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A CLASS OF IMPLICIT FIVE STEP BLOCK METHOD FOR GENERAL SECOND ORDER ORDINARY DIFFERENTIAL EQUATIONS

UMARU MOHAMMED

ABSTRACT. In this Paper, we extend the idea of collocation of linear multistep methods to develop a uniform Order 4 5step block methods. The single continuous formulation derived is evaluated at grid points of $x = x_{n+j},\ j = 5$ and its second derivative was also evaluated at some grid points, $x = x_{n+j},\ j = 2, 3,\ 4$ yielded the multi discrete schemes that form a Self starting uniform order 4 block methods. Two Numerical examples were used to demonstrate the efficiency of the methods

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[2000]Mathematics Subject Classification: ???

1. Introduction

It is well known that initial valued problems of ordinary differential equations often arise in many practical applications, such as chemical reactor, theory of fluid mechanics, automatic control and combustion etc. Aikeu [1]. The traditional methods for solving ODEs generally fall into two main classes: linear multistep (multivalue) and Runge-Kutta (multistage) methods Wright [10]. A linear multistep method with continuous coefficients is considered and applied to solve (ivps). The traditional multistep methods including the hybrid ones can be made continuous through the idea of multistep collocation Lie and Norsett [7] and Onumanyi etal [9]. Following Onumanyi [8,9], we

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identify a continuous formula (CF). The CF is evaluated at some distinct points involving step and off-step points along with its first and second derivatives, where necessary, to obtain multiple discrete formulae for a simultaneous application to the ODEs with initial conditions. This approach of using simultaneous discrete formulae (linked to a CF) both as corrector formula circumvent the requirement for special predictor in the use of single discrete formula as a corrector formula.

This paper is arranged as follows: in section two, we give the description of the method; we reformulate five step methods into the continuous form in section three. Convergence analysis, stability region and numerical examples are given in section four to show the accuracy of the method. Finally, the conclusion of the paper is discussed in section five.

2. The Method

Collocation solutions are desirable from practical and theoretical considerations and their advantages are now creating growing interest in continuous integration algorithms [7,9] for numerical solution of odes. In particular, collocation solutions of the odes by their nature are continuous.

In the spirit of onumanyi et al[9] we consider briefly the derivation of the continuous formula by the multistep collocation using constant mesh spacing h and give explicit representation for the coefficients. The value of k and m are arbitrary except for collocation at the mesh points, where $0 \prec m \leq k+1$.

Let Y_{n+j} be approximations to y_{n+j} where

$$y_{n+j} = y\left(x_{n+j}\right),\ j = 0, \ldots, k. \tag{2.1}$$

Then a k-step multistep collocation formula with m collocation points is constructed as follows.

We consider a given formula

$$Y\left(x_{n+k}\right)=\sum_{j=0}^{r-1}\bar{\alpha}_j Y_{n+j}+h\sum_{j=0}^{m-1}\bar{\beta}_j f\left(\bar{x}_j,Y\left(\bar{x}_j\right)\right) \tag{2.2}$$

where$\bar{\alpha}_j, j = 0,...,r-1$ *and* $\bar{\beta}_j, j = 0,...,m-1$ are the constant coefficients of discrete scheme. To obtain a continuous form of (2.2) we find the polynomial Y(x) of degree p=r+m-1, r>0, m>0 of the form

$$Y\left(x\right)=\sum_{j=0}^{r-1}\alpha_j\left(x\right)Y\left(x_{n+j}\right)+h\sum_{j=0}^{m-1}\beta_j\left(x\right)f\left(\bar{x}_j,Y\left(\bar{x}_j\right)\right) \tag{2.3}$$

such that it satisfies the conditions

$$\bar{\alpha}_j Y\left(x_{n+j}\right)=\bar{\alpha}_j Y_{n+j},\ \ j\in 0,....,r-1 \tag{2.4}$$

$$\bar{\beta}_j Y'\left(x_j\right)=\bar{\beta}_j f\left(\bar{x}_j,Y\left(\bar{x}_j\right)\right),\ \ j=0,....,m-1 \tag{2.5}$$

whereα_j *and* β_j are assumed polynomials of the form

$$\alpha_j\left(x\right)=\sum_{i=0}^{r+m-1}\alpha_{j,i+1}x^i;\quad h\beta_j\left(x\right)=\sum_{i=0}^{r+m-1}\beta_{j,i+1}x^i, \tag{2.6}$$

