Journal of the Nigerian Association of Mathematical Physics
Volume 46 (*May, 2018 Issue*), pp15 –22
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IRREVERSIBILITY RATIO OF A REACTIVE MHD THIRD GRADE FLUID IN A CONVECTIVECIRCULAR PIPE WITH VARIABLE THERMAL CONDUCTIVITY AND RADIALLY APPLIED MAGNETIC FIELD

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Abstract

The collective effects of variable thermal conductivity and radially applied magnetic field on irreversibility ratio of a steady reactive third grade magnetohydrodynamic fluid flowing through a uniformly circular pipe with convective boundary condition is presented. The governing equations were obtainable and the resulting non-linear dimensionless equations were solved numerically using Galerkin Weighted Residual Method. The entropy number was computed from the obtained velocity, temperature and concentration profile. A parametric study of all parameters involved are presented graphically and discussed. It was observed that irreversibility due to heat transfer dominates the flow compared to fluid friction, magnetic parameter, Dufourand Biot numbers inhibits the Bejan number while Brickman number and thermal conductivity, Joule heating parameter enhances the Bejan number.

Keywords: Magnetohydrodynamics, Thermal conductivity, Irreversibility ratio, Bejan number.

1.0 Introduction

A reasonable interest has been shown on the study of magnetohydrodynamics (MHD) flows incylindrical geometry. This interest is owned to the numerous important applications in biological and engineering industry such as reactive polymer flows, extraction of crude oil, synthetic fibres, paper production and also in absorption and filtration processes in chemical engineering [1]. The dynamics of reactive fluids through pipe at low Reynolds numbers has long been an important subject in the area of environmental engineering and science.

The steady flow of a reactive variable viscosity fluid in a cylindrical pipe with isothermal wall was studied in[2], reporting the dependence of the steady state thermal ignition criticality conditions on both Frank-Kamenetskii and viscous heating parameters. In[3], the closed-form solution using Homotopy Analysis method on the effect of variable viscosity and viscous dissipation on the thermal stability of a one-step exothermic reactive non-Newtonian flow in a cylindrical pipe assuming negligible reactant consumption was obtained. The effect of radiation on unsteady MHD flow of a chemically reacting fluid past a hot vertical porous plate using finite difference approach was examined in [4].

Thermal conductivity describes the tendency of a material to transfer heat and it plays a significant role in cooling systems which can be affected by thickness and temperature. The studies in [5], portrayed the effect of variable thermal conductivity,

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non-uniform heat source and dissipative heat in the presence of thermal radiation on flow and heat transfer in viscoelastic fluid over a stretching sheet with external magnetic field. It was noted that radiation accelerated the heat transfer. Thus radiation should be at its minimum in order to facilitate the cooling process. The researchers in [6],considered a viscoelastic model over a stretching plate and heat transfer with variable thermal conductivity for prescribed surface temperature and prescribed heat flux.A two dimensional non-Newtonian second grade fluid in [7] was studiedunder the influence of temperature dependent viscosity and thermal conductivity. In [8], an investigation into the effects of variable viscosity and thermal conductivity on MHD flow past a vertical plate were carried out. It was observed that the velocity profile increases with decrease of thermal conductivity parameter.

The thermodynamics second law analysis and its design-related concept of entropy generation minimization has been a cornerstone in the field transfer and thermal design. Several researchers were motivated to study fundamental and applied engineering problem based on second law analyses, due to the production of entropy resulting from combined effects of velocity and temperature gradient. Generating entropy is tied to thermodynamic irreversibility, which is common in all heat transfer process. Entropy generation rate in [9] were considered for a fluid-solid mixture flow in a pipe with Vogel viscosity model. The combined effects of Navier slip, convective cooling, variable viscosity and suction/injection on the entropy generation rate in an unsteady flow of an incompressible viscous fluid flowing through a channel with permeable wall was studied in [10].

