Derivation of Five Step Block Hybrid Backward Differential Formulas (HBDF) through the Continuous Multi-Step Collocation for Solving Second Order Differential Equation.

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ABSTRACT

The study aims to develop the theory of numerical methods used for the numerical solution of second order ordinary differential equations (ODEs). The method is derived by the interpolation and collocation of the assumed approximate solution and its second derivative at $x = x_{n+j},\ j = 1,2,.....k-1$ and $x = x_{n+k}$ respectively, where *k* is the step number of the methods. The interpolation and collocation procedures lead to a system of (*k*+1) equations, which are solved to determine the unknown coefficients. The resulting coefficients are used to construct the approximate continuous solution from which the Multiple Finite Difference Methods (MFDMs) are obtained and simultaneously applied to provide the direct solution to IVPs. The suggested approach eliminates requirement for a starting value and its speed proved to be up when computations with the block discrete schemes were used. One specific methods for *k*=5 is used to illustrate the process. The test problem was solved with the proposed numerical method and obtained numerical and analytical solutions were compared.

(Keywords: block methods, self-starting integration scheme, second order ordinary differential equations, backward differential formulas)

INTRODUCTION

Many scientific and engineering problems are described using apparatus of ordinary differential equations (ODEs), where the analytic solution is unknown. Much research has been done by the scientific community on developing numerical methods which can provide an approximate solution of the original ODE. In recent years many review articles and books have appeared on numerical methods for integrating ODEs, particularly in stiff cases Ibáñez *et al.*[1]. Stiff problems are very common problems in many fields of the applied sciences: control theory, biology, chemical engineering processes, electrical networks, fluid dynamics, plastic deformation etc.

Most of numerical methods for solving initial value problems (IVPs) for ODEs will become unbearably slow when the ODEs are stiff. The most popular multistep methods families for stiff ODEs are formed by the backward differentiation formulae (BDF or Gear methods) methods, Rosenbrock methods, implicit or diagonally implicit Runge-Kutta methods (Ibáñez *et al*.[1]; Jiaxiang and Cameron [2]; Butcher [3]).

In this paper we are suggested a construction of five step HBDF method, it is self-starting and can be applied for the numerical solution of IVPs (Cauchy problem) for second-order ODEs. Development of Hybrid Backward Differential Formulas Methods (HBDF) for solving ODEs can be generated using different methods. We use the collocation technique for the construction of implicit HBDF.

Block methods for solving ODEs have initially been proposed by Milne Miline [4].The Milne's idea of proceeding in blocks was developed by Rosser Roser [5] for Runge-Kutta method. Also block Backward Differeuntiation Formulas (BDF) methods are discussed and developed by many researchers (Mohammed [6]; Ibrahim et al.[7]; Majid and Suleiman [8]; Akinfenwa et al. [9]; Akinfenwa et al. [10]; Mohammed and Yahaya [11]; Yahaya and Mohammed [12];Yahaya and Mohammed [13]; Semenov et al. [14]). The

method of collocation and interpolation of the power series approximate solution to generate continuous LMM has been adopted by many researchers among them are (Houwen et al.[15]; Fatunla [16]; Awoyemi [17]; Jiaxiang eta al.[2]).
The paper is presented as follows: In section 2, we discuss the basic idea behind the algorithm and obtain a continuous representation *Y*(*x*) for the exact solution *y*(*x*) which is used to generate members of the block method for solving IVPs. In section 3, we briefly discuss the order and error constant and convergence analysis of the method. Finally, we present numerical results and concluding remarks.

DEVELOPMENT OF METHODS

The mathematical formulation of physical phenomena in science and engineering often leads to initial value problems of the form:

$$y'' = f(x, y', y), \quad y(a) = y_0 \ , \ y'(a) = \eta_0 \qquad \textbf{(1)}$$

We seek an approximation of the form:

$$Y(x) = \sum_{j=0}^{r+s-1} \ell_j x^j \qquad \textbf{(2)}$$

Where $x \in [a,b], \ell_j$ are unknown coefficients to be determined and $1 \le r \prec k$ and $s \succ 0$ are the number of interpolation and collocation points respectively. We then construct our continuous approximation by imposing the following conditions:

$$Y(x) = y_{n+j}, \ j = 0,1,2,\ldots,.k-1 \qquad \textbf{(3)}$$

$$Y''(x_{n+k}) = f_{n+k} \qquad \textbf{(4)}$$

We note that $y_{n+\mu}$ is the numerical approximation to the analytical solution $y(x_{n+\mu}), f_{n+\mu} = f(x_{n+\mu}, y_{n+\mu}, y'_{n+\mu})$.

