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COMBINED EFFECT OF MAGNETIC AND BUOYANCY FORCES ON MELTING FROM A VERTICAL PLATE HAVING VARIABLE TEMPERATURE EMBEDDED IN POROUS MEDIUM

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AUTHORS' CONTRIBUTIONS
This work was carried out in collaboration between all authors. All authors read and approved the final manuscript.

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ABSTRACT

The Combined effect of magnetic and buoyancy forces on melting from a vertical plate having variable temperature embedded in porous medium are investigated numerically. The similarity equations are integrated by use of the fourth-order Runge-Kutta method coupled together with shooting techniques to satisfy the boundary conditions. The effects of Magnetic number (Ha), Melting parameter (M), Mixed convection parameter (Gr/Re) and constant (λ) on the velocity temperature profiles are presented graphically. Heat transfer in the melting region has also been studied and the effect of melting parameter and magnetic parameter on Nusselt number are presented in graphical form.

Keywords: Liquid phase; mixed convection; magnetic effect; buoyancy forces effect; melting effect; porous medium.

Nomenclature

B_0: *Magnetic flux density [T];* C_ρ: *Liquid specific heat capacity [J/kg K];* C_s: *Solid specific heat capacity [J/kg K];* Da: *Darcy number,* $\frac{K}{x^2}$; f : *Dimensionless stream function;* G_r: *Grash of number* $\frac{Kg\beta_T(T_\infty - T_m)x}{\nu^2}$; h : *Heat transfer coefficient [W/m² K];* Ha : *Hartman number,* $\sqrt{\frac{\sigma B_0^2 K}{\rho\nu}}$; K : *Permeability of porous media [m²];* k : *Thermal conductivity [W/m K];* L :*Plate length [m];*

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M :Melting parameter, $\frac{C_p(T_\infty - T_m)}{1 + C_s(T_m - T_s)}$; Nu: Nusselt number $\frac{hx}{k}$; q_w : Heat flux [W/m^2];

Re: Reynolds number $\frac{U_\infty x}{\nu}$; T :Temperature [K]; T_m :Melting Temperature [K]; T_s :Solid temperature [K]; T_∞ :Liquid temperature [K]; u_∞ : External flow velocity [m/s]; U, V :Velocity in x and y direction [m/s]; x, y : Coordinate axes along and perpendicular to plate [m].

Greeks

α : Thermal diffusivity [m^2/s]; β : Coefficient of kinematic viscosity [m^2/s]; η : Dimensionless similarity variable; ρ : Liquid density [kg/m^3]; ν : Kinematic viscosity [m^2/s]; σ : Electrical conductivity of fluid [mho m^{-1}]; θ : Dimensionless Temperature, $\frac{T - T_m}{T_\infty - T_m}$; ψ : Dimensionless stream function [m^2/s]; λ : Constant defined in equation (4).

1 Introduction

The studies of boundary layer fluid flows and heat transfer over surfaces of different geometric shapes are great interest because of their applications in several industrial and physical fields. Of these, the flow over a plate though classic in nature, has been investigated extensively in literature, and still continuous to attract as an active area of research. In addition, heat transfer accompanied by melting and solidification has numerous thermal engineering applications. Phase change processes are considered to be very efficient in maintaining the system temperature within operating range. Several authors have studied the problem of melting effect about different surface geometries Kairi [1,2], Prasad et al. [3], Prasad and Hemalatha [4], Bakier [5], Gorla et al. [6], Tashtoush [7], Cheng and Chung [8,9], Jha and Mohammed [10], Jha et al. [11] and Hemalatha and Prasad [12]. The study of electrically conducting fluid in engineering application is of considerable interest, especially in metallurgical and metal working processes or in the separation of molten metals from non- metallic inclusion by the application of magnetic field. The phase change problem occurs in casting, welding, and melting purification of metals. Epstein and chao [13] studied melting heat transfer from a flat plate in a steady laminar case, while Kazamierczak et al. [14,15] consider melting from a vertical flat embedded in a porous medium in both natural and force convection approach . Hydro magnetic flows have stimulated considerable interest due to its important application in cosmic fluid dynamics, meteorology, solar physics, and in the motion of earth's core Cramer and Pai [16]. Many research papers have been published on this area. Hartman [17], Hartman and Freimut [18], Shercliff [19], Rosser [20] and Lykoudis [21], dealing with various phenomena in hydro magnetic boundary layers.

