



## Modelling of Vibration of Mobile Vehicle

**Abdulmalik D. M. <sup>a</sup>, Gursky N. N. <sup>b</sup>, Muhammad K. M. <sup>c</sup>, Ojerinde O. A. <sup>d</sup>, Ekundayo A. <sup>e</sup>**

<sup>a,c,d,e</sup> Department of Computer Science, School of Information and Communication Technology, Federal University of Technology, P.M.B 65, Minna, Niger State, Nigeria

<sup>b</sup> Department of Software Engineering, Belarussian National Technical University, Minsk, Belarus

Corresponding authors' email: [drmalik@futminna.edu.ng](mailto:drmalik@futminna.edu.ng), [nic\\_nic.by@mail.ru](mailto:nic_nic.by@mail.ru), [muhammad\\_kudu@futmnna.edu.ng](mailto:muhammad_kudu@futmnna.edu.ng), [o.ojerinde@futminna.edu.ng](mailto:o.ojerinde@futminna.edu.ng), [a.ekundayo@futminna.edu.ng](mailto:a.ekundayo@futminna.edu.ng)

### Article Info

#### Keywords:

*Object, Methods, Algorithms, Program, Multi-support mobile machine, Mathematical model, Calculation scheme, Models with passive and active control systems.*

Received 10 January 2021

Revised 22 January 2021

Accepted 24 January 2021

Available online 01 March 2021



<https://doi.org/10.37933/nipes/3.1.2021.5>

<https://nipesjournals.org.ng>

© 2021 NIPES Pub. All rights reserved.

### Abstract

*The movement of vehicle on uneven roads causes vibration of its sprung and unsprung parts. Vibrations of the sprung part of the car cause fatigue for drivers, creating uncomfortable and sometimes unsafe conditions for them. A systematic analysis of the vibration process occurring in a mobile vehicle can be helpful in evaluating the properties of the vehicle to protect the driver, passengers, transported goods and structural elements of the car from dynamic loads resulting from the interaction of wheels with road irregularities. This paper presents the mathematical model of vibration of mobile vehicle and the results of the computer simulation of vibrations of a mobile vehicle on different road surfaces in the MATLAB +Simulink software environment.*

## 1. Introduction

The study of vibration processes is aimed at finding out the general features and regularities of the processes in various dynamic systems and the conditions of their existence, i.e., the specific type of movements inherent in a certain class of systems. Such dynamic systems, in which vibration processes exist, are commonly called vibration systems [1]. Vibrations are repetitive limited movements relative to a certain average state, which in a particular case may be a state of equilibrium. All vibration processes can be classified according to their kinematic characteristics, i.e. by the nature of the law of change in time of a certain quantity involved in the process. It is almost impossible to name any branches of science and technology that do not have vibration processes. Radio engineering and electronics is entirely based on the use of vibration processes [2]. In mechanics, optics, acoustics and electrical engineering there exists vibration processes. The unbalance of rotating rotors causes vibrations of building structures; a number of technological machines, installations and tools are based on the use of vibrations. Even though the physical nature of the processes in which vibrations take place is different, the basic laws of vibration are the same in all cases.

The movement of a multi-wheeled vehicle on uneven roads causes vibrations of its sprung and unsprung parts [3,4]. Sprung parts include units and parts whose gravity is perceived by the springing system (suspension), i.e. the frame with all parts rigidly connected to it, and unsprung

parts – units and parts whose gravity is not perceived by the suspension, i.e. wheels, axles, etc. From all the variety of vibrations that are inherent in a wheeled car when it moves, it is customary to distinguish those that occur with a frequency of less than 22 Hz. Such vibrations are called vibrations. Vibrations with a frequency greater than 20 Hz are caused, as a rule, not so much by the movement of the machine over irregularities/uneven roads, but also by internal causes, mainly the work of various mechanisms. If all the parts of the machine are combined into just two groups- sprung and unsprung masses, then the number of possible movements of these masses remains large. The sprung part of a moving car has some connections imposed on it, so the number of its degrees of freedom is significantly reduced. Vibrations along the longitudinal axis (twitching) are largely eliminated by the suspension guides. In real conditions, they are insignificant in any case, they are more apparent in some cases when braking than when driving over rough terrain. These same devices minimize angular vibrations around the vertical axis and linear vibrations along the transverse axis, which can practically only be due to lateral malleability and lateral tire slippage. As a result, the sprung part of the machine performs mainly linear vertical, longitudinal-angular and transverse-angular vibrations. Unsprung masses make linear vertical fluctuations. With dependent suspension of wheels or with independent, but with a swing of the wheels relative to the longitudinal axis, in some cases, you can take into account the transverse-angular fluctuations of unsprung masses.