Equations (3) and (4) lead to a system of (*k*+1) equations which is solved by Cramer's rule to obtain ℓ_j. Our continuous approximation is constructed by substituting the values ℓ_j into equation (2). After some manipulation, the continuous method is expressed as:

$$Y(x) = \sum_{j=0}^{k-1} \alpha_j(x) y_{n+j} + \alpha_\mu(x) y_{n+\mu} + h^2 \beta_k(x) f_{n+k} \qquad \textbf{(5)}$$

Where $\alpha_j(x), \beta_k(x)$ and $\alpha_\mu(x)$ are continuous coefficients. We note that since the general second order ordinary differential equation involves the first derivative, the first derivative formula:

$$Y'(x) = \frac{1}{h}\left(\sum_{j=0}^{k-1} \alpha'_j(x) y_{n+j} + \alpha'_\mu(x) y_{n+\mu} + h^2 \beta'_k(x) f_{n+k} \right) \qquad \textbf{(6)}$$

$$Y'(x) = \delta(x) \qquad \textbf{(7)}$$

$$Y'(a) = \delta_0 \qquad \textbf{(8)}$$

Five Step Methods with One-Off-Step Point

To derive this methods, we use Equation (5) to obtained a continuous 5-step HBDF method with the following specification : r=6,s=1,k=5. We also express $\alpha_j(x), \alpha_\mu(x)$ and $\beta_k(x)$ as a functions of t, where $t = \dfrac{x - x_n}{h}$ to obtain the continuous form as follows:

$$y(x) = \alpha_0 y_n + \alpha_1 y_{n+1} + \alpha_2 y_{n+2} + \alpha_3 y_{n+3} + \alpha_4 y_{n+4} + \alpha_{\frac{9}{2}} y_{n+\frac{9}{2}} + h^2 \beta_5 f_{n+5} \qquad \textbf{(9)}$$

Where:

$$\alpha_0(t) = 1 - \frac{69019}{27828}t + \frac{388025}{166968}t^2 - \frac{179705}{166968}t^3 + \frac{44017}{166968}t^4 - \frac{5461}{166968}t^5 + \frac{5}{3092}t^6$$

$$\alpha_1(t) = \frac{33696}{5411}t - \frac{49860}{5411}t^2 + \frac{28409}{5411}t^3 - \frac{46951}{32466}t^4 + \frac{2091}{10822}t^5 - \frac{163}{16233}t^6$$

$$\alpha_2(t) = -\frac{31941}{3865}t + \frac{50565}{3092}t^2 - \frac{34021}{3092}t^3 + \frac{10425}{3092}t^4 - \frac{7491}{15460}t^5 + \frac{41}{1546}t^6$$

$\alpha_3(t)=\frac{6368}{773}t-\frac{122860}{6957}t^2+\frac{91819}{6957}t^3-\frac{61459}{13914}t^4+\frac{9463}{13914}t^5-\frac{91}{2319}t^6$ $\alpha_{\frac{9}{2}}(t)=\frac{716288}{243495}t-\frac{964480}{146097}t^2+\frac{770752}{146097}t^3-\frac{279872}{146097}t^4+\frac{233536}{730485}t^5-\frac{320}{16233}t^6$

$\alpha_4(t)=-\frac{20601}{3092}t+\frac{91515}{6184}t^2-\frac{72011}{6184}t^3+\frac{76873}{18552}t^4-\frac{4179}{6184}t^5+\frac{379}{9276}t^6$ $\beta_5(t)=\frac{h^2}{1546}\left(-216t+498t^2-415t^3+160t^4-29t^5+2t^6\right)$

Evaluating (9) at $x=x_{n+5}$ yields Hybrid Five step implicit method:

$$y_{n+5}=-\frac{98}{6957}y_n+\frac{605}{5411}y_{n+1}-\frac{316}{773}y_{n+2}+\frac{2270}{2319}y_{n+3}-\frac{1970}{773}y_{n+4}+\frac{140288}{48699}y_{n+\frac{9}{2}}+\frac{60}{773}h^2 f_{n+5} \quad \textbf{(10)}$$