In this paper, the motivation comes from a desire to gain more understanding into the combined effect of radially applied magnetic field and Hall current on the flow of chemically reactive third grade fluid. The relevant governing equation have been solved numerically by Galerkin Weighted Residual Method [11,12]. The effects of the various apposite parameters on the velocity, temperature and concentration are presented. In this work, entropy generation rate of a laminar MHD flow of a reactive third grade fluid is considered in a circular pipe, which is assumed electrically conducting and incompressible in the presence of an externally applied radially exponential magnetic field.

2.0 Mathematical Formulation

Considering a steady flow of electrically conducting, incompressible, third grade fluid in a non-conducting circular pipe in the absence of gravitational force. The z-axis is taken along the axis of flow. Radially exponential varying magnetic field $B_r = B_0 e^{-2R}$ is applied [13] and no electric field is applied. The flow is induced due to constant applied pressure gradient in the z-direction and electron atom collision frequency is assumed to be relatively high compared to the collision frequency of ions. The equations which governthe MHD flow are the continuity, momentum and Maxwell equations. In fluid dynamics studies, it is assumed that the Clausius-Duhem inequality and the specific Helmholtz free energy of fluid has a minimum at equilibrium holds [14]. The fluid thermal conductivity assumes the linear temperature function

$$
\begin{aligned}
k_r &= k_0(1+\alpha(T-T_0)) \\
&= k_0(1+\varepsilon\theta(\eta))
\end{aligned}
\qquad (2.1)
$$

where $\varepsilon = \alpha(T_w - T_0)$, α is a thermo-physical constant dependent on the fluid. Using the velocity field $V = (0,\ 0,\ w(r))$ as in[15, 16]and the stated assumptions give

$$\frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\mu\frac{\partial w}{\partial r}\right)+2\beta_3\frac{\partial}{\partial r}\left(r\left(\frac{\partial w}{\partial r}\right)^3\right)\right]-\frac{\partial p}{\partial z}-\sigma B_r^2 w=0 \qquad (2.2)$$

$$\frac{\mu}{\rho c_p}\left(\frac{\partial w}{\partial r}\right)^2+\frac{2\beta_3}{\rho c_p}\left(\frac{\partial w}{\partial r}\right)^4+\frac{k_r}{\rho c_p}\left[\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right]+\frac{Q_0}{\rho c_p}(T-T_0)-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial r}+\frac{\sigma B_r^2 w^2}{\rho c_p} \qquad (2.3)$$

$$+\frac{D_m Z_T}{\rho c_p c_s}\left(\frac{\partial^2 C}{\partial r^2}+\frac{1}{r}\frac{\partial C}{\partial r}\right)=0$$

$$u\frac{\partial C}{\partial r}=D_m\left(\frac{\partial^2 C}{\partial r^2}+\frac{1}{r}\frac{\partial C}{\partial r}\right)-k_c(C-C_0) \qquad (2.4)$$

with boundary conditions

$$
\begin{aligned}
&\frac{\partial w}{\partial r}=0,\quad T(r)=T_0,\qquad C(r)=C_0 \quad \text{at} \quad r=0 \\
&w(r)=0,\quad k\frac{dT}{dr}=-h(T-T_a),\ C(r)=C_w \quad \text{at} \quad r=R
\end{aligned}
\qquad (2.5)
$$

where $w, T, B_0, \hat{p} = -p + \alpha\left(\frac{dw}{dr}\right)^2, \sigma, k_0, q, D_m, \lambda_T, c_p, k_c, T_0, T_w, C_0, C_w$ are fluid velocity, fluid temperature, applied magnetic field strength, modified pressure, electrical conductivity, reference thermal conductivity, thermal radiation, molecular diffusivity, thermal diffusivity, specific heat capacity, chemical reaction rate constant, reference temperature, wall temperature, reference concentration and wall concentration.

