Numerical Prediction of Land Surface Temperature Using Shallow Water Equation

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ABSTRACT

In this paper, we formulated a model for the prediction of Land Surface Temperature from shallow water equation using the Explicit Finite Difference Method, that resulted in the discretization of shallow water equation, with the incorporation of atmospheric scale height to predict the Land Surface Temperature (LST) of a place at a particular altitude. The study centers on the attempt to predict and analyze the behaviour of LST for a particular area using shallow water equation. The result derived from this study re-affirms the efficiency of Shallow Water Equation in making accurate predictions of the Land Surface Temperature. The result also revealed that the Land Surface Temperature increases with decrease in altitude.

Keywords: Land surface temperature, zonal wind speed, meridional wind speed, scale height and atmospheric altitude.

1. INTRODUCTION

Land Surface Temperature (LST) is described as determining how hot or cold the surface of the Earth would feel to the touch in a particular location at a particular time. Land Surface Temperature is a key input or variables to various environmental models and applications, when studying the atmosphere. Researches in the field of land surface temperature prediction has being based on data extracted from weather stations and researchers used these data to investigate the ways the land surface temperature behaved and its effects on people and their immediate environment (AchutaRao and Sperber (2006); Ayoade, 2006)).

LST is an important variable in climate, Hydrologic, Ecological, Biophysical and Biochemical studies (Mildrexlex 2011). It also plays a key role in modeling the surface energy balance and has a substantial impact on analyzing the heat-related issues such as soil moisture, evapotranspiration and urban heat islands (Bu'hler, 1998).

In remote sensing land surface temperature data are usually obtained through the weather satellite. Weather satellites measure radiance from the atmosphere, the data is determined through a retrieval algorithm and analyzed with statistical methods. Some of the models formulated for the analysis of this data do not really present the real behavior of the land surface temperature. This necessitated a numerical approach for the prediction of land surface temperature (Bu'hler, 1998).

Zaharaddeen et al (2016), estimated land surface temperature of Kaduna metropolis, Nigeria using Landsat Images. In his study, Landsat images of 2001, 2006, 2009 and was obtained. Normalized Difference Vegetation Index (NDVI) image was developed. The digital number of thermal infrared band is converted into spectral radiance using the equation supplied by the Landsat. The effective at-sensor brightness temperature is obtained from the spectral radiance using Plank's inverse function. The results clearly show that the land surface temperature varies over space and time. His work fail to predict the degree of hot or coldness of the environment in degree Celsius, it only established the fact that the land surface temperature varies over space and time.

This paper focuses on the use of one-dimensional form of shallow water equation to predict the land surface temperature using the explicit finite difference discretization Method.

2. Model Formulation

Following Warner (2011), the one-dimensional shallow water equation is given as,

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} - fv + g\frac{\partial h}{\partial x} = 0 \quad (1)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + fu + g\frac{\partial H}{\partial y} = 0 \quad (2)$$

$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial x} + v\frac{\partial H}{\partial y} + h\frac{\partial u}{\partial x} = 0 \quad (3)$$

where

$$\frac{\partial H}{\partial y} = -f\frac{\overline{U}}{g} \quad (4)$$

u is zonal wind speed
v is meridional wind speed
x is x-component
y is y-component
H is Scale height
h is shallow height of the atmosphere
f is Coriolis force
t is Time
T is Temperature
g = Gravitational force
$\overline{U}$ is Average mean wind speed of the atmosphere.

Equation (1) is u-momentum equation along the x-axis, Equation (2) is v- momentum equation along the y-axis and Equation (3) is continuity equation.

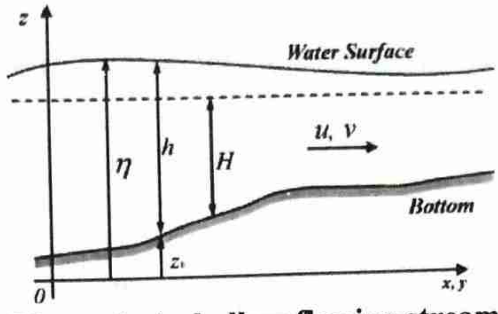


Figure 1: A shallow flowing stream

Figure 1 is the flow diagram for the derivation of our model equation. The water surfaces represent the layers of the atmospheric, above the troposphere. η is considered as the height of the troposphere and it is a "constant", H is the scale height at which the atmospheric pressure decreases with increase in altitude, h is taking as shallow height of the atmosphere and u and v is the zonal and meridional wind speed along the x-horizontal component and y-vertical component.