x_{n+j}in (2.3) are r(0<r<=k) arbitrarily chosen interpolation points taken from $\{x_n,...,x_{n+k-1}\}$and the collocation points $\bar{x}_j, j = 0,...,m-1$belong to the extended set

$$Q=\{x_n,...,x_{n+k-1}\}\bigcup\{x_{n+k-1},...,x_{n+k}\}.$$

From the interpolation conditions and the expression for $y\left(x\right)$ in (2.3) the following conditions are imposed on $\alpha_j\left(x\right)$ *and* $\beta_j\left(x\right)$

$$\left.\begin{array}{r}\alpha_j\left(x_{m+i}\right)=\delta ij,\ j=0,\ldots,t-1,\ i=0,\ldots,t-1\\ h^2\beta_j\left(x_{n+i}\right)=0, j=0,\ldots,m-1,\ i=0,\ldots,t-1\end{array}\right\}. \tag{2.7}$$

and

$$\left.\begin{array}{r}\alpha_j^{"}\left(\overline{x}_{n+i}\right)=0, j=0,\ldots,t-1,\ i=0,\ldots,m-1\\ h^2\beta_j\left(x_{n+i}\right)=\delta ij, j=0,\ldots,m-1;\ i=0,\ldots,m-1\end{array}\right\} \tag{2.8}$$

Next we write (2.7) and (2.8) in a matrix equation of the form.

$$AC=B \tag{2.9}$$

where

$$A=\begin{pmatrix} \bar{\alpha}_0 & \bar{\alpha}_0 x_n & \bar{\alpha}_0 x_n^2 \dots & \bar{\alpha}_0 x_n^{r+m-1} \\ \bar{\alpha}_0 & \bar{\alpha}_0 x_{n+1} & \bar{\alpha}_0 x_{n+1}^2 \dots & \bar{\alpha}_0 x_{n+1}^{r+m-1} \\ \vdots & \vdots & \vdots & \vdots \\ \bar{\alpha}_{r-1} & \bar{\alpha}_{r-1} x_{n+r-1} & \bar{\alpha}_{r-1} x_{n+r-1}^2 \dots & \bar{\alpha}_{r-1} x_{n+r-1}^{r+m-1} \\ 0 & \bar{\beta}_0 & 2\bar{\beta}_0 \bar{x}_0 \dots & (r+m-1)\,\bar{\beta}_0 x_0^{r+m-2} \\ \vdots & \vdots & \vdots & \vdots \\ 0 & \bar{\beta}_{m-1} & 2\bar{\beta}_{m-1}\bar{x}_{m-1} \dots & (r+m-1)\,\bar{\beta}_{m-1} x_{m-1}^{r+m-2} \end{pmatrix}$$

$$C=\begin{pmatrix} \alpha_{01} & \alpha_{11} \dots & \alpha_{r-1,1} & h\beta_{0,1} \dots & h\beta_{m-1,1} \\ \alpha_{02} & \alpha_{12} & \alpha_{r-1,2} & h\beta_{0,2} \dots & h\beta_{m-1,2} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \alpha_{0r+m} & \alpha_{1,r+m} & \alpha_{r-1,r+m} & h\beta_{0,r+m} & \alpha_{m-1,r+m} \end{pmatrix}$$

$$B=\begin{pmatrix} \bar{\alpha}_0 & 0 \dots & 0 & 0 \dots & 0 \\ 0 & \alpha_1 \dots & & & \\ \vdots & \vdots & & & \\ 0 & 0 \dots & \bar{\alpha}_{r-1} & & 0 \\ 0 & 0 \dots & 0 & \bar{\beta}_0 & 0 \\ 0 & 0 & 0 & 0 & \bar{\beta}_0 \end{pmatrix}$$

Pre-multiplying both sides of (2.9) by B^{-1}, we have that

$$B^{-1}AC = I$$

$$DC = I \tag{2.10}$$

where I is the identity matrix of dimension $r+m$ and

$$D = B^{-1}A = \begin{pmatrix} 1 & x_n & x_n^2 \dots & x_n^{r+m-1} \\ l & x_{n+1} & x_{n+1}^2 \dots & x_{n+1}^{r+m-1} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & x_{n+r-1} & x_{n+r-1}^2 \dots & x_{n+r-1}^{r+m-1} \\ 0 & 1 & 2x_0 \dots & (r+m-1)\,x_0^{r+m-2} \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 1 & 2x_{m-1} \dots & (r+m-1)\,x_{m-1}^{r+m-2} \end{pmatrix} \tag{11}$$