Most of the actual springing systems for wheeled vehicles are symmetrical with respect to the longitudinal axis. In this case, the longitudinal-angular vibrations become independent of the transverse-angular ones, and vice versa, which means that they can be studied separately. Longitudinal-angular vibrations of the sprung part of the machine are called galloping.

Vibrations have a harmful effect on the driver, passengers and transported goods, worsen the working conditions of units and parts, and destroy road surfaces. The vibrations of the sprung part of the car cause fatigue for drivers, creating uncomfortable and sometimes unsafe conditions for them. In addition, vibrations are accompanied by an increase in dynamic loads on components and parts, leading to a reduction in their service life. In order to protect the driver, passengers, transported goods and structural elements of the vehicle from dynamic loads arising/resulting from the interaction of wheels with road irregularities, it is important to evaluate the properties of the mobile vehicle through a careful system analysis of the vibration processes taken place in a moving vehicle. Thus, this paper presents the mathematical model and computer simulation of vibration of mobile vehicle for analyzing the behavior of mobile vehicle under the effect of vibration process.

## 2. General Mathematical Models

A mathematical model of an object is any mathematical description that reflects with the required accuracy the behavior of a real object under real conditions. The mathematical model [5] reflects the set of knowledge, ideas and hypotheses about the modeled object written in the language of mathematics. Since this knowledge is never absolute, the model only approximates the behavior of a real object.

The mathematical model can be considered as some operator that correspond with the internal parameters of the system,  $x_1, x_2, \dots, x_m$ , a set of functionally related external parameters  $y_1, y_2, \dots, y_n$ .

The state of a system at any arbitrary moment of time  $t$  from a given interval  $[t_0, t_1]$  can be characterized by a set of values  $z_1, z_2, \dots, z_n$  – characteristics of the state of the system. These can

be its output variables, their derivatives functions. When the system is functioning, these characteristics take on values that are functions of time:

$$\{z_1(t), z_2(t), \dots, z_n(t)\} = \bar{Z}(t) . \quad (1)$$

Where:

$\bar{Z}(t)$  is the vector of the system state.

Projections of this vector can be considered as coordinates of a point in an n – dimensional phase space, and the process of system functioning can be considered as a certain phase trajectory.

The system can be affected by the vector of input influences:

$$\bar{X}(t) = \{x_1(t), x_2(t), \dots, x_n(t)\} \quad (2)$$

on which the characteristics of the systems depend.

The system is characterized by a set of proper parameters:

$$A = \{a_1, a_2, \dots, a_k\} \quad (3)$$

which in a stationary system are constants, and in a non – stationary system are functions of time.

The system may also be affected by some random factors:

$$\xi = \{ \xi_1, \xi_2, \dots, \xi_m \} \quad (4)$$

The system can have a number of outputs (output changes):

$$\bar{Y}(t) = \{y_1(t), y_2(t), \dots, y_q(t)\} \quad (5)$$

Thus, a mathematical model of a system is a set of relations (formulas, inequalities, equations, logical relations) that determine the characteristics of the system's states depending on its internal parameters, initial conditions, input signals, random factors, and time.

### 3. Equation of Motion of Vehicle

Vehicles are complex mechanical systems [4]. Their complexity is due not only to the number of masses, but also to the variety of different connections between the main elements: sprung and unsprung masses, elastic and damping elements, guiding devices. To fully determine the state of such a system during vibration, it is necessary to know the movements of all points of the structure. In other words, we need to find an infinite number of values (coordinates) in the form of certain functions of time and position of points that determine these movements at any time. Such systems are systems with an infinite number of degrees of freedom. In many cases, the study of vibrations of systems with an infinite number of degrees of freedom is associated with great difficulties. There are various techniques for constructing simplified design schemes that can be used to conduct a systematic analysis of the vibration processes that accompany the objects under consideration. One of these methods is to replace a given complex system with an infinite number of degrees of freedom with another, simpler one with a different distribution of masses and stiffness, but close to the given

one in the sense that its calculation leads to the values of the desired quantities that do not differ too much from the actual values for this system. Such a simplified system is called a reduced system or a system with a finite number of degrees of freedom. A calculation based on such simplifications can only give approximate values of the desired values, and almost always they are also the boundaries of the corresponding values of the same values for a given non-simplified system. The main advantage of using these systems for dynamic calculations is saving time and material resources.