From the literature, it appears that the contribution found to melting phenomena accounting for a combination of magnetic effect and buoyancy effects on the melting from vertical plate embedded in a porous medium Tashtoush [7], The present work has been undertaken to analyze the problem of combined effect of magnetic and buoyancy forces on melting from a vertical plate having variable temperature embedded in a porous medium. The melting process is assumed to be a steady state and melting parameter is introduced to study effect of melting on the heat transfer characteristics. The effect of buoyancy as well as magnetic effect on the velocity and temperature profiles is also presented.

2 Mathematical Formulation

Let us consider the mixed convection flow in a porous medium beside a heated vertical plate with assisting external flow u_∞. The coordinate system and flow model are shown in Fig. 1. It is assumed that this plate constitutes the interface between the liquid and solid phases during melting inside the porous matrix. The x coordinate is measure along the plate and the y coordinate is normal to it. The temperature of the plate T_m, is the melting temperature of the material occupying the porous matrix. The liquid phase far from the plate is maintaining a constant temperature $T_\infty \left(T_\infty \succ T_m\right)$ the fluid and the porous matrix are assumed to be in local thermal equilibrium. A magnetic field of variable strength B_0 is applied in the y direction, which is normal to the flow direction.

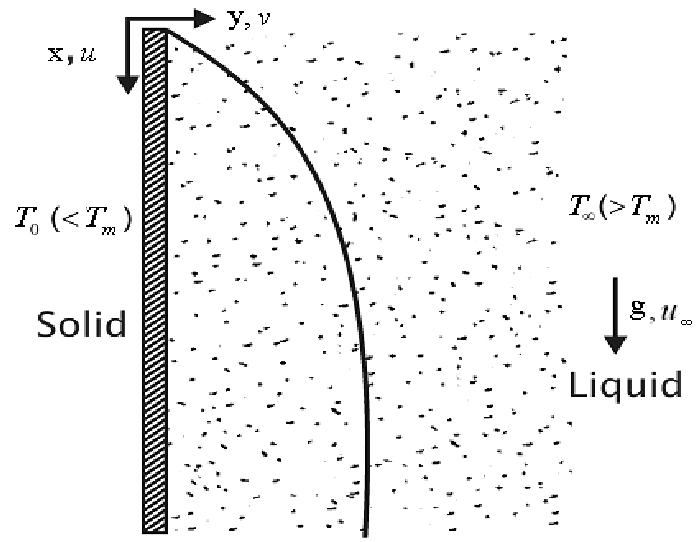

Fig. 1. Physical model investigated

Based on the above assumptions, the governing equations for steady laminar non-Darcy flow heat transfer in a porous medium can be written as follows Bakier [5], Gorla et al. [6], Tashtoush [7]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\left(1 + \frac{\sigma B_0 K}{\rho v}\right)\frac{\partial u}{\partial y} = \frac{-Kg\beta}{v}\frac{\partial T}{\partial y} \tag{2}$$

$$\rho C_\rho \left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} \right) = k\frac{\partial^2 T}{\partial y^2} \tag{3}$$

Where $K, k, \rho, \sigma, v, \beta, g, C\rho$ and T are the permeability, liquid thermal conductivity, liquid density, electric conductivity, kinematic viscosity, thermal expansion coefficient, acceleration due to gravity, liquid specific heat and temperature respectively.