There are limitations to the degree to which the system of equations (1) to (3) can represent the atmosphere, but one step toward more realism is to obtain a mathematical model for the land surface temperature using the set of equaions (1) to (3) is to define the fluid depth to be consistent with the boundary layer of the troposphere, so the temperature at this lower atmosphere can varies with altitude. A very common way to describe the behaviour of temperature and altitude at this atmosphere layer is by its 'scale height'. Scale height is related to the temperature (T) and mean molecular mass (m) of the atmosphere as given by the relation:

$$H = \frac{RT}{g} \tag{5}$$

where

H	=	Scale Height
T	=	Temperature
R	=	Universal gas constant
g	=	Gravitational force

We incorporate equation (5) in our model formulation due to the existent of various gaseous element within the atmosphere.

By differentiating equation (5) with respect to y

$$\frac{\partial H}{\partial y} = \frac{R}{g}\frac{\partial T}{\partial y} \tag{6}$$

using (6) in (4), we obtain

$$\frac{\partial T}{\partial y} = -f\frac{\overline{U}}{R} \tag{7}$$

Combining equation (2) and (3) then taking the shallow height of the atmosphere "h" to be constant, this leads to a new continuity equation.

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + fu = 0 \tag{8}$$

To represent the behaviour of the atmosphere in other to derived a model for the land surface temperature from equation (1) to (4), there is a need to consider the temperature lapse rate "Υ"at which temperature decreases with increase in altitude, by subtracting the product of the temperature lapse rate Υ and the atmospheric altitude φ from equation (8).

$$\frac{\partial T}{\partial y} = -f\frac{\overline{U}}{R} - \gamma\varphi \tag{9}$$

We therefore have the following equations:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} - fv = 0 \tag{10}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + fu = 0 \tag{11}$$

$$\frac{\partial T}{\partial y} = -f\frac{\overline{U}}{R} - \gamma\varphi \tag{12}$$

To obtain solution for our model equation, we discretize equation (10) to (12).

For the purpose of this study Finite Difference Method (FDM) will be adapted for our discretization process.

2.1 Discretization of the Model Equations

Discretizing equation (1.9);

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} - fv = 0$$

Using Forward Difference Formula for $\frac{\partial u}{\partial t}$ and Central Difference Formula for $\frac{\partial u}{\partial x}$

$$\frac{u_{i+1,j} - u_{i,j}}{\Delta t} + u_{i,j}\left(\frac{u_{i,j+1} - u^{n}_{i,j-1}}{2\Delta x}\right) - fv_{i,j} = 0 \tag{13}$$

$$u_{i+1,j} - u_{i,j} + \frac{\Delta t}{2\Delta x}u_{i,j}\left(u_{i,j+1} - u_{i,j-1}\right) - \Delta t\ fv_{i,j} = 0 \tag{14}$$

$$u_{i+1,j} = u_{i,j} - \frac{\Delta t}{2\Delta x}u_{i,j}\left(u_{i,j+1} - u_{i,j-1}\right) + \Delta t\ fv_{i,j} \tag{15}$$

Taking $k = \frac{\Delta t}{\Delta x}$

$$u_{i+1,j} = u_{i,j} - \frac{k}{2} u_{i,j}\left(u_{i,j+1} - u_{i,j-1}\right) + \Delta t\ fv_{i,j} \quad (16)$$

$$u_{i+1,j} = u_{i,j}\left(1 - \frac{k}{2}\left(u_{i,j+1} - u_{i,j-1}\right)\right) + \Delta t\ fv_{i,j} \quad (17)$$

Also discretizing equation (1.10)