The basis for drawing up a mathematical description of the design features of a mobile object is a design scheme, presented as a dynamic system consisting of a number of concentrated masses (chassis body, cab, container, supports, tires, driver's seat). During the movement, interactions of these masses connected by inertial elastic and damping elements are observed. When drawing up a spatial scheme of vibrations of a mobile machine we will make some assumptions:

- The body of the machine is a solid body, the center of mass of which makes translational movements along three coordinate axes and angular movements around them.
- The contact of wheels (rollers) with the support surface is point-based.
- The projection of the speed of the center of mass of the machine on the horizontal axis is constant.
- The route profile is non-deformable.
- The initial data for modeling are inertia, kinematic, power parameters of the car and road conditions.

The generalized design scheme [6] of a mobile multi-support ground vehicle is shown in Figure 1.

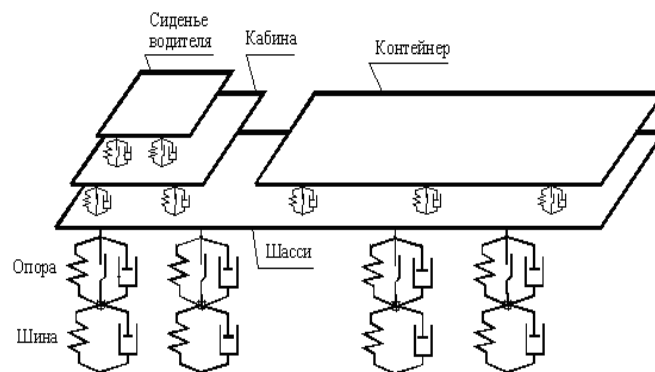


Figure 1-Generalized design scheme of a mobile object

The dynamic model, represented by masses concentrated in several discrete points, is reasonably used as the main one for solving such problems as smoothness of travel, stability, and cross-country mobility of mobile machines. In this case, the equations determine the main types of vibrations-vertical, longitudinal-angular, longitudinal, transverse-angular, etc.

The derivation of the differential equations of motion of a mobile machine consists in the use of scalar values of energy in the variation setting and can be formulated by Lagrange equations of the second kind, written as:

$$\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_k} \right) - \frac{\partial T}{\partial q_k} = - \frac{\partial P}{\partial q_k} - \frac{\partial \Phi}{\partial \dot{q}_k}, \quad (k = 1, n_1), \quad (5)$$

Where:

$T$  – is the kinetic energy of the system;

$P$  – the potential energy of the system;

$\Phi$  – the Rayleigh function;

$q_k$  – the generalized coordinate;

$n_1$  – the number of degrees of freedom.

Substituting expressions for kinetic and potential energy and the Rayleigh function in the Lagrange equation and carrying out differentiation using the general coordinates and velocities, we obtain a system of differential equations for a mobile machine, which in any case is generally nonlinear.

The spatial calculation scheme of the vibration of vehicle's wheel is shown in Figure 2.