Introducing the following non-dimensional quantities [17] into (2.2) to (2.5) and the boundary conditions

$w = w_0\bar{w},\quad r = R\eta,\quad T = (T_w - T_0)\theta + T_0,\quad C = (C_w - C_0)\chi + C_0$

$$\Lambda = \frac{2\beta_3 w_0^2}{\mu R^2}, c = \frac{R^2}{\mu w_0}\left(\frac{\partial \bar{p}}{\partial z}\right), M = \frac{\sigma B_0^2 R^2}{\mu}, Q_H = \frac{Q_s R}{k_0}, B_r = \frac{\mu w_0^2}{k_0(T_w - T_0)} \tag{2.6}$$

$$J_H = \frac{\sigma B_0^2 w_0^2}{k_0(T_w - T_0)}, K_R = \frac{K_c R^2}{D_m}, D_u = \frac{D_m \lambda_T (C_w - C_0)}{k_0 c_p (T_w - T_0)}, R_p = \frac{16\sigma_* T_0^3}{3\delta_* R^2}, S_c = \frac{w_0 R}{D_m}$$

and using Rosselands approximation

$$q_r = -\frac{4\sigma_*}{3\delta_*}\frac{\partial T^4}{\partial r} \tag{2.7}$$

$\Lambda, M, c, P_r, E_c, Q_H, \beta_*, D_u, R_p, S_c, K_R, \sigma_*, \delta_*$ denotes third grade parameter, magnetic parameter, pressure drop, Prandtl number, Eckert number, heat source/sink parameter, material constant parameter, Dufour number, radiation parameter, Schmidt number, chemical reaction parameter, Stefan-Boltzmann constant and mean absorption coefficient. For steady flow, the time dependent terms are set to zero and the following are equations were obtained respectively with the boundary conditions

$$\frac{d^2w}{d\eta^2} + \frac{1}{\eta}\frac{dw}{d\eta} + \frac{\Lambda}{\eta}\left(\frac{dw}{d\eta}\right)^3 + 3\Lambda\left(\frac{dw}{d\eta}\right)^2\frac{d^2w}{d\eta^2} - c - Me^{\eta}w = 0 \tag{2.8}$$

$$(1+R_p)\frac{d^2\theta}{d\eta^2} + \frac{1}{\eta}\frac{d\theta}{d\eta} + Br(1-\varepsilon\theta)\left(\frac{dw}{d\eta}\right)^2 + Br\Lambda(1-\varepsilon\theta)\left(\frac{dw}{d\eta}\right)^4 + Q_H(1-\varepsilon\theta)\theta \tag{2.9}$$

$$-J_H(1-\varepsilon\theta)e_c^{\eta}w^2 - R_p\varepsilon\theta\frac{d^2\theta}{d\eta^2} + D_u(1-\varepsilon\theta)\frac{d^2\chi}{d\eta^2} + \frac{D_u(1-\varepsilon\theta)}{\eta}\frac{d\chi}{d\eta} = 0$$

$$\frac{d^2\chi}{d\eta^2} = S_c w\frac{d\chi}{d\eta} - \frac{1}{\eta}\frac{d\chi}{d\eta} - K_R\chi \tag{2.10}$$

Boundary conditions

$$\frac{dw}{d\eta} = 0,\quad \theta(\eta) = 0,\qquad \chi(\eta) = 0 \quad \text{at} \quad \eta = 0$$
$$w(\eta) = 0,\quad \frac{d\theta}{d\eta} = -Bi(\theta - \theta_a),\quad \chi(\eta) = 1 \quad \text{at} \quad \eta = 1 \tag{2.11}$$

Equations (2.8), (2.9), (2.10) and (2.11) comprise the boundary value problem to now be solved.

3.0 Method of Solution (Galerkin Weighted Residual)

Suppose an approximate solution is to be determined for the differential equation of the form

$$L(\phi) + f = 0 \tag{3.1}$$

where $\phi(x)$ is an unknown dependent variable, L is a differential operator and $f(x)$ is a known function. Let $\psi(x) = \sum_{i=1}^{N} c_i u_i(x)$ be an approximate solution to (2.8). On substituting $\psi(x)$ into (2.8), it is unlikely that (2.8) is satisfied i.e.