Is the multistep collocation matrix of dimension $(r+m)\times(r+m)$. Then it follows from (2.11) that the column of matrix $C = D^{-1}$ give the continuous coefficient $\alpha_j(x)$ and $\beta_j(x)$

3. Derivation of the Continuous Method

We propose an approximate solution in the form

$$y\left(x\right)=\sum_{j=0}^{m+t-1}a_jx^i,\ \ i=0,(1)\left(m+t-1\right), \tag{3.1}$$

$$y''\left(x\right)=\sum_{j=0}^{m+t-1}i\left(i-1\right)a_jx^{i-2}, i=2,3,\ldots\left(m+t-1\right) \tag{3.2}$$

with $m=5, t=1$, and $p=m+t-1$, also
$\alpha_j,\ \ \beta_j\ ,\ j=0,1,(m+t-1)$ are the parameters to be determined, where p, is the degree of the polynomial. Specifically, we collocate equation (3.2) and interpolate equation (3.1) using the method described in section 2 of this paper; we obtain a continuous form for the solution

$$\sum_{j=0}^{t-1}\alpha_j\left(x\right)y_{n+j}=h^2\sum_{j=0}^{m-1}\beta_j\left(x\right)f\left(\overline{x}_{j+1},y\left(\overline{x}_{j+1}\right)\right) \tag{3.3}$$

This gives rise to the system of equations put in the matrix form below.

$$\begin{pmatrix} 1 & x_n & x_n^2 & x_n^3 & x_n^4 & x_n^5 \\ 1 & x_{n+1} & x_{n+1}^2 & x_{n+1}^3 & x_{n+1}^4 & x_{n+1}^5 \\ 1 & x_{n+2} & x_{n+2}^2 & x_{n+2}^3 & x_{n+2}^4 & x_{n+2}^5 \\ 1 & x_{n+3} & x_{n+3}^2 & x_{n+3}^3 & x_{n+3}^4 & x_{n+3}^5 \\ 1 & x_{n+4} & x_{n+4}^2 & x_{n+4}^3 & x_{n+4}^4 & x_{n+4}^5 \\ 0 & 0 & 2 & 6x_{n+5} & 12x_{n+5}^2 & 20x_{n+5}^3 \end{pmatrix}\begin{pmatrix}\alpha_0\\ \alpha_1\\ \alpha_2\\ \alpha_3\\ \alpha_4\\ \alpha_5\end{pmatrix}=\begin{pmatrix}y_n\\ y_{n+1}\\ y_{n+2}\\ y_{n+3}\\ y_{n+4}\\ f_{n+5}\end{pmatrix}. \tag{3.4}$$

Note,

$$\underline{D}\ \ \underline{a}=\underline{F}$$

$$\underline{a}=\underline{\underline{D}}^{-1}\underline{F}$$

where $\underline{F}=\left(y_1,y_2,\ldots,y_r,f_1,f_2,\ldots,f_s\right)^T$
Matrix D in equation (3.4), which when solved either by matrix inversion techniques or Gaussian elimination method to obtain the values of the parameters $a_j,\ j=0,1,m+t-1$ and then substituting them

into equation (3.1) give a scheme expressed in the form.

$$y(x) = \sum_{j=0}^{k-1} \alpha_j(x) y_{n+j} + h^2 \sum_{j=0}^{k-2} \beta_j(x) f_{n+j} \quad (3.5)$$

If we now let $k = 5$, after some manipulations we obtain a continuous form of solution