Taking the second derivative of Equation (9), thereafter, evaluating the resulting continuous polynomial solution at $x=x_{n+2}, x=x_{n+3}, x=x_{n+4}, x=x_{n+\frac{9}{2}}$ we generate four additional methods:

$$y_{n+2}=-\frac{653}{21222}y_n+\frac{4520}{8253}y_{n+1}+\frac{1576}{3537}y_{n+3}+\frac{251}{2358}y_{n+4}-\frac{5120}{74277}y_{n+\frac{9}{2}}+\frac{1546}{3537}h^2 f_{n+2}+\frac{16}{3537}h^2 f_{n+5}$$

(11)

$$y_{n+3}=\frac{589}{120864}y_n-\frac{1053}{17626}y_{n+1}+\frac{10989}{20144}y_{n+2}+\frac{26055}{40288}y_{n+4}-\frac{3632}{26439}y_{n+\frac{9}{2}}-\frac{6957}{20144}h^2 f_{n+3}+\frac{27}{20144}h^2 f_{n+5}$$

(12)

$$y_{n+4}=-\frac{1043}{459027}y_n+\frac{832}{39669}y_{n+1}-\frac{190}{1889}y_{n+2}+\frac{70208}{153009}y_{n+3}+\frac{2001920}{3213189}y_{n+\frac{9}{2}}-\frac{3092}{17001}h^2 f_{n+4}-\frac{8}{17001}h^2 f_{n+5}$$

(13)

$$y_{n+\frac{9}{2}}=\frac{308567}{18141440}y_n-\frac{122229}{907072}y_{n+1}+\frac{4443579}{9070720}y_{n+2}-\frac{5186391}{4535360}y_{n+3}+\frac{32135859}{18141440}y_{n+4}+\frac{146097}{566920}h^2 f_{n+\frac{9}{2}}-\frac{319221}{4535360}h^2 f_{n+5}$$

(14)

Since our method is design to simultaneously provide the values of:

$y_{n+1}, y_{n+2}, y_{n+3}, y_{n+4}, y_{n+\frac{9}{2}}, y_{n+5}$ at a block point $x_{n+1}, x_{n+2}, x_{n+3}, x_{n+4}, x_{n+\frac{9}{2}}, x_{n+5}$,
the five Equations (10)-(14) are not sufficient to provide the solution for six unknown

$y_{n+1}, y_{n+2}, y_{n+3}, y_{n+4}, y_{n+\frac{9}{2}}, y_{n+5}$. Thus, we obtain an additional method from (8), given by:

$$973980h\delta_0+2415665y_0-6065280y_1+8049132y_2-8023680y_3+6489315y_4-2865152y_{\frac{9}{2}}=-136080h^2 f_5$$

(15)

The derivatives are obtained from (7) by imposing that $\delta(x_{n+\mu})=\delta_{n+\mu}, \mu=\{j,v\}, j=0,\ldots 5,$ thus, we have:

$$h\delta_{n+1} = -\frac{2233}{13914}y_n - \frac{43177}{32466}y_{n+1} + \frac{10269}{3865}y_{n+2} - \frac{9233}{4638}y_{n+3} + \frac{6685}{4638}y_{n+4} - \frac{150272}{243495}y_{n+\frac{9}{2}} + \frac{21}{773}h^2 f_{n+5}$$

$$h\delta_{n+2} = \frac{2515}{83484}y_n - \frac{6100}{16233}y_{n+1} - \frac{2571}{3865}y_{n+2} + \frac{10460}{6957}y_{n+3} - \frac{7655}{9267}y_{n+4} + \frac{242944}{730485}y_{n+\frac{9}{2}} - \frac{10}{773}h^2 f_{n+5}$$

$$h\delta_{n+3} = -\frac{92}{6957}y_n + \frac{1341}{10822}y_{n+1} - \frac{2556}{3865}y_{n+2} - \frac{865}{4638}y_{n+3} + \frac{864}{773}y_{n+4} - \frac{92672}{243495}y_{n+\frac{9}{2}} + \frac{9}{773}h^2 f_{n+5}$$

$$h\delta_{n+4} = \frac{223}{27828}y_n - \frac{1136}{16233}y_{n+1} + \frac{1089}{3865}y_{n+2} - \frac{200}{2319}y_{n+3} - \frac{3775}{9276}y_{n+4} + \frac{255488}{243495}y_{n+\frac{9}{2}} - \frac{12}{773}h^2 f_{n+5}$$