The above problem is solved subject to the following boundary conditions.

$$y=0,\quad T=T_m=T_\infty+Ax^{\lambda}\psi(\eta),\ k\frac{dT}{dy}=\rho\left[1+C_s\left(T_m-T_s\right)\right]v \tag{4}$$

$$y\to\infty,\ T\to T_\infty,\ u\to U_\infty \tag{5}$$

Where A>0, we will designated as aiding flows when the buoyancy force has a component in the direction of free stream velocity i.e. $T = T_m = T_\infty + Ax^{\lambda}\psi(\eta)$ the first boundary conditions on the melting surface simply started that the temperature of the interface equals the melting temperature of the materials saturating the porous matrix. The second boundary condition at y=0 is a direct result of heat balance. It is started that the heat conducted to the melting surface is equal to the heat of melting plus the sensible heat required raising the temperature of the solid T_0 to its melting temperature T_m (Baker [5], Epstein and Cho [13]). To seek similarity solution to equation (4) and (5) with boundary condition (6) and (7), we introduce the following dimensionless variables

$$\eta=\sqrt{\frac{u_\infty}{\alpha}}\ yx^{\frac{n-1}{2}}\quad \psi=\sqrt{\alpha u_\infty}\ x^{\frac{n+1}{2}}f(\eta),\quad \theta(\eta)=\frac{T-T_m}{T_\infty-T_m} \tag{6}$$

Substituting equation (6) into equation (2) and (3), we obtain the following transform governing equations:

$$\left(1+Ha^2\right)f''+\frac{Gr}{\mathrm{Re}}\theta'=0 \tag{7}$$

$$\theta''+\frac{1}{2}\left(1+\lambda\right)f\theta'-\lambda f'\theta=0 \tag{8}$$

The boundary conditions are

$$\eta=0,\theta=0,\ f(0)+2M\theta'(0)=0 \tag{9}$$

$$\eta\to\infty,\theta=1,\ f'=1 \tag{10}$$

Where $M=\frac{C_p\left(T_\infty-T_m\right)}{1+C_s\left(T_m-T_s\right)}$ is the melting parameter, $\mathrm{Re}=\frac{U_\infty x}{\nu}$ is the Reynolds number, $Gr=\frac{Kg\beta_T\left(T_\infty-T_m\right)x}{\nu^2}$ is the Grashof number $Da=\frac{K}{x^2}$ is the Darcy number and $Ha=\sqrt{\frac{\sigma B_0^2 K}{\rho\nu}}$ is the Hartman number.

The quantity $\frac{Gr}{\mathrm{Re}}$ in equation (7) is a measure of the relative importance of free and force convection and is the controlling parameter for the present problem.

The heat transfer rate along the surface of the plate q_w can be computed from the Fourier heat conduction law.

$$q_w = -k \left. \frac{\partial T}{\partial y} \right|_{y=0} \tag{11}$$

The heat transfer results can be represented by the local Nusselt number Nu, which can be defined as

$$Nu = \frac{hx}{k} = \frac{q_w x}{k(T_m - T_\infty)} \tag{12}$$

Where h denotes the local heat transfer coefficient and k represent the liquid phase thermal conductivity. Substituting equation (6) and (11) into equation (12) we obtain

$$\frac{Nu}{Ra^{\frac{1}{2}}} = \theta'(0). \tag{13}$$

3 Numerical Solution

The system of non-linear ordinary differential equation (7) - (9) together with the boundary conditions (10) is solved numerically using the shooting iteration technique by Adam and Hashim [22] together with sixth order Runge-Kutta integration scheme. In a shooting method, the missing (unspecified) initial condition at the initial point of the interval is assumed, and the differential equation is then integrated numerically as an initial value problem to the terminal point. The accuracy of the assumed missing initial condition is then checked by comparing the calculated value of the dependent variable at the terminal point with its given value there. If a difference exists, another value of the missing initial condition must be assumed and the process is repeated. This process is continued until the agreement between the calculated and the given condition at the terminal point is within the specified degree of accuracy. For this type of iterative approach one naturally inquires whether or not there is a systematic way of finding each succeeding. The present results are validated by direct comparison with those obtained by Bakair [5], Gorla et al. [6] and Tashtoush [7] as shown in Table 1. It seen from this table that both results are in excellent agreement, which gives confidence in the numerical results obtained for the present problem.

