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Analysis and Application of N-Person Games to Politics Using The Principle of Coalitions

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Abstract

The mathematical theory of *n*-person games and related solution concepts aims to model and analyze problems arising in various disciplines such as from operations research, management science, decision analysis, to economics, sociology and political science; voting power. We analyse and apply the concept of games in characteristic function form; we further apply the principle of coalition to real life situation in a political environment.

Key words: Zero-sum, Payoff, Characteristic function, Coalition, Prudential strategy, Security level, PDP, ANPP, AC, Minimal winning.

Introduction

In [9], we dealt only with games played between two players. In our modern interconnected world, such games are rare. Most important economic, social, and political games involve more than two players. In n-person games n is assumed to be at least three, with three or more players, new and interesting difficulties appear [1], [6].

In our analysis we consider a three - person 2 x 2 x 2 zero - sum as described below:

Kingsley A

Kaka \ Gali	A	B
A	(1, 1, -2)	(-4, 3, 1)

Kingsley A

Kaka \ Gali	A	B
A	(3, -2, -1)	(-6, -6, 12)

Figure 1: A 2 x 2x 2 Zero-sum Game

The three players are Kaka, Gali and Kingsley (Kingsley chooses the "layer"). Each outcome is a triple of numbers giving the payoff to these three players in that order. There are 2 x 2 x2 = 8 possible outcomes, which could be positioned in a three-dimensional array [6],[10]. For convenience on a two-dimensional page, the outcomes for Kingsley A and Kingsley B are given in two separate two-dimensional tables. The game is zero-sum since the three payoffs in each outcome add to zero.

As in two-player games we can search for pure strategy equilibria by drawing a movement diagram [6],[11]. Two possible form are shown in Figure 2.

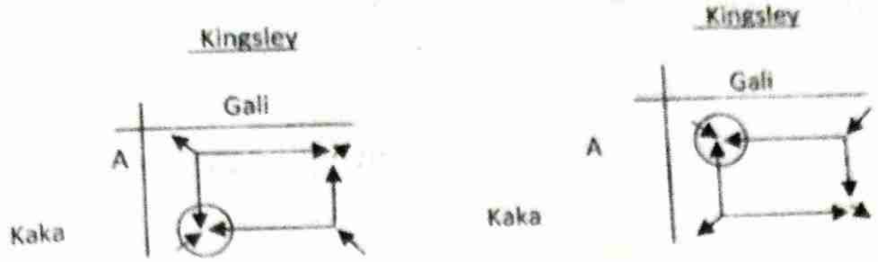

Figure 2: Movement diagram for Figure 1

In the two – dimensional diagrams, arrows point out of Kingsley A and into Kingsley B when Kingsley prefer his payoff for B to his payoff for A. If Kaka and Gali both play A, Kingsley prefers his payoff -1 at AAB to his payoff -2 at AAA. Hence at Kaka A-Gali A the arrow points out of the Kingsley A diagram and into the Kingsley B diagram.

The movement diagram indicates that none of the three players has dominant strategy, and that there are two pure strategy equilibria, at BAA = (2, -4, 2) and AAB = (3, -2, -1). These equilibria are not equivalent and not interchangeable. Kaka and Gali would prefer the equilibrium at AAB, while Kingsley would prefer the equilibrium at BAA. If Kaka plays A to try for her favorite equilibrium and Kingsley plays A to try for his favourite, the result will be AAA, which is not equilibrium.

If the players were allowed to communicate there may be a strong temptation for two of the players to form a coalition against the third player [2],[6]. From figure 1 suppose Gali and Kingsley from a coalition and agree to coordinate their play against Kaka. The result can be represented as a two- player game of Kaka against the combined player Gali – and – Kingsley. Since this game is zero sum, we can just give the payoffs to Kaka, obtaining the 2 x 4 game in figure 3a.

Gali and Kingsley

		AA	BA	AB	BB
Kaka	A	1	-4	3	-6
	B	2	-5	2	-2
Gali and Kingsley Optimal			4/5	1/5	

Kaka optimal:

3/5

Figure 3a: Gali and Kingsley Coalition

The solution for this game has Kaka playing $\frac{3}{5}A, \frac{2}{5}B$ and receiving an expected payoff of -4, 4. Since this is the best Kaka can do in the worst possible situation, when Gali and Kingsley gang up and play against Kaka, we call this strategy Kaka's prudential strategy, and the payoff -4, 4 Kaka's security level. The analysis also tells us what Gali and Kingsley should do if they decide to form a coalition with the goal of winning as much as possible from Kaka. Gali should always play B, and Kingsley should play $\frac{4}{5}A, \frac{1}{5}B$. The coalition will win an expected payoff of 4.4 from Kaka.