$L(\psi) + f \neq 0$ therefore

$$L(\psi) + f = R \tag{3.2}$$

where $R(x)$ is a measure of error called the Residual [12, 18]. Multiplying (3.2) by an arbitrary weight function $u(x)$ and integrating over the domain to obtain

$$\int_D u(x)\left[L(\psi) + f\right]dD = \int_D u(x)R(x)dD \neq 0 \tag{3.3}$$

Galerkin Weighted Residual method ensures equation (3.3) vanishes over the solution domain and the weight function is choosing from the basis functions $u(x) = u_i(x) \quad (i = 0, \ldots, N)$ hence

(3.4)

$$(u,R) = \int_D u(x)R(x)dD = \int_D u(x)\left[L\left(u_0(x) + \sum_{i=1}^{n} c_i u_i(x)\right) - f\right]dD = 0$$

These are a set of n-order linear equations to be solved to obtain all the c_i coefficients. The trial functions can be polynomials, trigonometric functions etc. The trial functions are usually chosen in such that the assumed function $\psi(x)$ satisfies the global boundary conditions for $\phi(x)$ though this is not strictly necessary and certainly not always possible [11]. To apply the method to (2.8)-(2.10), we select an approximate solutions of the form $\psi_u(\eta) = a_0 + a_1\eta + a_2\eta^2$, $\psi_\theta(\eta) = b_0 + b_1\eta + b_2\eta^2$, $\psi_\chi(\eta) = c_0 + c_1\eta + c_2\eta^2$ for the velocity, temperature and concentration respectively, which satisfies the boundary conditions (2.11). Applying the boundary conditions on the approximate solution we obtain the following:

$u(\eta) = a_0(1-\eta^2)$, $\theta(\eta) = \dfrac{Bi\theta a\eta^2}{Bi+2} + b_1\left(\eta - \dfrac{(Bi+1)\eta^2}{Bi+2}\right)$, $\chi(\eta) = \eta^2 + c_1(\eta - \eta^2)$ and $u_1 = (1-\eta^2)$, $u_2 = \left(\eta - \dfrac{(Bi+1)\eta^2}{Bi+2}\right)$, $u_3 = (\eta - \eta^2)$ are the weighting functions u_i, where a_1, b_1, c_1 are the coefficients to be determined.

The residue R for (2.7)-(2.9) respectively and taking into account the orthogonality of the residues, we have are given by

(3.5)

$$21Ma_0 - 8Mca_0 + \frac{2}{3} - \frac{64\Lambda a_0^3}{15} - \frac{8a_0}{3} = 0$$