$$\begin{aligned} y(x) = & \frac{1}{1080h^5}\Big[1080h^5 - 2418h^4(x-x_n) + 1925h^3(x-x_n)^2 \\ & -695h^2(x-x_n)^3 + 115h(x-x_n)^4 - 7(x-x_n)^5\Big] y_n \\ & +\frac{1}{1350h^2}\Big[6384h^4(x-x) - 7900h^3(x-x_n)^2 \\ & +3460h^2(x-x_n)^3 - 635h(x-x_n)^4 + 41(x-x_n)^5\Big] y_{n+1} \\ & +\frac{1}{900h^2}\Big[-3876h^4(x-x) + 6725h^3(x-x_n)^2 \\ & -3515h^2(x-x_n)^3 + 715h(x-x_n)^4 - 49(x-x_n)^5\Big] y_{n+2} \\ & +\frac{1}{1350h^2}\Big[3216h^4(x-x) - 6100h^3(x-x_n)^2 \\ & +3640h^2(x-x_n)^3 - 815h(x-x_n)^4 + 59(x-x_n)^5\Big] y_{n+3} \\ & +\frac{1}{5400h^2}\Big[-3054h^4(x-x) + 6025h^3(x-x_n)^2 \\ & -3835h^2(x-x_n)^3 + 935h(x-x_n)^4 - 71(x-x_n)^5\Big] y_{n+4} \\ & +\frac{1}{450h^2}\Big[24h^4(x-x) - 50h^3(x-x_n)^2 + 35h^2(x-x_n)^3 \\ & -10h(x-x_n)^4 + (x-x_n)^5\Big] f_{n+5} \quad (3.6) \end{aligned}$$

Evaluating (3.6)at $x = x_{n+5}$ yields five step implicit method see Jain [5]

$$y_{n+5} - \frac{2}{3}y_n + \frac{61}{45}y_{n+1} - \frac{52}{15}y_{n+2} + \frac{214}{45}y_{n+3} - \frac{154}{45}y_{n+4} = \frac{4}{15}h^2 f_{n+5} \quad (3.7)$$

Taking the second derivative of equation (3.6), thereafter, evaluating the resulting continuous polynomial solution at $x = x_{n+4}$, $x = x_{n+3}$

and $x = x_{n+2}$, yields respectively three integrator below:

$$y_{n+4} + \frac{29}{173}y_n - \frac{176}{173}y_{n+1} + \frac{438}{173}y_{n+2} - \frac{464}{173}y_{n+3} = \frac{108}{173}h^2 f_{n+4} - \frac{24}{173}h^2 f_{n+5}$$

$$y_{n+3} + \frac{5}{568}y_n - \frac{8}{568}y_{n+1} - \frac{282}{568}y_{n+2} - \frac{283}{568}y_{n+4} = -\frac{270}{568}h^2 f_{n+3} - \frac{6}{568}h^2 f_{n+5}$$

$$y_{n+2} + \frac{1}{30}y_n - \frac{16}{30}y_{n+1} - \frac{16}{30}y_{n+3} + \frac{1}{30}y_{n+4} = -\frac{12}{30}h^2 f_{n+2}. \quad (3.7)$$

The first derivative of equation (3.6) at $x = x_n$ is used along with the schemes in (3.7) and (3.8) to start the integration process, that is

$$900hz_n + 2015y_n - 4256y_{n+1} + 3876y_{n+2} - 2144y_{n+3} + 509y_{n+4} = 48h^2 f_{n+5} \quad (3.9)$$

equations (3.7), (3.8), and (3.9) constitute the member of a zero-stable block integrators of order $(4, 4, 4, 4, 4)^T$ with $C_6 = \left(-\frac{137}{675}, -\frac{391}{15}, -\frac{227}{30}, -\frac{2}{15}, -\frac{2798}{15}\right)$ the application of the block integrators with $n = 0$, give the accurate values of y_1, y_2, y_3, y_4 and y_5 as shown in tables 4.1-4.3 of Section 4.

To start the IVP integration on the sub interval $[X_0, X_5]$, we combine (3.7),(3.8) and (3.9), when $n = 0$, i.e the 1-block 5-point method as given in equation (4.1). Thus produces simultaneously values for $y_1, y_{2,}y_3, y_4$ along with y_5 without recourse to any predictor.

4. Convergence Analysis

Recall, that, it is a desirable property for a numerical integrator to produce solution that behave similar to the theoretical solution to a problem at all times. Thus several definitions, which call for the method to possess some "adequate" region of absolute stability, can be found in several literatures. See Lambert[6], Fatunla [2,3] etc. following Fatunla [2,3], the five integrator proposed in this report in equations (3.7),(3.8) and (3.9) are put in the matrix-equation form and for easy analysis the result was normalized to obtain;