Table 1. Values of $\theta'(0)$ and $f(0)$ for different values of buoyancy parameter and melting parameters, $Ha=0, \lambda=0$

M	Gr/Re	Bakair [5],		Gorla et al. [6]		Tashtoush [7]		Present work	
		$\theta'(0)$	$f(0)$	$\theta'(0)$	$f(0)$	$\theta'(0)$	$f(0)$	$\theta'(0)$	$f(0)$
0.4	0.0	0.4570	-0.3656	0.4570	-0.3656	0.4571	-0.3657	0.4570	-0.3656
	1.4	0.6278	-0.5022	0.6278	-0.5022	0.6278	-0.5023	0.6278	-0.5022
	20	1.6865	-1.3492	1.6865	-1.3492	1.6866	-1.3493	1.6866	-1.3493
2.0	0.0	0.2799	-1.1197	0.2799	-1.1197	0.2743	-1.097	0.2743	-1.097
	1.4	0.3823	-1.5291	0.3823	-1.5291	0.3807	-1.5231	0.3808	-1.5232
	3.0	0.4754	-1.9016	0.4754	-1.9016	0.4747	-1.8988	0.4747	-1.8988
	8.0	0.6902	-2.7611	0.6902	-2.7611	0.6902	-2.7607	0.6902	-2.7607
	10.0	0.7594	-3.0370	0.7594	-3.0370	0.7587	-3.0290	0.7593	-3.0375
	20.0	1.0383	-4.1530	1.0383	-4.1530	1.0382	-4.1529	1.0382	-4.1529

Table 2. Values of $\theta'(0)$, $f(0)$ and $f'(0)$ for different values of magnetic parameter, $\frac{GR}{\text{Re}}=2, M=0.5\ \lambda=0.2$

Ha	$\theta'(0)$	$f(0)$	$f'(0)$
0	0.479215	-0.479215	3.
1.0	0.407553	-0.407553	1.999999
2.0	0.357581	-0.357581	1.399999
3.0	0.339318	-0.339318	1.199999
4.0	0.331518	-0.331518	1.117647
5.0	0.327595	-0.327595	1.076923

4 Results and Discussion

Numerical computations were carried out for a range of values of the buoyancy parameter Gr/Re, melting parameter M and Hartman number Ha on the dimensionless stream function f, velocity f' and θ' at the plate are indicated in Tables 1-2. In the absence of the Hartman Number, the results are in accordance with Bakeir [5], Gorla et al. [6] and Tashtoush [7] for λ = 0 as shown in table 1. Figs. 2, 3 reveal the velocity distribution for different values of buoyancy parameter Gr/Re for λ = 0 and λ = 1/5. It is observed that the velocity increases as buoyancy increases for both λ = 0 and λ = 1/5. The temperature variation against η is shown in Figs. 4, 5. As the buoyancy parameter increases, there is a rise in the temperature for fixed values of M (see Figs. 4 and 5). Fig. 6 shows that effect of the melting parameter, M, for buoyancy parameters Gr/Re = 2.0 and 15. As the melting parameter increases the velocity increases for both free (Gr/Re = 15) and forced (Gr/Re = 2.0) convection flows. Increasing the melting parameter will increases the velocity inside the boundary layer and, therefore, decreases the boundary layer thickness, which results decrease in the temperature as shown in Fig. 7. The effects of the melting parameter M on the stream function *f* for fixed buoyancy parameter is given in Fig. 8. It is observed that f decreases as the melting parameter increases.