$$h\delta_{n+\frac{9}{2}} = -\frac{11585}{890496}y_n + \frac{18225}{173152}y_{n+1} - \frac{98469}{247360}y_{n+2} + \frac{25445}{24736}y_{n+3} - \frac{367605}{98944}y_{n+4} + \frac{728648}{243495}y_{n+\frac{9}{2}} + \frac{945}{24736}h^2 f_{n+5}$$

$$h\delta_{n+5} = -\frac{1591}{41742}y_n + \frac{1393}{4638}y_{n+1} - \frac{4191}{3865}y_{n+2} + \frac{35149}{13914}y_{n+3} - \frac{27439}{4638}y_{n+4} + \frac{439552}{104355}y_{n+\frac{9}{2}} + \frac{257}{773}h^2 f_{n+5}$$

Error Analysis and Zero Stability

Following Fatunla [16] and (Lambert [18] we define the local truncation error associated with the conventional form of (5) to be the linear difference operator:

$$L[y(x);h] = \sum_{j=0}^{k}\{\alpha_j y(x+jh)\} + \alpha_v y(x+vh) + h^2\beta_v y''(x+jh)$$

(16)

Assuming that y(x) is sufficiently differentiable, we can expand the terms in (16) as a Taylor series about the point x to obtain the expression:

$$L[y(x);h] = C_0 y(x) + C_1 hy' + ..., + C_q h^q y^q(x) + ...,$$

(17)

where the constant coefficients $C_q, \quad q = 0,1,...$ are given as follows: $C_q, \quad q = 0,1,...$

$$C_0 = \sum_{j=0}^{k}\alpha_j,$$

$$C_1 = \sum_{j=1}^{k} j\alpha_j,$$

.

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$$.C_q = \left[\frac{1}{q!}\sum_{j=1}^{k} j^q \alpha_j - q(q-1)\sum_{j=1}^{k} j^{q-2}\beta_j\right].$$

According to (Henrici, 1962), method (5) has order p if:

$$C_0 = C_1 = ... = C_P = C_{P+1} = 0, \quad C_{P+2} \neq 0$$

Therefore, C_{p+2} is the error constant and $C_{p+2}h^{p+2}y^{(p+2)}(x_n)$ the principal local truncation error at the point x_n. It is establish from our calculations that the HBDF have higher order and relatively small error constants as displayed in the Table 1.

Table 1: Order and Error Constants for the HBDF Methods.

Step number	Method	order	Error constant
5	(10)	5	$-\frac{257}{32460}$
	(11)	5	$\frac{3177}{5}$
	(12)	5	$\frac{11601}{20}$
	(13)	5	$-\frac{23967}{10}$
	(14)	5	$-\frac{1733553}{10}$
	(14)	5	$\frac{13137}{108220}$

In order to analyze the methods for zero stability, we normalize the HBDF schemes and write them as a block method from which we obtain the first characteristic polynomial $\rho(R)$ given by:

$$\rho(R) = \det\left(RA^{(0)} - A^{(1)}\right) = R^k(R-1) \quad \textbf{(18)}$$

Where $A^{(0)}$ is the identity matrix of dimension k+1, $A^{(1)}$ is the matrix of dimension k+1

Case k=5. It is easily shown that (10)-(15) are normalized to give the first characteristic polynomial $\rho(R)$ given by:

$$\rho(R) = \det\left(RA^{(0)} - A^{(1)}\right) = R^5(R-1)$$

Where $A^{(0)}$ an identity matrix of is dimension six and $A^{(1)}$ is a matrix of dimension six given by:

$$A^{(1)} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Following Fatunla [16] the block method by combining k+1 HBDF is zero-stable, since from (18), $\rho(R) = 0$ satisfy $|R_j| \le 1 \; j = 1\ldots, k$ and for those roots with $|R_j|$=1, the multiplicity does not exceed 2. The block method by combining k+1 HBDF is consistent since HBDF have order $P > 1$. According to Henrici [19], we can safely ascertain the convergence of HBDF method.