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + fu = 0$$

Using Forward Difference Formula for $\frac{\partial v}{\partial t}$ and Central Difference Formula for $\frac{\partial v}{\partial x}$

$$\frac{v_{i+1,j} - v_{i,j}}{\Delta t} + u_{i,j}\left(\frac{v_{i,j+1} - v_{i,j-1}}{2\Delta x}\right) + fu_{i,j} = 0 \quad (18)$$

$$v_{i+1,j} - v_{i,j} + \frac{\Delta t}{2\Delta x} u_{i,j}\left(v_{i,j+1} - v_{i,j-1}\right) + \Delta t\ \ fu_{i,j} = 0 \quad (19)$$

$$v_{i+1,j} = v_{i,j} - \frac{\Delta t}{2\Delta x} u_{i,j}\left(v_{i,j+1} - u_{i,j-1}\right) + \Delta t\ \ fu_{i,j} \quad (20)$$

$Taking\ k = \frac{\Delta t}{\Delta x}$

$$v_{i+1,j} = v_{i,j} - \frac{k}{2} u_{i,j}\left(v^n_{i,j+1} - v^n_{i,j-1}\right) - \Delta t\ fu_{i,j} \quad (21)$$

$$v_{i+1,j} = v_{i,j} - u_{i,j}\left(\frac{k}{2}\left(v_{i,j+1} - v_{i,j-1}\right) - \Delta t\ f\right) \quad (22)$$

Discretizing equation (1.11)

Using Central Difference Formula for $\frac{\partial T}{\partial y}$

$$\frac{\partial T}{\partial y} = -f\frac{\overline{U}}{R} - \gamma y \quad (23)$$

$$\frac{T_{i,j+1} - T_{i,j-1}}{2\Delta y} = -\frac{1}{R} f\bar{u}_{i,j} - \gamma y_{i,j} \quad (24$$

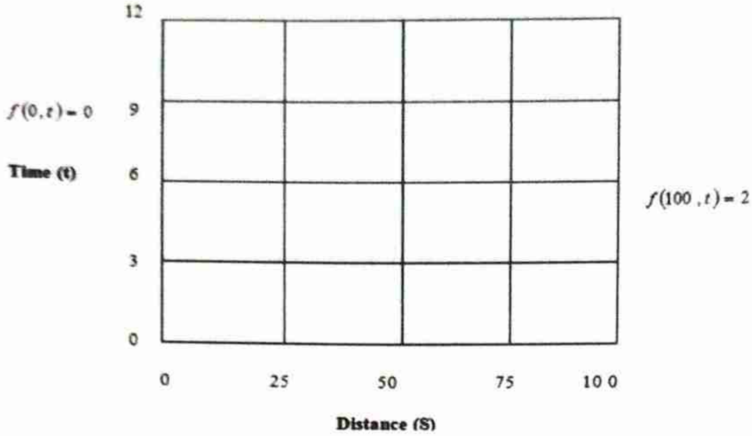
$$T_{i,j+1} - T_{i,j-1} = 2\Delta y\left(-\frac{1}{R} f\bar{u}_{i,j} - \gamma y_{i,j}\right) \quad (25)$$

$$T_{i,j+1} = T_{i,j-1} - 2\Delta y\left(\frac{1}{R} f\bar{u}_{i,j} + \gamma y_{i,j}\right) \quad (26)$$

Now these discretized equations are no longer a continuous function but a discrete one, that can be handled by a computer, to avoid error and to obtain good numerical result for this study math labs software were used, with the following initial and boundary conditions.

2.2 Boundary conditions for the simulation of Zonal Wind Speed

We need to consider a grid consisting of (M+1) and (N+1) nodes, i.e. 5 x 5=25 nodes:-



The value of the zonal velocity "$u_{i,j}$" at the following boundaries of the grid is calculated based on the appropriate boundary conditions.