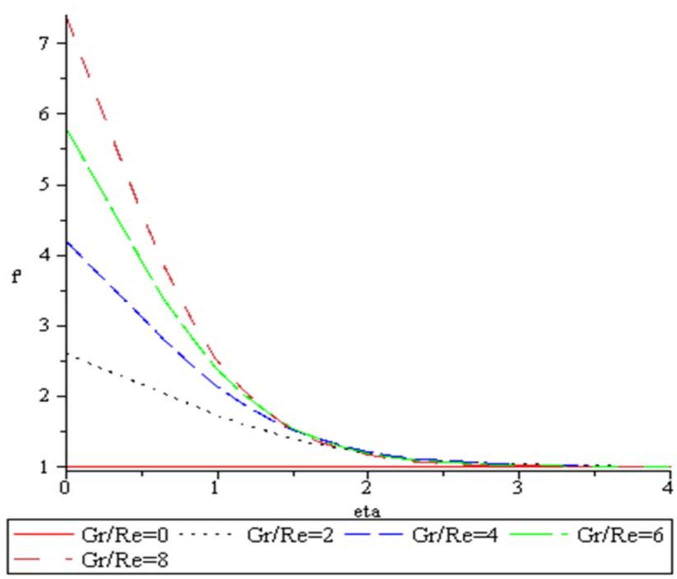

Fig. 2. Velocity profile for different values of Gr/Re, λ=0, M=1.0 and Ha=0.5

The effect of the magnetic parameter on the velocity and temperature profile for buoyancy parameter Gr/Re = 2.0 are shown in Figs .9 and 10. It can be seen that as magnetic number increases, the velocity and the temperature profile decrease, and the thermal boundary layer increases. The effects of constant (λ) on velocity and temperature profile are shown in Figs. 11 and 12. As the λ increases the velocity profile increases while in the other hand, the temperature profile decreases in Fig. 12.