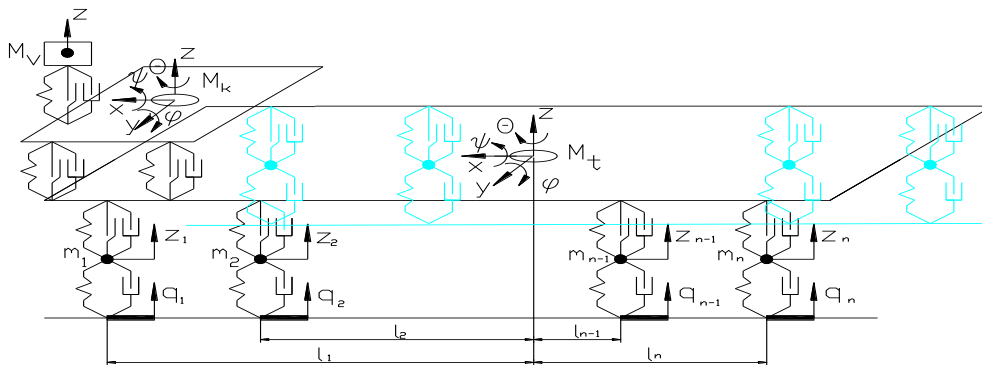


Figure 2-Spatial calculation scheme of transport vehicle vibrations

The equations of spatial motion of the system describing the rectilinear course movement of a wheeled vehicle taking into account the vibrations of the wheel on irregular road have the form:

$$\left. \begin{aligned}
 M\ddot{z} + \sum_{l=1}^2 \sum_{i=1}^n \sum_{j=1}^3 P_{jil} &= 0, \\
 J_y \ddot{\phi} + \sum_{l=1}^2 \sum_{i=1}^n \sum_{j=1}^3 l_{il} P_{jil} &= 0, \\
 J_x \ddot{\psi} + \sum_{l=1}^2 \sum_{i=1}^n \sum_{j=1}^3 b_{il} P_{jil} &= 0, \\
 J_z \ddot{\theta} + \sum_{l=1}^2 \sum_{i=1}^n \sum_{j=6}^7 l_{il} P_{jil} &= 0, \\
 m_{il} \ddot{z}_{il} - (P_{1il} + P_{2il} + P_{3il}) + P_{4il} + P_{5il} &= 0, \\
 l = 1, 2; i = \overline{1, n}; j = 1, 2, 3 &
 \end{aligned} \right\} \quad (6)$$

with initial conditions:

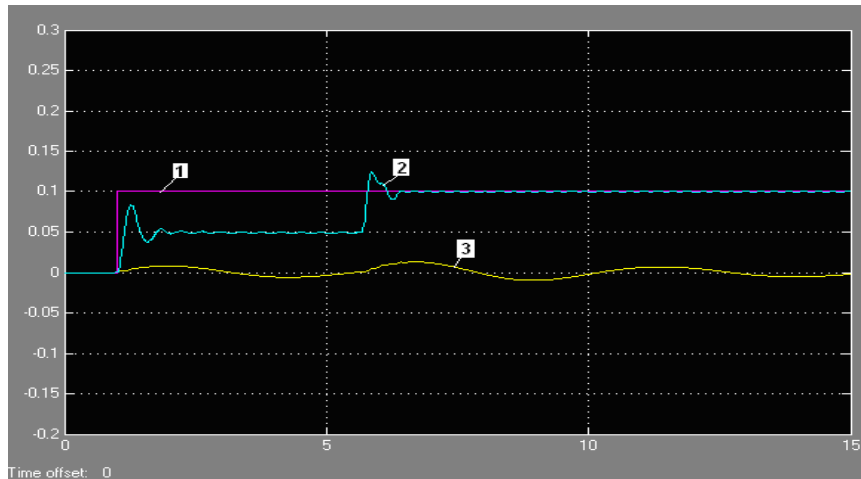
$$\begin{aligned}
 t \geq t_0 : \\
 z \Big|_{t=t_0} &= z_0, \quad \dot{z} \Big|_{t=t_0} = \dot{z}_0, \\
 \varphi \Big|_{t=t_0} &= \varphi_0, \quad \dot{\varphi} \Big|_{t=t_0} = \dot{\varphi}_0, \\
 \psi \Big|_{t=t_0} &= \psi_0, \quad \dot{\psi} \Big|_{t=t_0} = \dot{\psi}_0, \\
 \theta \Big|_{t=t_0} &= \theta_0, \quad \dot{\theta} \Big|_{t=t_0} = \dot{\theta}_0, \\
 z_{il} \Big|_{t=t_0} &= z_{0il}, \quad \dot{z}_{il} \Big|_{t=t_0} = \dot{z}_{0il}; \quad i = \overline{1, n}, \quad l = 1, 2.
 \end{aligned}$$

