCD-C1;	$1/(f_{c,28}) = 0.2996 - 0.00147587 * \text{Cement} + 0.00000234307 * \text{Cement}^2$
CCD-F1;	$1/(f_{c,28}) = 0.23053 - 0.00101814 * \text{Cement} + 0.00000149142 * \text{Cement}^2$
CCD-C2;	$1/(f_{c,28}) = 0.29822 - 0.00141862 * \text{Cement} + 0.00000209428 * \text{Cement}^2$
CCD-F2;	$1/(f_{c,28}) = 0.25648 - 0.00117542 * \text{Cement} + 0.00000169111 * \text{Cement}^2$

Table 4(b) Response prediction for 7-day strength: CCD method

CCD-0;	$1/(f_{c,7}) = 0.27845 - 0.000995268 * \text{Cement} + 0.00000121943 * \text{Cement}^2$
CCD-C1;	$1/(f_{c,7}) = 0.44452 - 0.00217823 * \text{Cement} + 0.00000339415 * \text{Cement}^2$
CCD-F1;	$1/(f_{c,7}) = 0.2871 - 0.0011235 * \text{Cement} + 0.00000147466 * \text{Cement}^2$
CCD-C2;	$1/(f_{c,7}) = 0.39451 - 0.00176367 * \text{Cement} + 0.00000239979 * \text{Cement}^2$
CCD-F2;	$1/(f_{c,7}) = 0.35948 - 0.00146593 * \text{Cement} + 0.00000190695 * \text{Cement}^2$

Table 4(c) Response prediction for Cost: CCD method

CCD-0;	$\text{Cost} = 22.97628 + 0.064321 * \text{Cement} - 0.00398027 * \text{Laterite}$
CCD-C1;	$\text{Cost} = 22.88414 + 0.064355 * \text{Cement} - 0.00395016 * \text{Lat} + 10\% \text{ C-Sand}$
CCD-F1;	$\text{Cost} = 23.15873 + 0.064292 * \text{Cement} - 0.00402267 * \text{Lat} + 10\% \text{ F-Sand}$
CCD-C2;	$\text{Cost} = 22.91487 + 0.064335 * \text{Cement} - 0.0039638 * \text{Lat} + 20\% \text{ C - Sand}$
CCD-F2;	$\text{Cost} = 23.09909 + 0.064282 * \text{Cement} - 0.00401697 * \text{Lat} + 20\% \text{ F-Sand}$

**4.3 Decision on choice of number of variables**

Sand, as a replacement of a quantity of laterite is not considered as a single variable, this is permitted (Simon et al, 1999; Montgomery, 2001). This obviously helps to reduce experimental cost. Constraints here take the form of upper and lower bounds as stated in equation (5) derived within the limits of 8-20 percent cement content. This defines the experimental domain. To achieve other user-defined requirements of cost for example, they may be added. Building constraints are essentially to provide flexibility in defining the experimental region of interest.

**4.4 Comparative results using Central Composite Design**

It can also be validated that the measured properties of bricks produced are largely dependent on the quantity of cement and compactive effort (Hydraform, 2014; Osunade and Fajobi, 2000; Aguwa, 2009; Awoyera and Akinwumi, 2014; Guetalla et al, 2004) and this is shown in Table 5. Similarly, production of bricks within 8 - 20 percent cement content design domain has shown reasonable results that would guide against bricks that would be durable.