It would also be possible for Kaka and Kingsley to form coalition gainst Gali, or for Kaka and Gali to form coalition against Kingsley.

Figure 3b and c show these possibilities.

Kaka and Kingsley

		AA	BA	AB	BB
	A	1	-4	-2	2
Gali	B	3	-5	-6	3
Kaka and Kingsley Optimal			1	0	

Gali optimal:

1

Figure 3b: Kaka and Kingsley Coalition

Kaka and Gali

		AA	BA	AB	BB
	A	-2	2	1	10
Kingsley	B	-1	-4	12	-1
Kaka and Gali Optimal		6/7	1/7		

Kingsley optimal:

3/7

Figure 3c: Kaka and Gali Coalition

It is clear from this analysis that if coalitions are possible, each of the players would like to be in one. Being left out is costly.

A Game in Characteristic function form

A game in characteristic function is a set N of players, together with a function v which for any subset $S \subseteq N$ gives a number $v(S)$. [4],[6].

The number $v(S)$, called the value of S, is to be interpreted as the amount that the players in S could win if they formed a coalition. The function v is the *characteristic function* of the game. It is traditional to take the value of the empty coalition ϕ (the coalition of no players at all) to be zero. [3], [6].

Any game in normal form can be translated into a game in characteristic function form by taking $v(S)$ to be the security level of S. To calculate $v(S)$, assume that the coalition S forms and then plays optimally under the worst possible condition, which is that all the other players from an opposing coalition $N - S$ and play to hold down the pay – off to S. [5], [6]. The characteristic function for the game in figure 3a, using the symbol K, G and Kn for Kaka, Gali and Kingsley, is

$$v(\phi) = 0$$

$$v(K) = -4.4 \qquad v(G) = -4 \qquad v(Kn) = -1.43$$

$$v(GKn) = 4.4 \qquad v(KKn) = 4 \qquad v(KG) = 1.43$$

$$v(KGKn) = 0$$

Methodology

In a democracy with more than two major parties, it is possible that no single party will have a majority of seats in a particular instance. Hence a majority government must be formed by a coalition of parties.

In our application we consider the result of election conducted in Niger State, Nigeria into State of House of Assembly, with the following results:

Party	Number of seats
People Democratic Party (PDP)	14
All Nigeria Peoples Party (ANPP)	11
Action Congress (AC)	2
	27

It takes 18 members to form a majority party in the house.

In the weighted voting game

[18; 14, 11, 2]
A B C

There is one coalition which is a minimal winning, in the sense that it is winning, but would not be winning if a player is omitted. This is AB.

Generalizing this principle we consider a parliamentary election, with the following results,[6], [8];

	Party	Number of seats
A.	N1	68
B.	N2	13
C.	N3	18
D.	N4	18
E.	N5	31
		148

If it takes 75 members to form a coalition government, then;

In the weighted voting game

[75; 68, 13, 18, 18, 31]
A B C D E

In this case there are five coalitions which are minimal winning, in the sense that they are winning, but would not be winning if any player were omitted. These are;

AB, AC, AD, AE, BCDE.

If AB forms and governs, A receives 68/81 of the cabinet posts and B gets 13/81. Alternatively if AC forms, A gets 68/86 and C gets 18/86. Notice that A prefers the larger fraction 68/81 to the smaller 68/86, and hence would rather form coalition with B than with C.

In general, parties want to belong to a wining coalition with as few votes as possible, since this maximizes their share of the cabinet posts, [6], [7].

Therefore;

Minimal wining coalition:	AB	AC	AD	AE	BCDE
Number of votes:	81	86	86	99	80

Hence BCDE is the most likely coalition to form, with AB as second most likely

Conclusion

In a situation where the parties believe that cabinet posts will be divided equally among coalition members, putting into consideration that minimal wining coalition becomes losing if any member defects, one can argue that all members are equally important. In this case, parties would maximize their share of the cabinet posts by belonging to a wining coalition with as few members as possible. We would predict that AB, AC, AD, or AE would all be more likely than BCDE

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