$$\frac{1}{1260(Bi+2)^3}\Big(-2280960(Bi+2)J_H a_0^2\Big(\big(\frac{6}{377} + (b_1 - \frac{335}{377}\vartheta_a)\varepsilon\big)Bi^2 + \big(\frac{113}{2262} + (\frac{838}{377}b_1 - \frac{2231}{2262}\vartheta_a)\varepsilon\big)Bi + \frac{2797}{2262}\varepsilon b_1 + \frac{41}{1131}\Big)\varepsilon + (((-9Q_H + 84R_p)b_1^2 + (-80\Lambda Bra_0^4 + (-48Br$$
$$+61979400J_H)a_0^2 + (-24Q_H + 42R_p)\vartheta_a + (63c_1 - 168)Du)b_1 + 147\vartheta_a(-\frac{40}{21}\Lambda Bra_0^4$$
$$+(42Q_H - 420R_p - 210)b_1 + 480\Lambda Bra_0^4 + (252Br + 986580J_H)a_0^2 + (63Q_H + 420R_p$$
$$+840)\vartheta_a - 210Du(c_1 - 4)Bi^3 + (((-9Q_H + 672R_p)b_1^2 + (-1040\Lambda Bra_0^4 + (-528Br$$
$$+261727200J_H)a_0^2 + (-216Q_H + 294R_p)\vartheta_a + (672c_1 - 1512)Du)b_1 + 1281\vartheta_a$$
$$(-\frac{160}{61}\Lambda Bra_0^4 + (-\frac{400}{427}Br - \frac{8156160}{61}J_H)a_0^2 + (-\frac{80}{427}Q_H - \frac{36}{61}R_p)\vartheta_a + Du(c_1 - \frac{96}{61})$$
$$))\varepsilon + (378Q_H - 2940R_p - 1470)b_1 + 5760\Lambda Bra_0^4 + (2520Br + 5070240J_H)a_0^2 +$$
$$(504Q_H + 2520R_p + 5040)\vartheta_a + (-2310c_1 + 6720)Du)Bi^2 + (((-342Q_H + 1932R_p)b_1^2$$
$$+(-5440\Lambda Bra_0^4 + (-2256Br + 352175040J_H)a_0^2 + (-696Q_H - 588R_p)\vartheta_a + (2652c_1$$
$$-5040)Du)b_1 + 1974\vartheta_a(-\frac{400}{141}\Lambda Bra_0^4 + (-\frac{320}{329}Br - \frac{2910960}{47}J_H)a_0^2 + Du(c_1 - \frac{72}{47})))\varepsilon$$
$$+(1260Q_H - 5880R_p - 1680)b_1 + 17280\Lambda Bra_0^4 + (7056Br + 8442000J_H)a_0^2 + (756Q_H$$
$$+3360R_p + 6720)\vartheta_a - 6720(c_1 - \frac{5}{2})Du)Bi - 576((Q_H - \frac{7}{3}R_p)b_1 + \frac{115}{9}\Lambda Bra_0^4 + (\frac{29}{6}Br$$
$$-266105J_H)a_0^2 + Du(-\frac{511}{96}c_1 + \frac{28}{3}))b_1\varepsilon + (1344Q_H - 3360R_p + 480)b_1 + 15360\Lambda Bra_0^4$$
$$+(6048Br + 4495680J_H)a_0^2 - 5880(c_1 - \frac{15}{7})Du\Big)$$

(3.6)

(3.7)

$$\frac{1}{60S_c}\left((-2(2\Phi + 5)c_1 - 3\Phi + 40)S_c - a_0(c_1 + 6)\right) = 0$$

The symbolic calculation software MAPLE 2016 is used to compute the values of a_0, b_1, c_1 and the approximate velocity, temperature and concentration solutions.

4.0 Entropy Generation

Inherent irreversibility in a pipe flow occurs owing to exchange of energy and momentum within the fluid and the solid boundaries. The entropy generation is owed to heat transfer and the effects of fluid friction. The equation for rate of entropy generation per unit volume[10, 19] is given

$$S''' = \frac{k}{T_0^2}\left(\frac{dT}{dr}\right)^2 + \frac{\mu}{T_0}\left(\frac{dw}{dr}\right)^2 + \frac{2\beta_3}{T_0}\left(\frac{dw}{dr}\right)^4 + \frac{\sigma B_0^2 w^2}{T_0} \qquad (4.1)$$

where the first term in () is the irreversibility due to heat transfer,the second and third term are entropy generation due to viscous dissipation. Introducing the dimensionless quantities in (2.6) to (4.1), we have

$$N_s = \frac{r^2 S'''}{k} = \frac{\eta^2}{\Omega^2}\left(\frac{d\theta}{d\eta}\right)^2 + \frac{B_r \eta^2}{\Omega}\left(\frac{dw}{d\eta}\right)^2 + \frac{\beta_r \eta^2}{\Omega}\left(\frac{dw}{d\eta}\right)^4 + \frac{B_r M \eta^2 w^2}{\Omega} \qquad (4.2)$$

$$\Omega = \frac{T_0}{T_w - T_0},\ B_r = \frac{\mu w_0^2}{k(T_w - T_0)},\ \beta_r = \frac{\beta_3 w_0^4}{kR^2(T_w - T_0)}$$

are temperature difference parameter, Brickman number and third grade parameter and

$$N_1 = \frac{\eta^2}{\Omega^2}\left(\frac{d\theta}{d\eta}\right)^2,\ N_2 = \frac{B_R \eta^2}{\Omega}\left(\frac{dw}{d\eta}\right)^2 + \frac{\beta_r \eta^2}{\Omega}\left(\frac{dw}{d\eta}\right)^4 + \frac{B_r M \eta^2 w^2}{\Omega} \qquad (4.3)$$