Table 5: Comparative compressive strength results using Central Composite Design

(1)	(2)	(3)	Hydraform (2014)	Aguwa (2009)	Guetalla et al (2005)	Awoyera & Akinwumi (2014)	CCD - 0	CCD - C1	CCD - F1	CCD - C2	CCD - F2
(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
S/No.	Cement Content (%)	Compactive effort MN/m <sup>2</sup>	10	4	15	2	10	10	10	10	10
1	4	COM PRESSIVE STRENGTH	-	1.9	-	-	-	-	-	-	-
2	5		3	-	15.4	-	-	-	-	-	-
3	6		-	3.5	-	-	-	-	-	-	-
4	7		5	-	-	-	-	-	-	-	-
5	8		-	5.1	-	2.3	8.78	7.46	8.84	8.76	8.22
6	10		8	6.1	18.4	3.49	10.35	9.23	10.63	10.54	10.09
7	12		-	6.5	-	3.86	12.14	11.27	12.69	12.58	12.37
8	14		-	7.1	-	-	14.03	13.25	14.82	14.69	14.89
9	15		10	-	-	-	-	-	-	-	-
10	16		-	8.3	-	-	15.8	14.61	16.6	16.5	17.27
11	18		-	9.2	-	-	17.13	14.88	17.55	17.51	18.85
12	20		12	9.6	-	-	-	-	-	-	-
13	25		14	-	-	-	-	-	-	-	-

\* The highlighted header row represents the compactive effort in MN/m<sup>2</sup>  
 \* The serial numbers 5 through 11; columns (8) through (12) are estimated using the example in Section 4.16  
 \* CCD represents Central Composite method  
 \* The Letters C1, F1 and C2, F2 immediately after the hyphen represents Coarse and Fine sand, 10 percent and 20 percent blends respectively

**4.5 Optimization formulation**

Minimize  $f(x) = ccd(x_i)$

Subject to inequalities:

$x_{il} \leq x_i \leq x_{iu}$  (upper and lower levels on the variables)

$\sum_{i=2}^3 a_i x_i \leq c_i$  (cost)

Equalities:

$-a_1 x_{2i} + x_{3i} = 0$  (ratio of cement: laterite)

$x_{1i} = y_{1i}$  (water requirement)  
 $i = 1, 2, \dots, n$

## 5. Conclusions and Recommendations

Based on the CCD method, it has been shown that statistically designed composite bricks satisfying user-defined requirement is practicable. Similarly, in using this method, responses capable of achieving target mean strengths can be developed and thus specification writing for site production is possible. The GA and the approximate procedures are implementable computationally. Strength and compactive effort still represent major factors in predicting the properties of the bricks moulded. Either fine or coarse sands within grading zones 2 and 3 can be used, the blends are suitable and yielding nearly same results within the domain of cement:laterite considered.

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### APPENDIX A.1: Example of GA Optimization of component mixes to meet user defined requirement using CCD formulated design

Problem statement: To obtain mix proportions for laterite cement brick (100% laterite with no sand replacement, which is coded as CCD-0. The input data for this requirement are as stated:

- i) Cement content should be 8% representing ratio 1:12.5 of cement to laterite
- ii) The total cost of a brick is not to exceed-N30 per brick or N1,200 per m<sup>2</sup>.

The objective function for strength at 28 days from Table 5(a) is:

$$CCD - 0; \frac{1}{f_{c_{28}}} = 0.21262 - 0.000856443 * cement + 0.00000116686 * cement^2$$

The response prediction for cost also for CCD-0 from Table 5(c) is:

$$22.97628 + 0.064321 * cement - 0.00398027 * laterite \leq 30$$

or rearranging gives,  $0.064321 * cement - 0.00398027 * laterite \leq 7$

Since water is usually priced under preliminary, therefore, water is not included in the estimate of cost.

The build-up of constraint for the ratio of cement to laterite which is to be ratio 1:12.5 is constructed as described:

$$\frac{x_2}{x_3} = 12.5; \quad \text{where } x_2 = \text{cement and } x_3 = \text{laterite. This can be re-written as:}$$

$$x_3 = 12.5x_2 \quad \text{and re-arranging gives } -12.5x_2 + x_3 = 0$$

The linear equality for water is obtained from equation (7). The required mixing water for CCD - 0;  $water = 269.5 - 36.93X$ ; where  $X = \frac{\text{cement}}{\text{laterite}}$  which is 1:12.5. Substituting gives:  $water = (269.5 - (36.93 * (1/12.5))) = 266.5\text{kg/m}^3$ . The upper limit of  $266\text{kg/m}^3$  will be used