i. At the left hand boundary, the values are calculated from $f(0,t)=0$,

ii. At the right hand boundary, the values are calculated from $f(100,t)=2$,

iii. At the top boundary, the values are calculated from $f(s,12)=0$

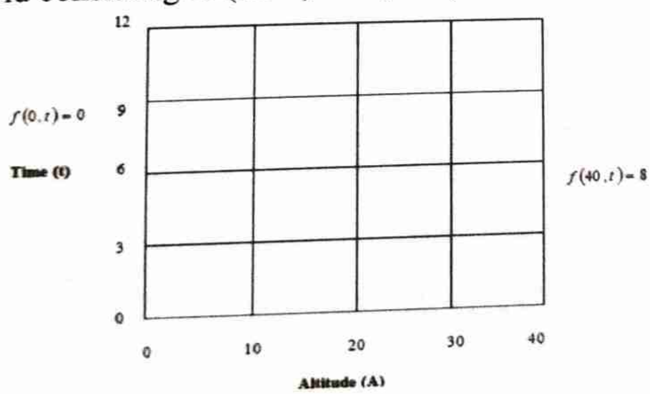
Our aim is to find the value of the zonal wind speed at the bottom of the grid within the interval $0 \le S \le 100$ The values so aimed at are obtained from the routine operation of equation (3.10) derived above.

$h = \frac{S_{max}}{M} = \frac{100}{4} = 25$ (considering M= 4 steps in the interval ($0 \le S \le 100$))

$k == \frac{T_{max}}{N} \frac{12}{4} = 3$ (Considering N= 4 steps in the interval ($0 \le t \le 12$))

2.3 Boundary conditions for the simulation of Meridional Wind Speed

We need to consider a grid consisting of (M+1) and (N+1) nodes, i.e. 5 x 5=25 nodes: -



The value of the meridional wind velocity "v_{ij}" at the following three boundaries of the grid is calculated based on the appropriate boundary conditions.

i. At the left hand boundary, the values are calculated from $f(0,t)=0$,

ii. At the right hand boundary, the values are calculated from $f(40,t)=8$,

iii. At the top boundary, the values are calculated from $f(s, 12) = 0$.

Our aim is to find the value of the Meridional wind speed at the bottom of the grid within the interval $0 \le A \le 40$ .the values so aimed at are obtained from the routine operation of equation (3.11) derived above, i.e. which operates forward in time starting from i=1 and proceed to i =2,3,4.

$h = \frac{A_{max}}{M} = \frac{40}{4} = 10$ (considering M= 4 steps in the interval $(0 \le A \le 40)$)

$k = \frac{T_{max}}{N} = \frac{12}{4} = 3$ (Considering N= 4 steps in the interval $(0 \le t \le 12$

3. Data Analysis

Wind is an important natural resource that have been used for centuries for navigation and agriculture. The knowledge of wind speed and direction is vital not only for its meteorological significance. The Nigerian Meteorological Agency is the primary outlet of meteorological data in the country. The Agency started operations in 1937 as the agency responsible for all forms of weather-observations in the country.

Table 1: Meridional Mean Wind Speed

Altitude(m)	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150
Wind speed	5.4	5.9	6.2	6.7	6.75	6.8	7.0	7.25	7.37	7.50	7.60	7.68	7.74	7.82	8.0

Source (Nimet Abeokuta, Ogun State, 2009)

Table 1: Shows a long range of mean value of meridional wind speed data, extracted from Nimet Abeokuta, Ogun State for the year 1990-2008, taking at a height of 10m above the sea level.

Table 2: Zonal Mean Wind Speed

Distance (m)	25	50	75	100	125	150	175	200	225	250	275	300	325
Wind speed	0.8	0.82	1.0	1.1	1.18	1.94	1.2	1.27	1.3	1.34	1.42	1.44	1.61

Source (Nimet Abeokuta, Ogun State, 2009)

Table 2: Shows a long range of mean value of zonal wind speed data, extracted from Nimet Abeokuta, Ogun State for the year 1990-2008, taking at a height of 4 m, above the sea level.

4. Results

Table 3: Simulation result generated for zonal wind velocity.

0.0000	5.5400	5.9000	6.2000	8
0.0000	5.1171	5.8610	6.0744	8
0.0000	4.8997	58521	5.9322	8
0.0000	4.8670	5.8474	5.6844	8
0.0000	4.8639	5.8410	5.2348	8

Time: 12, 9, 6, 3

Altitude (A): 0, 10, 20, 30, 40

Table 3 is the solution obtained from the simulation of the zonal wind speed dataset in table 1.2, with the difference equation (1.16) using Matlab software. This solution shows the pass predicted behavior of the zonal wind velocity along the horizontal distance with time. This result shows gradual increase in zonal wind speed along the horizontal distance at a particular time. It is incorporated into our model equation to validate the model for the prediction of land surface temperature.

