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RESEARCH PAPER

Boundary Layer Flow of Rarefied Gas Over a Flat Plate with Constant Heat Flux Boundary Conditions

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ABSTRACT

In the current work, Boundary layer flow of rarefied gas over a flat plate with constant heat flux boundary conditions is presented and solved numerically. The first-order slip boundary condition is adopted in the derivation. By using appropriate similarity variables, the fundamental equations of the boundary layer are transformed to ordinary differential equations. These ordinary differential equations are solved numerically using a fourth order Runge-Kutta and shooting method. The dimensionless velocity, temperature and shear stress profiles are plotted and discussed. Consequently, the velocity profiles, temperature profiles and the wall shear stress exhibit a dependence on the slip coefficient. It is found that an increase in slip parameter leads to an increase in velocity and a fall in skin-friction.

Keywords: Heat Flux, Skin-friction, Velocity, Momentum, Radiation

INTRODUCTION

It was Blasius who solved the boundary layer problem for a free stream past a fixed flat plate using a similarity transformation technique (White, F. M., 1991). Klemp and Acrivos (1972) studied the boundary layer flow for a free stream past a moving semi-infinite flat plate. However, in many engineering applications in micro-scale such as in Micro-Electro-Mechanical Systems (MEMS), compared to the characteristic length of the micro-devices, the fluid behavior might be treated as a rarefied gas (Gad-el-Hak, M., 1999). On the other hand, for large-scale problems with low density, the fluid is also modeled as a rarefied gas, for example, in outer space applications (Shidlovskiy, V. P.,1967). The behavior of a rarefied gas is determined by the Knudsen number, Kn, which is defined as the ratio of the mean free path of the fluid molecules to a characteristic length of the flow. The flow can be classified into four regimes according to the magnitude of the Knudsen number. If $Kn>10$ it is the free molecule flow, if $10 > Kn > 0.1$ it is the transition flow, if $0.1 > Kn > 0.01$ it is the slip flow, and if $Kn<0.01$ it is the conventional viscous flow. For the flow in the slip regime, the fluid motion still obeys the Navier–Stokes equations. The Blasius boundary layer flow with slip condition at the wall was discussed in (Martin, M. J. and Boyd, I. D., 2000) . In many problems, particularly those involving the cooling of electrical and nuclear components, the wall he at flux is specified. In such problems, over heating burnout and meltdown are very important issues. From practical stand point, an important wall model is considered with constant heat flux. In many applications, the wall heating effect is the result of radiation heating (the constant heat flux condition applies to nuclear radiation heating) from the other side or, as in the case of electronic components, the result of resistive heating (Bejan, A., 1995). The problems with prescribed heat flux are special cases of the vast analytically accessible class of problems. Sparrow et al (1958), Merkin et al. (1989), Lee.et al. (1992), Malarvizhi et al. (1994), Burak et al.(1995) and Pantokratoras, A.(2003) are some of the researchers who have investigated the convection flow with prescribed heat flux conditions.

Therefore, in this paper, Boundary layer flow

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of rarefied gas over a flat plate with constant heat flux boundary conditions and will be solved and discussed.

MATHEMATICAL FORMULATION

The two dimensions, steady laminar, external fluid over a surface with slip boundary conditions, which move with constant velocity u_w in a viscous incompressible fluid, the ambient fluid or the far flow from the plate moves with constant velocity $u = u_\infty + \in$ in the direction of the plate velocity. It is assumes that the no slip condition on the flat plate is replaced with partial slip condition on the form $\bar{u}(\bar{x},0) = \bar{u}_w + A\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)_{\hat{y}=0}$ (Zhu, j. et al); when $\bar{x}$ and $\bar{y}$ are the Cartesian coordinates along the plate and normal to it, respectively, u is the velocity component along the $\bar{x}$ –direction and A is the slip coefficient .

The continuity, momentum and energy equations are given by

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \qquad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = v\frac{\partial^2 u}{\partial y^2} \qquad (2)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \qquad (3)$$

With associated boundary conditions:

$$y = 0, u(x,0) = u_w + A\left(\frac{\partial u}{\partial y}\right)_{y=0}, v = 0\,, -k\frac{\partial T}{\partial y}(x,0) = q"$$

$$y \to \infty, u = u_\infty + \epsilon\,, T = T_\infty$$

We assume that $A = \left(\frac{u_w}{\nu x}\right)^{1/2}\lambda$, where ν is the kinematic viscosity of the fluid and λ is the slip parameter, which is non-negative constant.

We define the similarity variable $\eta = y\sqrt{\frac{u_\infty}{\nu x}}, u = u_\infty + f'(\eta), v = \frac{1}{2}\sqrt{\frac{u_\infty}{\nu x}}(\eta f'(\eta) - f)$ and $\theta = \frac{T - T_\infty}{\frac{q"}{k}\left(\frac{\nu x}{u_\infty}\right)^{\frac{1}{2}}}$ which was introduce by Bejan [6]; then the Nervier-Stokes equations in dimensions under boundary layer approximations reduces to the following governing differential equation and the boundary conditions for this problem and are given by Bejam, A. (1995).

$$f''' + \frac{1}{2}ff" = 0 \qquad (4)$$

$$\theta" + \frac{1}{2}Pr(f\theta' - f'\theta) = 0 \qquad (5)$$

$$\eta = 0, f(0) = 0, f'(0) = 1 + \lambda f"(0), \theta'(0) = -1 \qquad (6)$$

$$\eta \to \infty, f'(\infty) = 1 + \epsilon, \theta(\infty) = 0 \qquad (7)$$

Where $\epsilon = \frac{(u_\infty - u_w)}{u_w}$ and $\lambda = \frac{U_{slip}}{U_\infty} = \left(\frac{2}{\sigma} - 1\right)K_{n,x}Re_x^{\frac{1}{2}}$ is dimensionless parameter with $K_{n,x} = \frac{1}{x}, Re_x^{\frac{1}{2}} = \frac{u_\infty}{2v}$.