Arguments for characteristics are defined by the following relations:

$$\begin{aligned}
 \Delta_{il} &= z + l_i \varphi + b_k \psi - y_{il}, \quad \dot{\Delta}_{il} = \dot{z} + l_i \dot{\varphi} + b_k \dot{\psi} - \dot{y}_{il}, \\
 y_{il} &= y_{mi} + b_{kil} \theta_{mi}, \quad \dot{y}_{il} = \dot{y}_{mi} + b_{kil} \dot{\theta}_{mi}, \\
 \delta_{il} &= y_{il} - q_{il}; \quad \dot{\delta}_{il} = \frac{d}{dt} \delta_{il}
 \end{aligned} \quad (7)$$

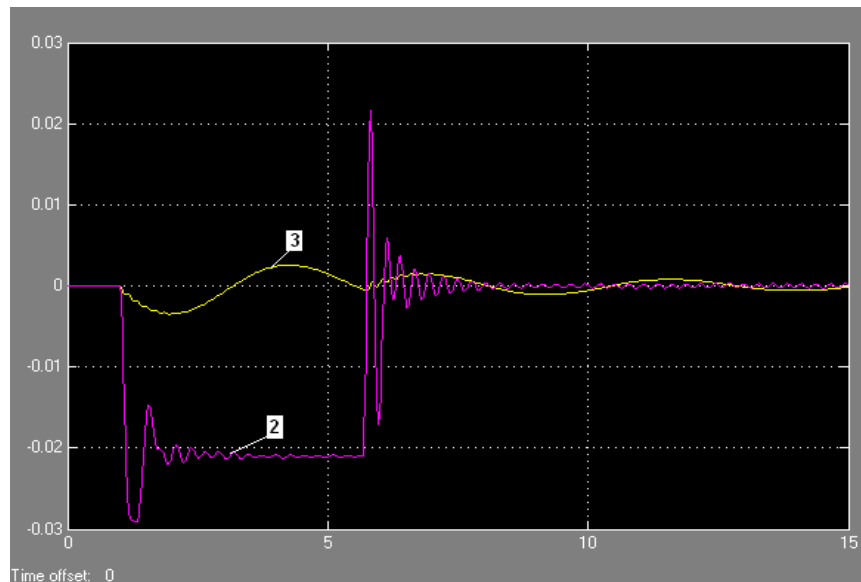
#### 4. Results and Discussion

The results of vibration processes in the center of mass of the car with passive and active springing supports when the front and rear wheels hit a step is shown in Figure 3 The mechatronic support system, controlled by the controller, stabilizes the position of the machine's center of mass and ensure its slight deviation from the static equilibrium position.



**Figure 3. Results of moving the car's centre mass when hitting a step (1-perturbation, 2-passive springing, 3-active springing)**

Changes in the longitudinal angle when the wheels hit the step sequentially are shown in Figure 4. Curve 2 shows that the deviation of the angle of inclination of the car body reaches 0.03 rad or 1.76 degree . At the same time, the active oscillation control system practically maintains the original position of the housing (curve 3).

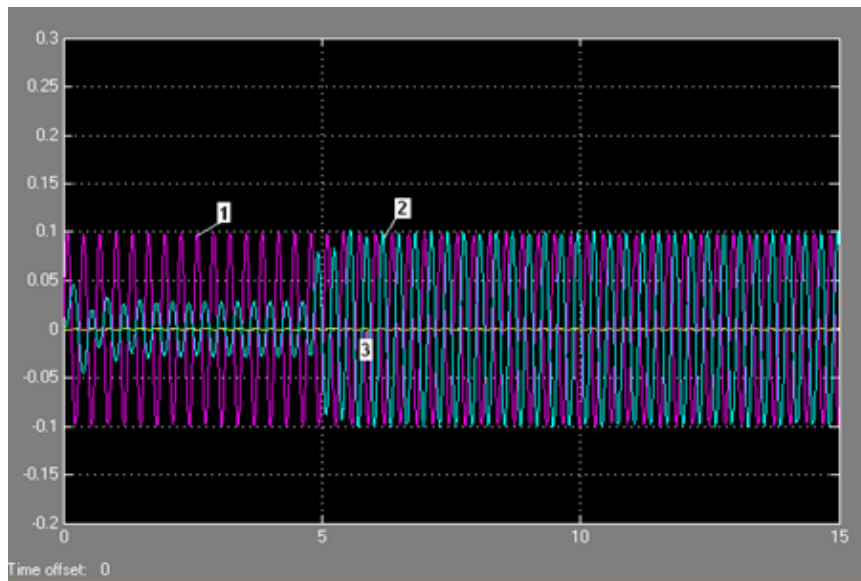


**Figure 4. Results of angular vibrations of the sprung mass of the car when hitting a step (1-not shown, 2-passive springing, 3-active springing)**