Table 4: Stimulation result generated for meridional wind velocity.

0.0000	5.5400	5.9000	6.2000	8
0.0000	5.1171	5.8610	6.0744	8
0.0000	4.8997	58521	5.9322	8
0.0000	4.8670	5.8474	5.6844	8
0.0000	4.8639	5.8410	5.2348	8

Time: 12, 9, 6, 3

Altitude (A): 0, 10, 20, 30, 40

Table 4 is the solution obtained from the stimulation of the meridional wind speed dataset in table 1.1, with the difference equation (1.21) using Matlab software. This solution shows the pass predicted behavior of the meridional wind velocity along the atmosphere altitude with time. This result shows gradual increase in zonal wind speed along the vertical distance at a particular time.it is incorporate into our model equation to validate the model for the prediction of land surface temperature.

Graphical solution to zonal velocity and meridional velocity at 3hours

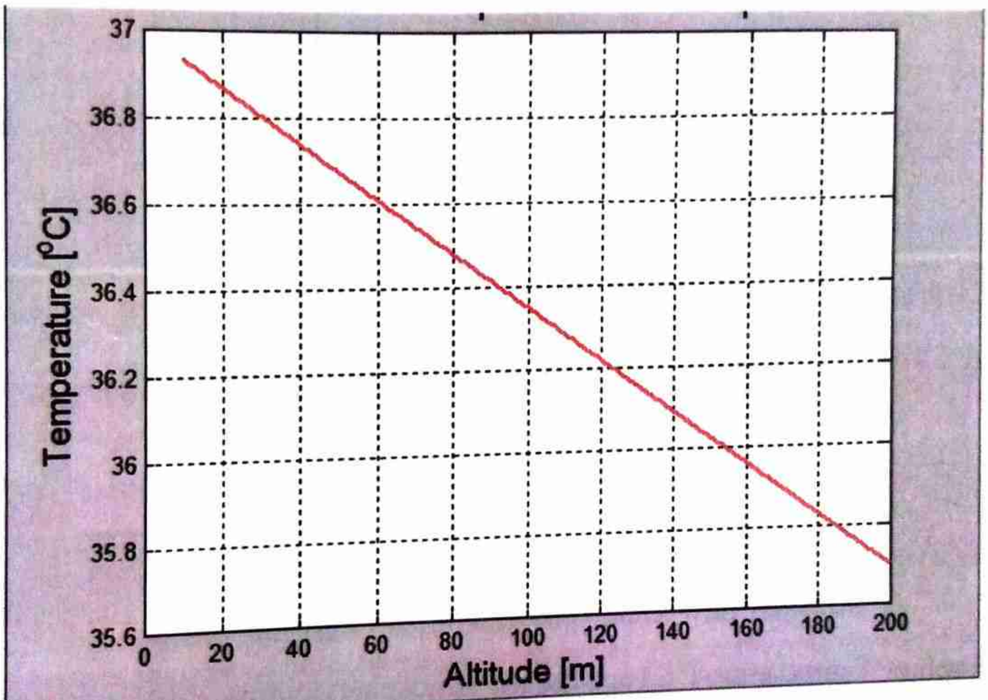


Figure 1: Graph of Temperature (^{0}C) against the atmospheric Altitude difference of 20m.

Figure 1: is the graphical solution to the simulation of the difference equation, (17), (1.22) and (26) with the mean value of the zonal and meridional wind speed presented in table (3) and (4) at three hours with altitude difference of 20 m, using Mathlab software. This shows the relationship between the land surface temperature and the atmospheric altitude for places with the same zonal and meridional wind speed with altitude differences of 20m.

Table 5: Predicted temperature extracted from Fig 1

Altitude (m)	15	25	35	45	55	75	85	95	105
Temperature (^{0}C)	36.9	36.8	36.69	36.67	36.61	36.50	36.42	36.38	36.26

It can be observed from figure 1.5, that the land surface temperature increases as the atmospheric altitude decreases, also the temperature drops as the altitude increases. It can be see how the temperature gradually decreases from 37⁰C to 35.7⁰C for the altitude 0m to 200m.