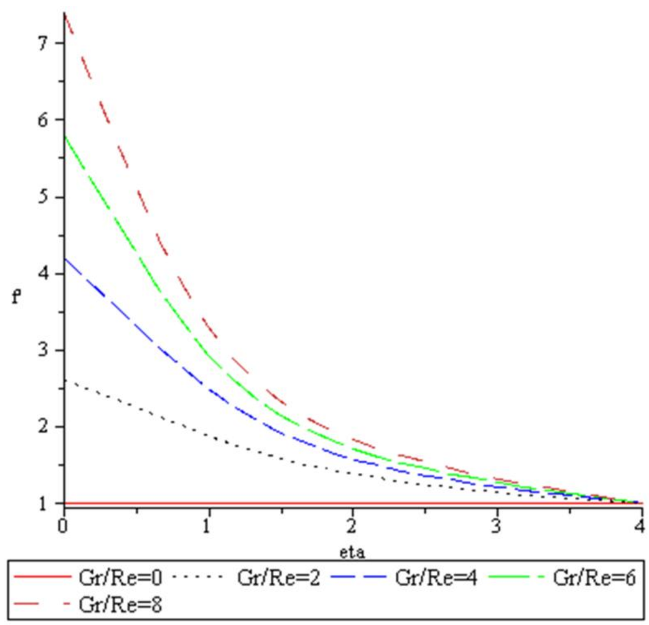

Fig. 3. Velocity profile for different values of Gr/Re, λ=1/5, M=1.0 and Ha=0.5

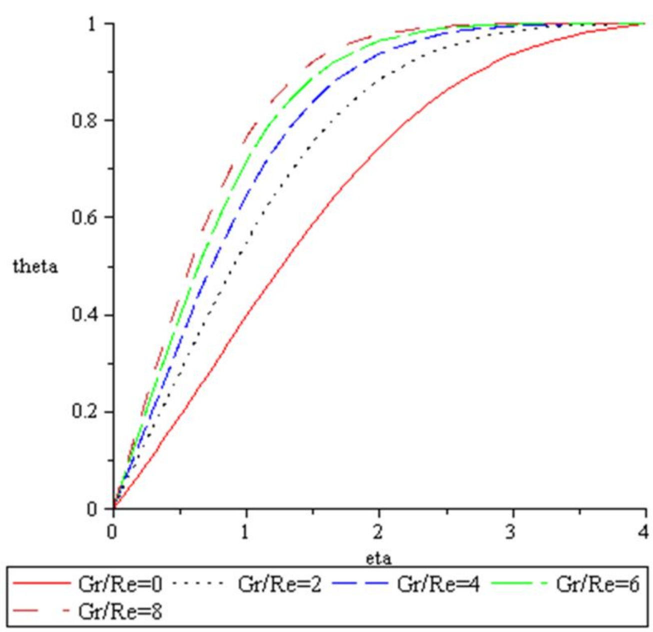

Fig. 4. Temperature profile for different values of Gr/Re, λ=0, M=1.0 and Ha=0.5

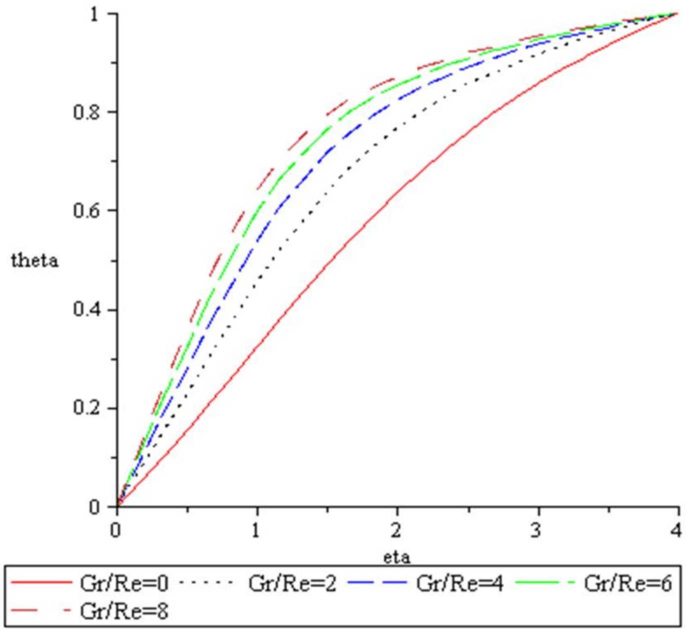

Fig. 5. Temperature profile for different values of Gr/Re, λ=1/5, M=1.0 and Ha=0.5

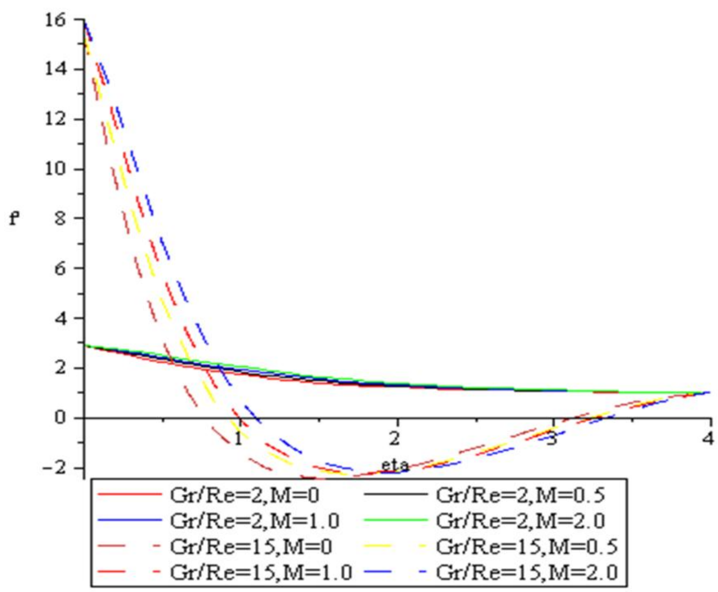

Fig. 6. Velocity profile for different values of Gr/Re,M, λ=1/10 and Ha=0.2

Fig. 13 shows the local Nusselt number varying with the Mixed convection parameter Gr/Re in the different melting parameter. Increasing the values of mixed convection parameter led to increase in average heat transfer rate while increase in melting parameter led to decrease in average heat transfer rate.

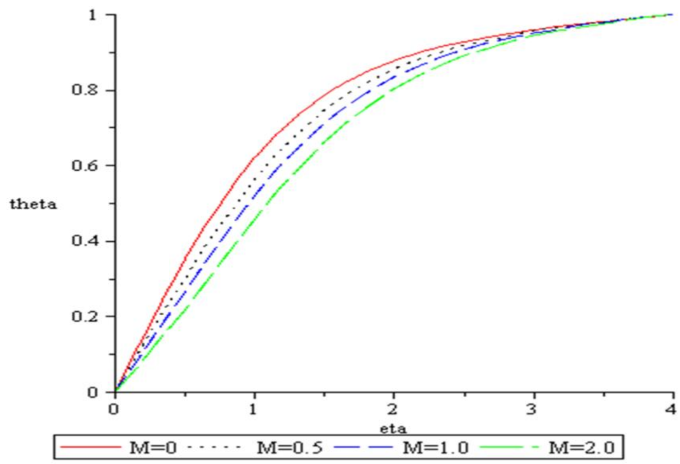

Fig. 7. Temperature profile for different values of M, Gr/Re=0.2 λ=1/10 and Ha=0.2