$$\begin{pmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\1&0&0&1&0\\0&0&0&0&1\end{pmatrix}\begin{pmatrix}y_{n+1}\\y_{n+2}\\y_{n+3}\\y_{n+4}\\y_{n+5}\end{pmatrix}=\begin{pmatrix}0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\end{pmatrix}\begin{pmatrix}y_{n-4}\\y_{n-3}\\y_{n-2}\\y_{n-1}\\y_{n}\end{pmatrix}+h^2$$

$$\left\{\begin{pmatrix}0&\frac{1387}{360}&\frac{109}{15}&\frac{637}{120}&\frac{1159}{900}\\0&\frac{538}{45}&\frac{106}{5}&\frac{76}{5}&-\frac{842}{225}\\0&\frac{849}{40}&\frac{711}{20}&\frac{1017}{40}&-\frac{157}{25}\\0&\frac{1376}{45}&\frac{736}{15}&\frac{536}{15}&\frac{1984}{225}\\0&\frac{2875}{72}&\frac{125}{2}&\frac{375}{8}&\frac{202}{180}\end{pmatrix}\begin{pmatrix}f_{n+1}\\f_{n+2}\\f_{n+3}\\f_{n+4}\\f_{n+5}\end{pmatrix}+\begin{pmatrix}0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\end{pmatrix}\begin{pmatrix}f_{n-4}\\f_{n-3}\\f_{n-2}\\f_{n+1}\\f_{n}\end{pmatrix}\right\} \quad (4.1)$$

Equation (4.1) is the 1-block 5 point method. The first characteristic polynomial of the proposed 1-block 5- step method is

$$P(R)=det\left|RA^{(0)}-A^{(1)}\right|$$

$$\rho(R)=\det\left[R\begin{pmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\1&0&0&1&0\\0&0&0&0&1\end{pmatrix}-\begin{pmatrix}0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\end{pmatrix}\right] \quad (4.2)$$

$$=\det\begin{pmatrix}R&0&0&0&-1\\0&R&0&0&-1\\0&0&R&0&-1\\0&0&0&R&-1\\0&0&0&0&R-1\end{pmatrix}$$

$$=\left[R^4(R-1)\right]$$

$P(R)=R^4(R-1)$, This implies, $R_1=R_2=R_3=R_4=0$ or $R_5=1$

The 1 block 5-point is zero stable and is also consistent as its order $(4,4,4,4,4)^T$ >1, thus, it is convergent, following Henrici [4]

4.1 Stability Region of Block Method

To compute and plot absolute stability region of the block methods, the method of section three are reformulated as general linear methods expressed as

$$\begin{pmatrix} Y \\ y_{n+1} \end{pmatrix} = \begin{pmatrix} A & U \\ B & V \end{pmatrix} \begin{pmatrix} hf(y) \\ y_{i-1} \end{pmatrix}$$

Where $A = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{3}{6} \\ 0 & 0 & -\frac{2}{5} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{135}{284} & 0 & -\frac{3}{284} \\ 0 & 0 & 0 & 0 & \frac{108}{73} & -\frac{24}{173} \\ & & & & & \end{pmatrix}$

$$B = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{4}{5} \\ 0 & 0 & 0 & 0 & \frac{108}{173} & -\frac{24}{173} \\ 0 & 0 & 0 & -\frac{135}{284} & 0 & -\frac{3}{284} \\ 0 & 0 & -\frac{2}{5} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{3}{266} \end{pmatrix}$$

$$U = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ \frac{509}{4256} & -\frac{67}{133} & \frac{51}{56} & 0 & \frac{2015}{4256} \\ -\frac{1}{30} & \frac{8}{15} & 0 & \frac{8}{15} & -\frac{1}{30} \\ \frac{283}{568} & 0 & \frac{141}{284} & \frac{1}{71} & -\frac{5}{568} \\ 0 & \frac{464}{173} & -\frac{438}{173} & \frac{176}{173} & -\frac{29}{173} \\ \frac{154}{45} & -\frac{214}{45} & \frac{52}{15} & -\frac{61}{45} & \frac{2}{9} \end{pmatrix}$$

$$V = \begin{pmatrix} \frac{154}{45} & -\frac{214}{45} & \frac{52}{15} & -\frac{61}{45} & \frac{2}{9} \\ 0 & \frac{464}{173} & -\frac{438}{173} & \frac{176}{173} & -\frac{29}{173} \\ \frac{283}{568} & 0 & \frac{141}{284} & \frac{1}{71} & -\frac{5}{568} \\ -\frac{1}{30} & \frac{8}{15} & 0 & \frac{8}{15} & -\frac{1}{30} \\ \frac{509}{4256} & -\frac{67}{133} & \frac{51}{56} & 0 & \frac{2015}{425} \end{pmatrix}$$