Graphical solution to zonal velocity and meridional velocity at 6hours

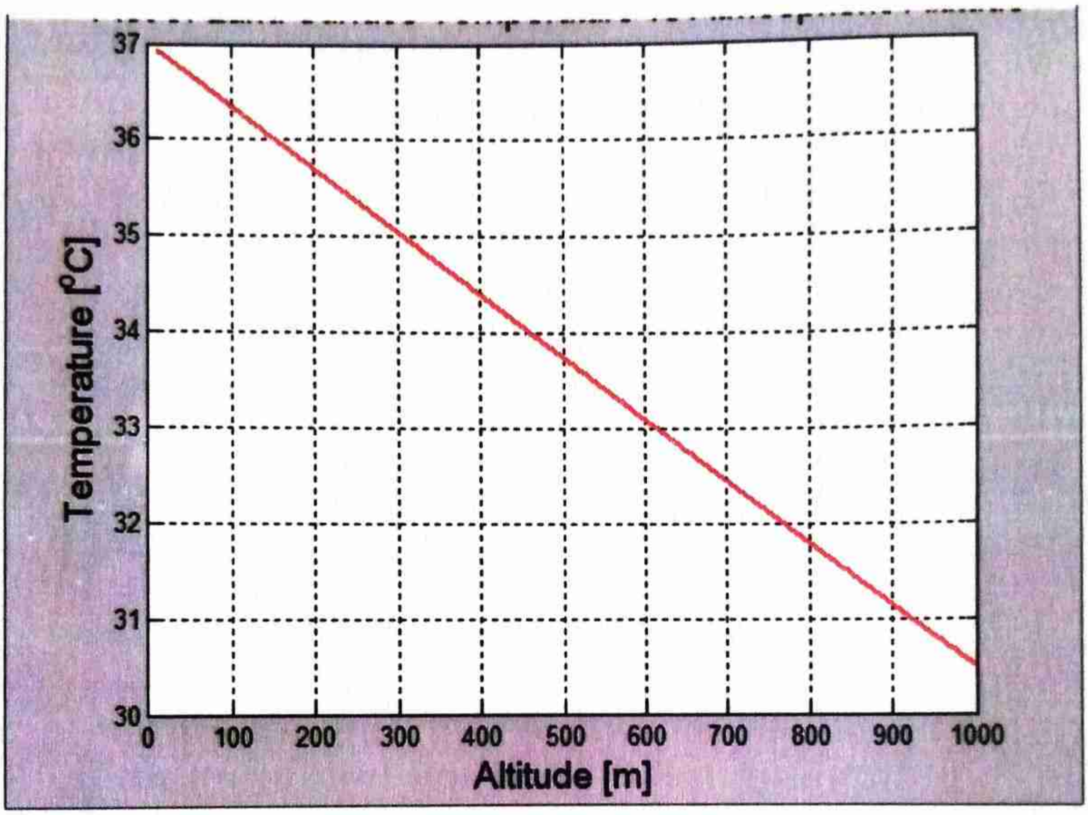


Figure 2: Graph of Temperature (⁰C) against the atmospheric altitude difference of 100 m.

Figure 2: is the graphical solution to the stimulation of the difference equation (17), (22) and (26) with the mean value of the zonal and meridional wind speed presented in tables (3) and (4) with altitude difference of 100m, using Mathlab software. This shows the relationship between the land surface temperature and the atmospheric altitude for places with the same zonal and meridional wind speed with altitude differences of 100m.

Table 6: Predicted temperature extracted from Fig 2

Altitude (m)	120	150	180	210	240	270	300	330	360
Temperature (⁰C)	36.2	36.0	35.8	35.5	35.3	35.2	35.0	34.8	34.6

It can be observed from figure 2, that the land surface temperature increases as the atmospheric altitude decreases, also the temperature drops as the altitude increases. It can be see how the temperature gradually decreases from 37⁰C to 30.5⁰C for the altitude 0m to 1000m.