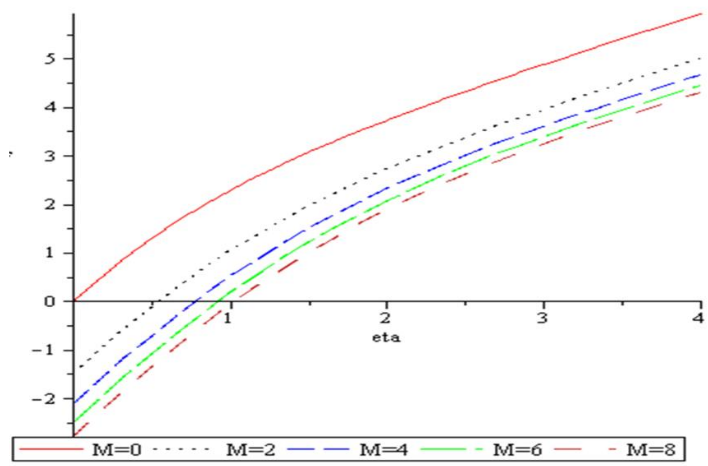

Fig. 8. Stream function for different values of M, Gr/Re=0.2 λ=1/10 and Ha=0.2

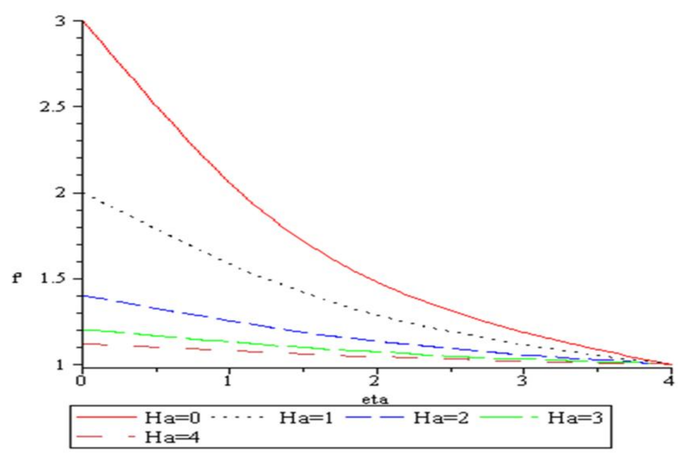

Fig. 9. Velocity profile for different values of Ha, Gr/Re=2.0,M=0.5 and λ=1/3

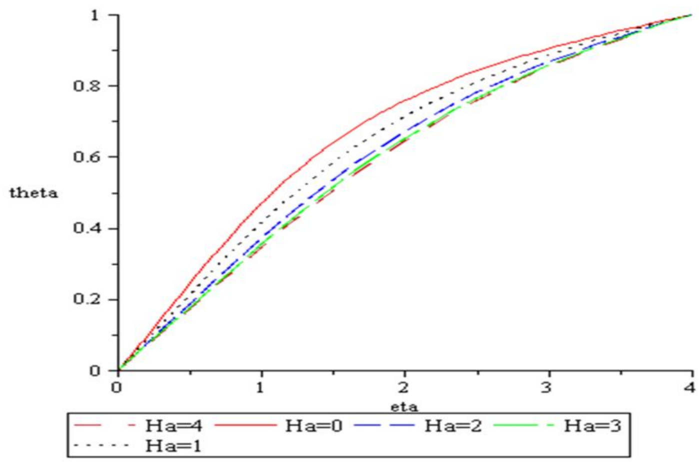

Fig. 10. Temperature profile for different values of Ha, Gr/Re=2.0,M=0.5 and λ=1/3