N_1 denotes irreversibility due to heat transfer and N_2 gives entropy generation due to viscous dissipation and Joule heating. The Bejan number is defined as

$$B_e = \frac{N_1}{N_s} \qquad (4.4)$$

such that $0 \le B_e \le 1$ denoting $B_e = 1$ is the limit at which heat transfer irreversibility dominates , $B_e = 0$ is the limit at which fluid friction irreversibility dominates, and $1/2$ connotes equal contribution [20].

5.0 Results and Discussion

The dimensionless nonlinear coupled Eqs. (2.8)-(2.10) subject to boundary conditions (2.11), which describe heat and mass transfer flow in a cylindrical pipe in the presence of variable thermal conductivity, convective cooling and Joule heating under the influence of radially applied magnetic field are solved numerically by GWRM. In order to get physical insight to the problem, the influence of the following physical quantities on Bejan profile were plotted.

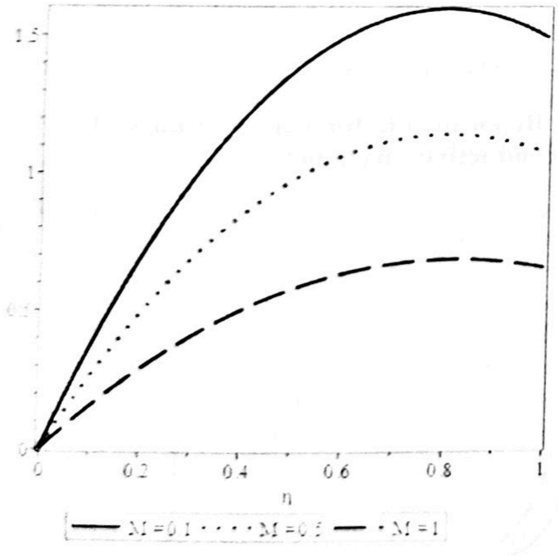

Figure 1: Temperature profile for various values of Magnetic parameter

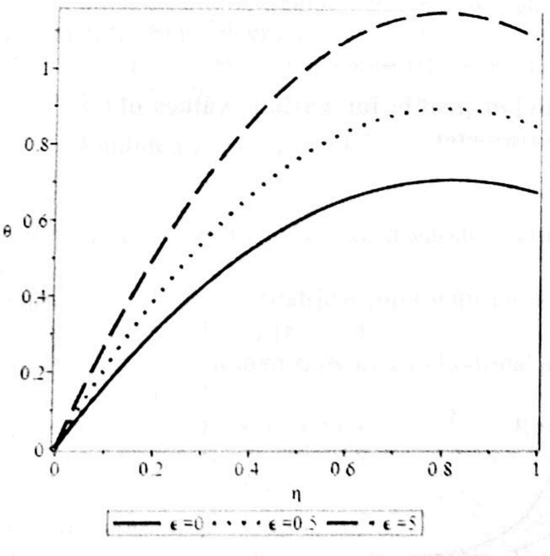

Figure 2: Temperature profile for various values of Thermal conductivity

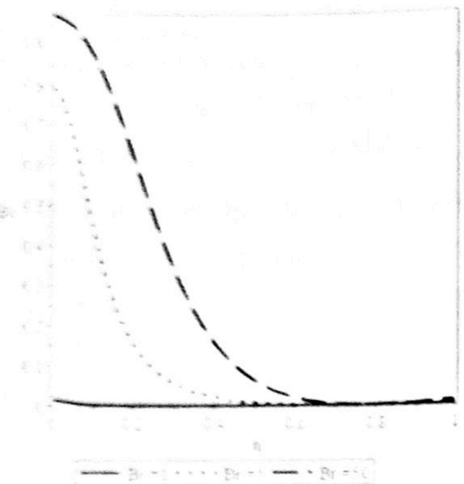

Figure 3: Temperature profile for various values of Biot number