Graphical solution to zonal velocity and meridional velocity at 9hours

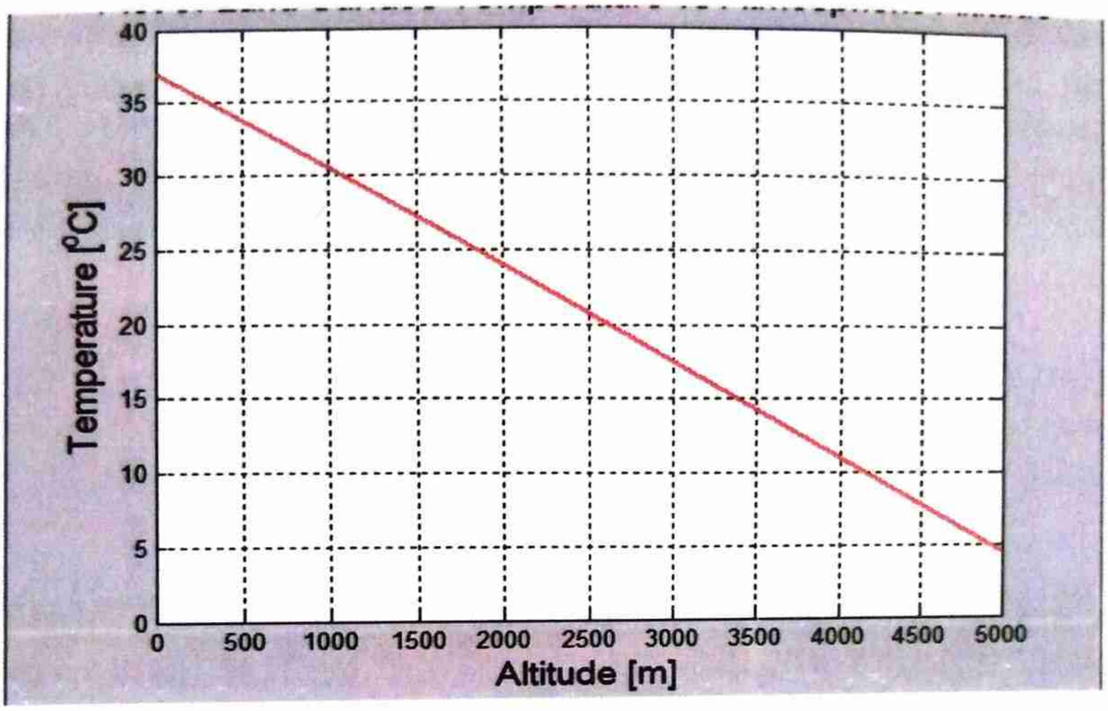


Figure 3: Graph of Temperature (^{0}C) against the atmospheric Altitude difference of 500 km

Graph 1.3: is the graphical solution to the simulation of the difference equation, (17), (22) and (26) with the mean value of the zonal and meridional wind speed presented in table (3) and (4) with altitude difference of 500m using Mathlab software. This shows the relationship between the land surface temperature and the atmospheric altitude for places with the same zonal and meridional wind speed with altitude differences of 500m.

Table 7: Predicted temperature extracted from fig 3

Altitude (m)	400	800	1200	1600	2000	2400	2800	3200	3600
Temperature (^{0}C)	30.9	30.1	29.0	27.0	24.0	21.0	18.0	16.0	14.0

It can be observed from figure 1.3, that the land surface temperature increases as the atmospheric altitude decreases, also the temperature drops as the altitude increases. It can be seen how the temperature gradually decreases from 37^{0}C to 4.8^{0}C for the altitude 0 to 5000m.