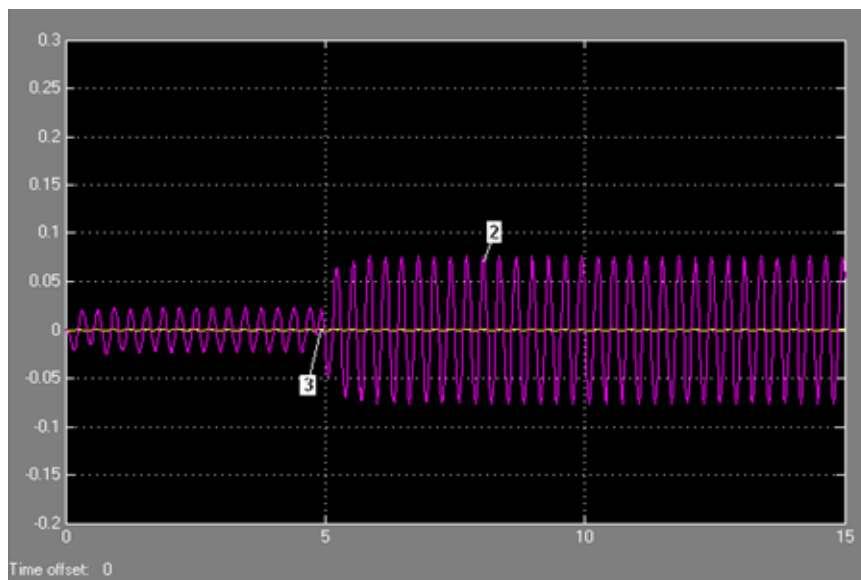
Simulation of vibrations of the sprung part of the car (vertical movements of the CM and longitudinal-angular position) when the vehicle is moving in a straight line on a road with a sinusoidal profile is shown in Figures 5 and 6.

As can be seen from Figure 5, the vibrating system with passive suspension of the chassis practically repeats (curve 2) the profile of the road. Active springing does not allow the sprung mass to make large deviations from the initial state (curve 3). It can be seen that the amplitude of vertical vibrations with a passive springing system is about 15 times higher than the corresponding parameter with

active vibration control. Approximately the same results are observed for longitudinal-angular vibrations of the machine (Figure 6).



**Figure 5. Results of movement of the machine's CM when moving on a sinusoidal surface (1-perturbation, 2-passive springing, 3-active springing)**



**Figure 6. Results of angular vibrations of the sprung mass of the machine when moving on a sinusoidal surface (1-not shown, 2-passive springing, 3-active springing)**

#### 4. Conclusion

In this paper, the mathematical model of vibration of a mobile vehicle is presented. We analyzed the vibration process in mobile vehicle with a view to evaluating the properties of the vehicle that will ensure not only the safety of the driver, the passenger but also the transported goods. Computer simulation of local support vibrations of the vehicle when driving on different support surfaces was performed and the results of vibrations with both passive and active suspension systems were



compared. The Results obtained showed that the use of the controller in the support loop will help to achieve a significant reduction in the vibration levels of the car.

## References

- [1] Ilyin M. M., Kolesnikov K. S., (2003). Theory of vibrations. Moscow State Technical University, page272
- [2] Theory of oscillations, <http://elib.ispu.ru/library/lessons/Munitcin/index.html>. Accessed: 2009-11-27
- [3] Smirnov G. A., (1990). Theory of motion of wheeled machines. Moscow, page345
- [4] Furunzhiev R. I., (1997). Computer-aided Design of Vibration Systems: Advance Textbook. Belarussian National Technical University, Minsk, page454.
- [5] Tarasik V. P., (1997). Mathematical Modeling of Technical Systems: A Textbook on Specialized Technical Education. Minsk, p640
- [6] Gursky N. N., Furunzhiev R. I., (2008). Modeling and optimization of vibrations of multi-support machines: Monograph. Belarussian National Technical University, Minsk, page 296.