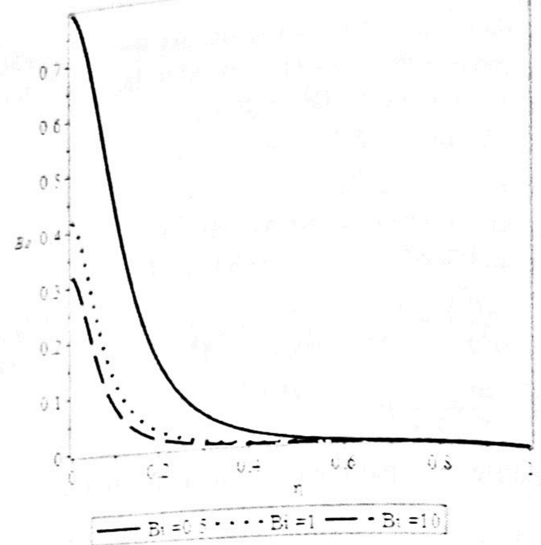

Figure 4: Temperature profile for various values of Joule heating parameter

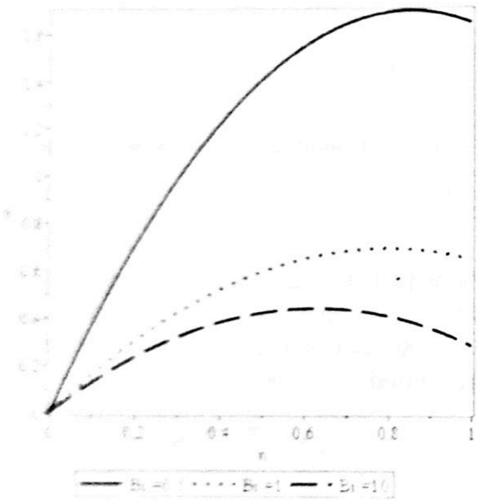

Figure 5: Bejan profile for various values of Magnetic parameter

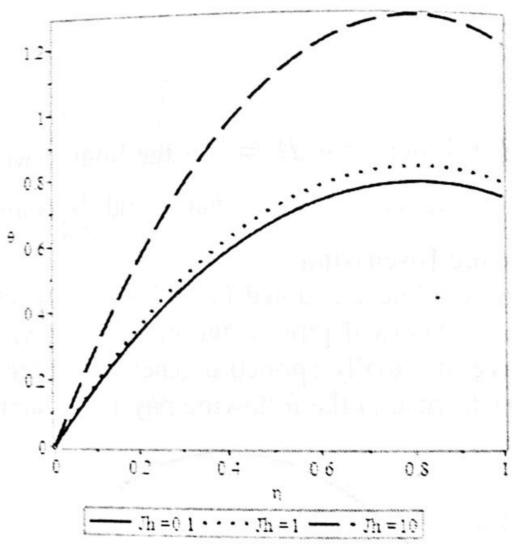

Figure 6: Bejan profile for various values of Thermal conductivity parameter

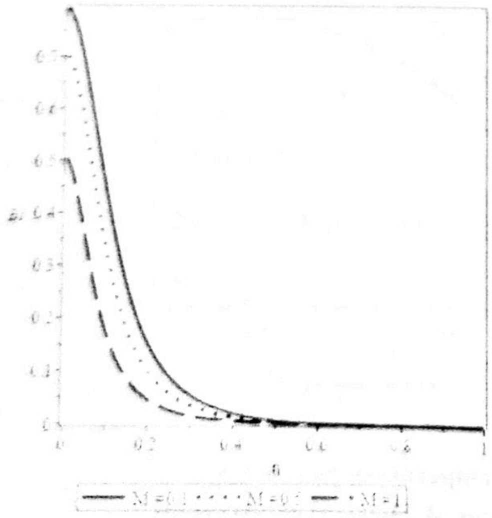

Figure 7: Bejan profile for various values of Brickman number

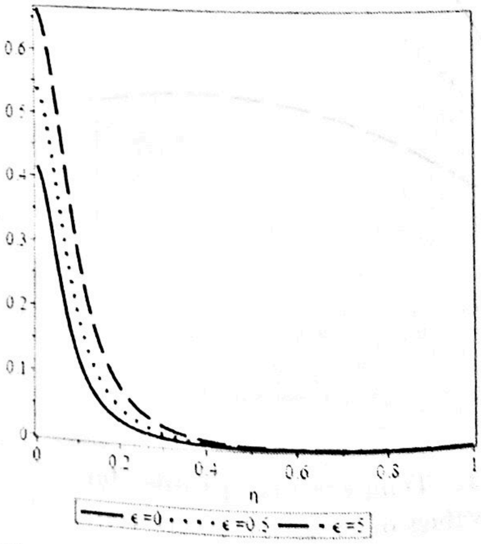

Figure 8: Bejan profile for various values of Biot number

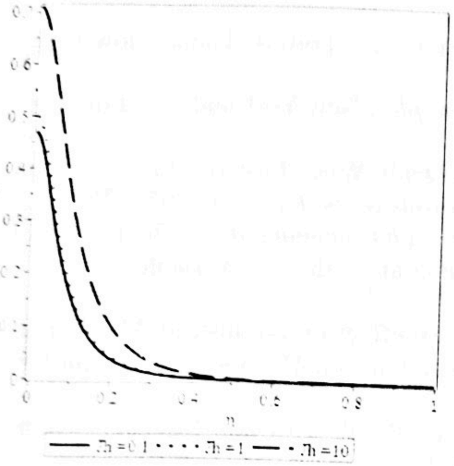

Figure 9: Bejan profile for various values of Joule Heating parameter

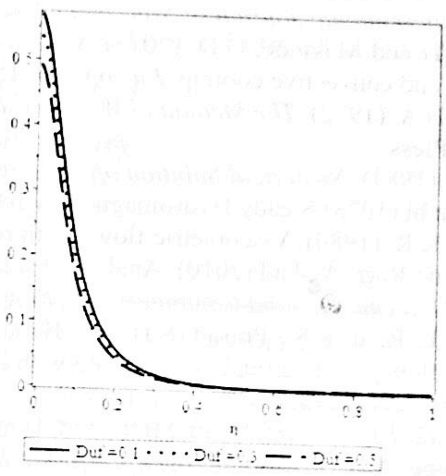

Figure 10: Bejan profile for various values of Dufour number

Figures 1 to 4presents the effect of magnetic parameter, thermal conductivity parameter, Biot number and Joule heating parameter on the temperature profile, increasing values of magnetic parameter and Biot number inhibits the temperature profile while increasing thermal conductivity and Joule heating parameters enhances the temperature profile. Figures 5 to 10 shows the effects of magnetic parameter, thermal conductivity parameter, Brickman number, Biot number, Joule heating parameter and Dufour number on the irreversibility ratio profile. Irreversibility due to heat transfer dominates from the pipe centreline towards the pipe wall while fluid friction dominates at the wall pipe. Decreasing values of magnetic parameter, Biotand Dufour numbers enhances the irreversibility ratio while increasing the thermal conductivity parameter, Brickman number and Joule heating parameter enhances the irreversibility ratio profile.

6. Conclusion

In this numerical investigation, the irreversibility ratio of steady reactive magneto hydrodynamic third grade fluid flow in a circular pipe is presented using the Galerkin method. Numerical expression for the velocity, temperature and concentration was obtained which were used to compute the entropy generation rate. Special emphasis has been focused here to the variations of pertinent parameter of physical significance on the Bejan number. The main findings of the present analysis are:

i. The temperature is enhanced for values of ε, J_H and inhibited for M, Bi

ii. ε, Br and J_H enhances the entropy generation rate while it is inhibited for M, Bi and Du.

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