Numerical Examples:

The HBDF methods are implemented as simultaneously numerical integration for IVPs without requiring starting values and predictors. We proceed by explicitly obtaining initial conditions at x_{n+k}, n=0,k,….,N-k using the computed values $Y(x_{n\_k}) = y_{n+k}$ and $\delta(x_{n\_k}) = \delta_{n+k}$ over sub-intervals $[x_0, x_k], \ldots, [x_{N-K}, x_N]$ which do not overlap. We give examples to illustrate the efficiency of the methods. We report here a numerical example taken from the literature.

Problem 1

$y'' - y' = 0,\; y(0) = 0, y'(0) = -1, h = 0.1$

Exact Solution $y(x) = 1 - e^x$

Source: Mohammed [6]

Problem 2

$y'' + 1001y' + 1000y = 0,\; y(0) = 1, y'(0) = -1, h = 0.1$

Exact Solution $y(x) = e^{-x}$

Source: Jator [20]

Table 2: Showing Exact Solutions and the Computed Results from the Proposed Methods for Problem 1.

x	Exact Solution	Proposed Method
0	0	0
0.1	-0.105170918	-0.1051711184
0.2	-0.221402758	- 0.2214032966
0.3	-0.349858808	- 0.3498596920
0.4	-0.491824698	- 0.4918259277
0.5	-0.648721271	- 0.6487228462
0.6	-0.822118800	- 0.8221207204
0.7	-1.013752707	- 1.013755213
0.8	-1.225540928	- 1.225544034
0.9	-1.459603111	- 1.459606816
1.0	-1.718281828	- 1.718286132

Table 3: Comparing the Absolute Errors for Five Step (HBDF) to Errors in Mohammed [6] for Problem 1.

x	Error in Proposed Method	Error in K=5(BDF) Mohammed [6]
0	0.000000000E+00	0.000000000E+00
0.1	2.004000000E-07	2.198000000E-05
0.2	5.386000000E-07	6.070400000E-06
0.3	8.840000000E-07	1.005100000E-05
0.4	1.229700000E-06	1.402530000E-05
0.5	1.575200000E-06	1.799340000E-05
0.6	1.920400000E-06	2.161620000E-05
0.7	2.506000000E-06	2.799300000E-05
0.8	3.106000000E-06	3.456100000E-05
0.9	3.705000000E-06	4.111400000E-05
1.0	4.304000000E-06	4.765600000E-05

Table 4: Showing Exact Solutions and the Computed Results from the Proposed Methods for Problem 2.

x	Exact Solution	Proposed Method
0	1	1
0.1	0.9048374180	0.9048374165
0.2	0.8187307531	0.8187307535
0.3	0.7408182207	0.7408182212
0.4	0.6703200460	0.6703200458
0.5	0.6065306597	0.6065306596
0.6	0.5488116361	0.5488116337
0.7	0.4965853038	0.4965853041
0.8	0.4493289641	0.4493289647
0.9	0.4065696597	0.4065696591
1.0	0.36787944	0.3678794413

Table 5: Comparing the Absolute Errors for Five Step (HBDF) to Errors in Jator (2007) for Problem 2.

x	Error in K=5 Proposed Method	Error in K=2 (BDF) Jator (2007)	Error in K=3 (BDF) Jator (2007)
0	0.000000000E+00	0.000000000E+00	0.000000000E+00
0.1	1.499999902E-09	2.940180000E-04	1.111124000E-05
0.2	4.000000331E-10	5.571550000E-04	5.749050000E-05
0.3	4.999999303E-10	7.512790000E-04	9.210130000E-05
0.4	1.999999055E-10	9.202740000E-04	4.078390000E-05
0.5	1.000000083E-10	10.29514000E-04	2.530190000E-05
0.6	2.400000088E-09	11.26415000E-04	4.725860000E-05
0.7	3.000000248E-10	11.80252000E-04	1.893470000E-05
0.8	5.999999941E-10	12.27376000E-04	4.288120008E-05
0.9	5.999999941E-10	12.42326000E-04	7.966800000E-05
1.0	1.299999997E-09	12.54553000E-04	2.941190000E-05

CONCLUSION

A Collocation technique which yields a method with Continuous Coefficients has been presented for the approximate Solution of second Order ODEs with initial conditions. Two test examples have been solved to demonstrate the efficiency of the proposed methods and the Results Compare Favourably with the exact Solution and existing methods in the literature. Interestingly, all the discrete schemes used in the Block formulation were from a single continuous formulation (CF).

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