#### Input for the Matlab Genetic Algorithm solver

The input for the optimization process using GA solver can be obtained thus:

Function file name extension: `function z=@ccd0`

Function `z=ccd0(x)`

Fitness function construction is:

`function z = ccd0(x)`

`z = (0.21363-(x(2).*0.000856443)+(x(2)^2.*0.00000116686))`

linear inequality for cost is:

$$0.064321 * cement - 0.00398027 * laterite \leq 7$$

linear equality for cement 8% cement content representing ratio 1:12.5 is:

$$-12.5x_2 + x_3 = 0 \quad \text{and for water is obtained from equation (7) } CCD - 0; \quad water = 269.5 - 36.93X; \quad water = (269.5 - (36.93 * (1/12.5))) = 266.5\text{kg/m}^3$$

Number of variables: 3

Constraints:

Linear inequalities: A: 0,0.064321,-0.00398027 b: 7

Linear equalities: Aeq: 0,-12.5,1 ; 1, 0, 0 beq: 0;266

-equation 2.1 and ratio of laterite cement constraint

Bounds: Lower: 261,144.9,1671.12 Upper: 266,333.9,1816.32

- limits in equation 6(a)

Population type: Double vector

Creation function: Constraint dependent

The solution satisfying the fitness function is:  $x_1 = 266, x_2 = 145.306, x_3 = 1816.32$

with functional evaluation  $\frac{1}{f(x)} = 0.11382$  and the inverse is  $8.786\text{N/mm}^2$ .

## APPENDIX A.2: Optimization of component mixes to meet user defined requirement using approximate CCD design

This method starts as an iterative process by initially selecting a cement quantity and thus obtaining the desired strength. The procedure is stated thus:

- i) Start by calculating the quantity of cement from within the limits suggested
- ii) Substitute the cement quantity in the equation expressing the compressive strength
- iii) Calculate the inverse or reciprocal of the value obtained in (ii)
- iv) Calculate the corresponding quantity of laterite from the equation relating the calculated cement quantity
- v) Calculate the corresponding quantity of water from the equation relating the calculated cement/laterite ratio
- vi) Calculate the cost per brick or per m<sup>2</sup>
- vii) Calculate cement: laterite ratio

Using the same problem statement:

- i) Starting with the lowest limit of cement (absolute volume = 0.046) represents 145kg of cement, that is  $(0.046 \times 3150 = 145\text{kg})$ , where unit weight of cement is  $3150\text{kg/m}^3$
- ii) Substituting the cement quantity in the equation  $\frac{1}{f_{c_{28}}} = 0.21363 + 0.000856443 * 145 + 0.00000116686 * 145^2 = 0.113979$
- iii) The inverse is  $8.77\text{N/mm}^2$
- iv) The corresponding quantity of laterite from equation (8) relating the calculated cement quantity is:  $= 1927 - 0.7767 * \text{cement}$ ; gives  $(1927 - (0.7767 * 145)) = 1814.3785\text{kg/m}^3$ .
- v) The corresponding quantity of water from the equation relating the calculated cement/laterite ratio is:  $\text{water} = 269.5 - 36.93 * \frac{\text{cement}}{\text{laterite}}$ , this substitution gives  $= (269.5 - (36.93 * (145/1814.3785))) = 266.55\text{kg/m}^3$
- vi) The cost for CCD-0 is:  $\text{Cost} = 22.97628 + 0.064321 * \text{cement} - 0.00398027$  which can be substituted to yield:  $\text{cost} = (22.97628 + (0.064321 * 145 - (0.00398027 * 1814.3785))) = \text{N}25.08$  per brick < N30.00
- vii) The cement:laterite ratio is  $145/1814.3785 = 1:12.5$