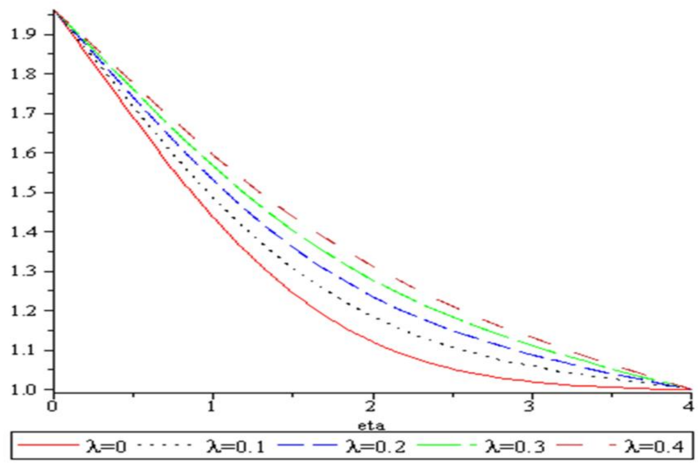

Fig. 11. Velocity profile for different values of λ, Ha=0.2, Gr/Re=1.0 and M=0.5

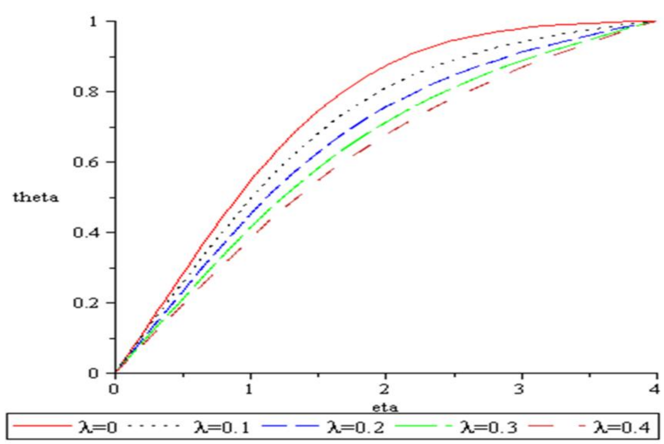

Fig. 12. Temperature profile for different values of λ, Ha=0.2, Gr/Re=1.0 and M=0.5

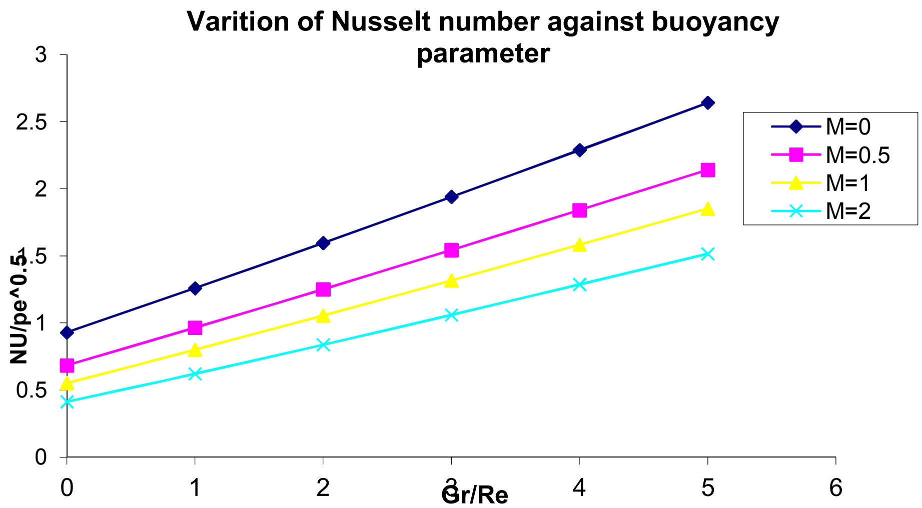

Fig. 13. The local Nusselt number varying with the mixed convection parameter Gr/Re in the different melting parameter

5 Conclusion

The Combined effect of magnetic and buoyancy forces on melting from a vertical plate having variable temperature embedded in porous medium are considered. The governing equations are derived using the usual boundary layer flow and accounting for the buoyancy forces and magnetic field effects. A boundary condition to account for melting is used at the interface between the solid and liquid phase. These equations are transformed using a similarity transformation and then solved numerically by the Runge-Kutta algorithm. Graphical results for the velocity and temperature profiles as well as the Nusselt number are presented and discussed for different parametric conditions. It was found that the temperature decreases as the melting and magnetic parameters increase.

Competing Interests

Authors have declared that no competing interests exist.

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