Graphical solution to zonal velocity and meridional velocity at 12hours

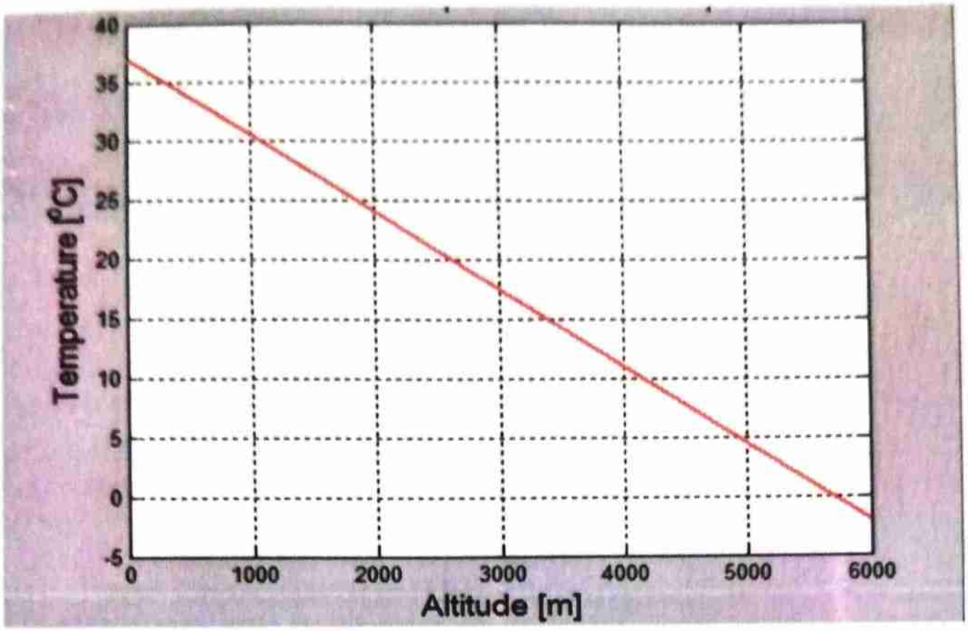


Figure 4: Graph of Temperature (^{0}C) against the atmospheric altitude difference of 1000m

Figure 4: is the graphical solution to the simulation of the difference equation, (17), (22) and (26) with the mean value of the zonal and meridional wind speed presented in table (3) and (4) with altitude difference of 1000m using Mathlab software. This shows the relationship between the land surface temperature and the atmospheric altitude for places with the same zonal and meridional wind speed with altitude differences of 1000m.

Table 8: Predicted temperature extracted from Fig 4

Altitude (m)	3800	4000	4200	4400	4600	4800	5000	5200	5800
Temperature (^{0}C)	13.1	11.0	9.2	7.3	6.4	5.2	4.9	3.0	0.00

It can be observed from figure 4, that the land surface temperature increases as the atmospheric altitude decreases, also the temperature drops as the altitude increases. It can be seen how the temperature gradually decreases from 37^{0}C to 3^{0}C for the altitude 0 to 6000m.

4.1 Analysis of Result

From figure 1 and table 5 the land surface's temperature varies with the atmospheric altitude and it can also be observed in figure 1.1that the temperature gradually decreases from 37^{0}C to35.7^{0}C for the altitude 0m to 200m. However, in figure 2 and table 1.6, the temperature decreases from 37^{0}C to 31.5^{0}C for the altitude 0m to 100m while in figure 1.3 and table 7 the temperature decreases from 37^{0}C to 4.8^{0}C for the altitude 0m to 5000m. From Figure 4 and table 8 , the temperature decreases from 37^0C to -3^0C for the altitude 0m to 6000m at this point, any further increases in atmospheric altitude, implies that the temperature will become freezing. As exhibited by figure 4, it is clearly seen that the temperature decreases from 37^0C to -3.4^0C for the altitude 0m to 12000m, which is more freeze than what was observed other figures 1 to 3. However, the behaviors' exhibited by both the atmosphere altitude and the land surface temperature in this work shows that at any given time at a giving space on the earth surface, "the higher the location of a particular area, the lower the temperature of people living in the area".

5. CONCLUSION

This paper presents a numerical method for the prediction of Land Surface Temperature using the shallow water equation. The atmospheric behaviour was carefully studied and most important parameters were identified and well represented accordingly as required in the prediction of Land Surface Temperature using the shallow water equation.

The demographic profile of Nigeria meridional and zonal wind speed were used to study the behaviour of this model.

Comparing our work with that of Zaharaddeen et al (2016), it shows that the land surface temperature varies over space and time. This implies that the model obtained in this study can be adopted in the prediction of land surface temperature of any place if both the zonal and meridional wind speed of a place are known. The results also revealed how the Land Surface Temperature increases as the altitude decreases.

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