Substituting the value of A, B, U and V into the stability matrix $M(z) = V + zU(I - zA)^{-1} B$ and stability function $\rho(\eta, z) =$

det $(\eta I - M(z))$ and using maple software yields the stability polynomial of the block method as;

$$\begin{aligned}\rho : &= \frac{1}{12768}\Big[-691092000\eta^4 z^4 + 1489259520\eta^5 z^4 \\ &+16206752160 z^3\eta^4 - 1114186752\eta^5 z^3 - 7242994080 z^3\eta^3 \\ &+38507582952\eta^3 z^2 - 19914581568\eta^5 z^2 - 25172306928\eta^2 z^2 \\ &+55561846038\eta^4 z^2 - 733585440\eta^5 z - 124426168557\eta^4 z \\ &-358540785471\eta^2 z + 454981156530\eta^3 z + 79966133928\eta z \\ &+785215862041\eta^3 + 302134115239\eta - 357926809780\eta^4 \\ &-46382877304 + 47048803200\eta^5 - 730089093396\eta^2\Big] \times \\ &\Big[(540z^2 - 889z - 4260)(2z+5)(108z-173)\Big]^{-1}\end{aligned}$$

Using a matlab program stability produces the absolute stability region of the block method as shown in fig 4.1

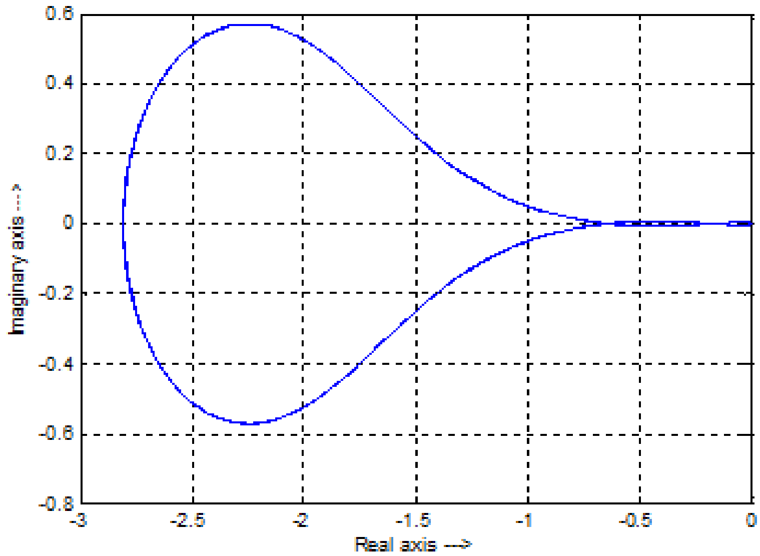

Fig 4.1

4.2 Numerical Experiment

In what follows, we present some numerical results on some problems.

Problem 1; from Yahaya and Badmus [13] ;

$y'' - y' = 0, \quad y(0) = 0, \quad y'(0) = -1, \quad h = 0.1$ Exact solution;

$y(x) = 1 - e^x$

Table of Results 4.1

N	x	Exact value	Approx value	Error
1	0.1	-0.105170918	-0.10516872	2.198E-06
2	0.2	-0.221402758	-0.221396688	6.0704E-06
3	0.3	-0.349858808	-0.349848757	1.0051E-05
4	0.4	-0.491824698	-0.491810673	1.40253E-05
5	0.5	-0.648721271	-0.648703278	1.79935E-05
6	0.6	-0.8221188	-0.822097184	2.16162E-05
7	0.7	-1.0137527	-1.013724707	2.7993E-05
8	0.8	-1.225540928	-1.225506367	3.4561E-05
9	0.9	-1.459603111	-1.459561997	4.1114E-05
10	1.0	-1.718281828	-1.718234172	4.7656E-05

Table 4.2

N	Exact value	Yahaya[11] error	Yahaya and Badmus[13]	Present Error
1	-0.1051709180	5.008136E-03	8.79316E-05	2.198E-06
2	-0.2214027580	1.101918E-02	3.26718E-04	6.0704E-06
3	-0.3498588080	1.9041146E-02	2.215564E-03	1.0051E-05
4	-0.4918246980	2.8374166E-02	4.857093E-03	1.40253E-05
5	-0.6487212710	4.0041949E-02	9.097734E-03	1.79935E-05
6	-0.82211880	5.339556E-02	1.4391394E-02	2.16162E-05
7	-1.0137527000	6.9481732E-02	2.1437918E-02	2.7993E-05
8	-1.2255409280	8.7709919E-02	2.9898724E-02	3.4561E-05
9	-1.4596031110	1.09158725E-01	4.0300719E-02	4.1114E-05
10	-1.7182818280	1.33295713E-01	5.255213E-02	4.7656E-05

REMARKS:Yahaya and Badmus [13] proposed order $(4, 3, 3)^T$ method, while the present method is 1-block 5-points that produces simultaneously, y, y_1, y_2, y_3, y_4 and y_5 (see Table 4.2)

Problem 2 (Yahaya and Badmus [12])

$$y'' = x\left(y'\right)^2 \quad y(0) = 1, \quad y'(0) = \frac{1}{2}, \quad h = \frac{1}{30}.$$

The exact solution is

$$y(x) = 1 + \frac{1}{2}\ln\left(\frac{2+x}{2-x}\right).$$

Table 4.3

X	**Exact value**	**Approx value**	**Present Error**	**Yahaya and Badmus[12]**
0.1	1.050041729	1.050041724	5.00E-09	5.891E-06
0.2	1	1.100318692	1.67E-05	8.2399E-05
0.3	1.151140436	1.151028384	1.12E-04	3.46421E-04
0.4	1.202732554	1.202585545	1.47E-04	7.52101E-04
0.5	1.255412817	1.255265756	1.47E-04	1.380283E-03

Conclusion

In this paper we developed a uniform order 1-block 5- point integrators of orders (4,4,4,4,4) and the resultant numerical integrators posses the following desirable properties.

(I) zero- stability i.e. stability at the origin

(II) cheap and reliable error estimates

(III) Facility to generate the solution at five point simultaneously.

(IV) It is a convergent scheme.

Hence, an improvement over other cited works.

References

[1] Aikeu. R.C.(1985) Stiff computation. New York, Oxford: Oxford University Press. Equations.

[2] Fatunla S.O (1992). Parallel methods for second order ODE‘s computational ordinary differential equations proceeding of computer conference (Fatunla eds)Pp87-99

[3] Fatunla S.O (1994) Higher order parallel methods for second order ode‘s. scientific computing Pp61-67.proceeding of fifth international conference on scientific computing (eds fatunla)

[4] Henrici. P(1962). Discrete variable methods for ODE‘s john wiley &sons New York, USA.pp276

[5] Jain M.K (1984). Numerical Solution of Differential Equations. John Wiley. pp 150.

[6] Lambert J.D (1973): Numerical methods for ordinary differential systems. John wiley & Sons, inc USA pp278.

[7] Lie I and Norset S.P (1989): Supper Convergence for Multistep Collocation. Math. Comp 52, pp 65-79.

[8] Onumanyi P, Sirisena U.W and Jator S.N (1999): Continuous Finite difference Approximate for Solving Differential Equations. Inter.J. Comp Maths 72, No 1, pp 15-27.

[9] Onumanyi, p. Awoyemi, D.O, Jator, S.N and Siresina, U.W (1994). New Linear multistep methods with continuous coefficients for the first order initial value problem, *Journal. Nig.Maths* soc 13, Pp37-51

[10] Wright. W.M (2002). Explicit general linear method with inherit runge-Kutta Stability. Numerical Algorithms 31:381-399

[11] Yahaya . Y.A. (2004). Some theory and application of Continuous linear multi-step methods for ordinary differential equations Ph.D thesis (unpublished) University of Jos, Nigeria.

[12] Yahaya.Y.A. and Badmus .A.M.(2009). An Accurate Uniform Order 6 Block Method for Direct Solution of General Second Order Ordinary Differential *Pacific Journal of Science and Technology.* 10 (2), 248-254.

[13] Yahaya.Y.A. and Badmus .A.M.(2009). A class of collocation methods for general second order ordinary differential equations *African Journal of Mathematics and Computer Science Research Vol.* 2 (4), (2009) 069-072.

Mathematics/Statistics Department, Federal University of Technology, Minna, Niger State

E-mail address: digitalumar@yahoo.com