The slip velocity at an isothermal wall can be obtained based on Maxell's first order approximation as (White, 1999 and Gad-el-Ha, 1999),.

$U_{slip} = \left(\frac{2}{\sigma} - 1\right)l\frac{du}{dy}$ where σ is the tangential momentum accommodation coefficient and *l* is the mean free path.

Numerical methods

Eqs. (3) and (4) along with boundary conditions are solved using shooting method by converting them to an initial value problem.

We set

$$f' = z$$

$$z' = p$$

$$p' = -\frac{fp}{2}$$

$$\theta' = q$$

$$q' = -\frac{1}{2}Prfq + z\theta$$

with the boundary conditions

$$f(0) = 0, z(0) = 1 +^{\lambda} p(0), q(0) = -1$$

In order to integrate (17) and (18) as an initial value problem we require a value for p(0) i.e.$f"(0)$ and $\theta(0)$ but no such values are given in the boundary. The suitable guess values for$f"(0)$ and $\theta(0)$ are chosen and then integration is carried out. We compare the calculated f' and $\theta at \eta = 4$ (say) with the given boundary conditions $f'(4) = 1 + \epsilon$ and $\theta(4) = 0$ and adjust the estimated values, $f''(0)$ and $\theta(0)$to give a better approximation for the solution. We take the series of values for $f''(0)$ and $\theta(0)$, and apply the fourth order classical Runge–Kutta method with step-size$\Delta\eta = 0.01$. The above procedure is repeated until we get the converged results within a tolerance limit of 10^{-6}.

Table.1: Effects of Pandtlnumbers on the skin friction and temperature at =0

Pr	Λ	ϵ	$f''(0)$	$\theta(0)$
0.01	1	0.2	0.0759057962	2.926230422
0.05	1	0.2	0.0759057962	2.669318505
0.1	1	0.2	0.0759057962	2.414296225
1	1	0.2	0.075089692	1.067059526
2	1	0.2	0.075905795	0.760090631
3	1	0.2	0.075905794	0.621894645
4	1	0.2	0.075905794	0.539236910

Table.2: Effects of ε on the skin friction and temperature at η=0

Pr	λ	ϵ	$f''(0)$	$\theta(0)$
0.71	1	-0.6	-0.192216656	1.604056639
0.71	1	-0.4	-0.133843477	1.502614995
0.71	1	-0.2	-0.069692518	1.413898806
0.71	1	0	4.10362646153964	1.335932554
0.71	1	0.2	0.074890280	1.267091044
0.71	1	0.4	0.154584485	1.206011630
0.71	1	0.6	0.238683358	1.151545785

Table.3: Effects of slip constant on the skin friction and temperature at η=0

Pr	Λ	ϵ	$f''(0)$	$\theta(0)$
0.71	3	0.2	0.043050464	1.246521918
0.71	5	0.2	0.030137967	1.238635283
0.71	10	0.2	0.017201574	1.230974699
0.71	15	0.2	0.012031379	1.227977356
0.71	20	0.2	0.009250157	1.226379715
0.71	25	0.2	0.007513135	1.225387057
0.71	30	0.2	0.006325266	1.224710488

RESULTS AND DISCUSSION

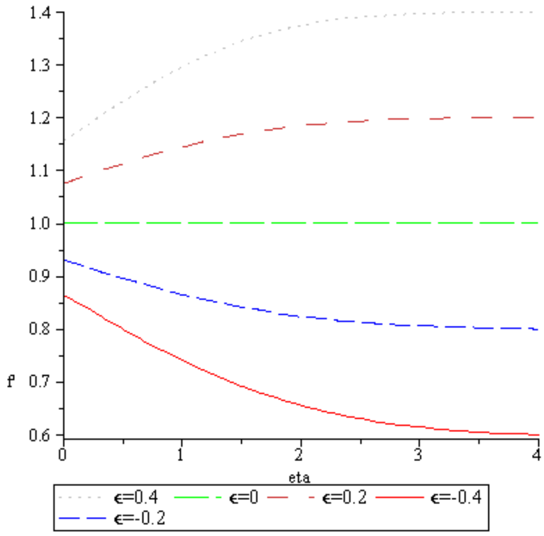

Fig.1. Velocity distribution as a function of η for various values of ε

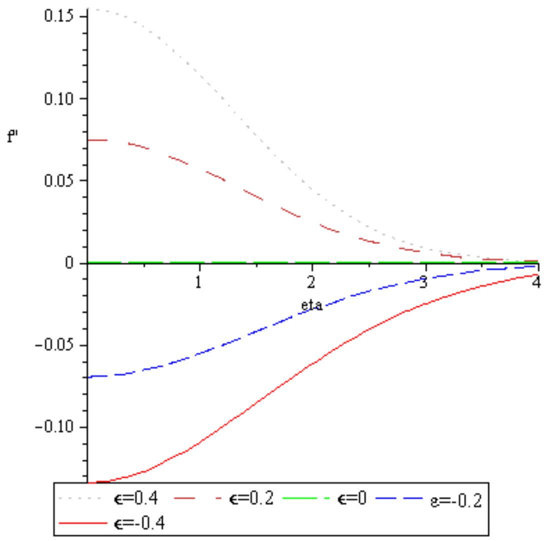

Fig.2. Variation o f the velocity gradient as function of η for various values of ε

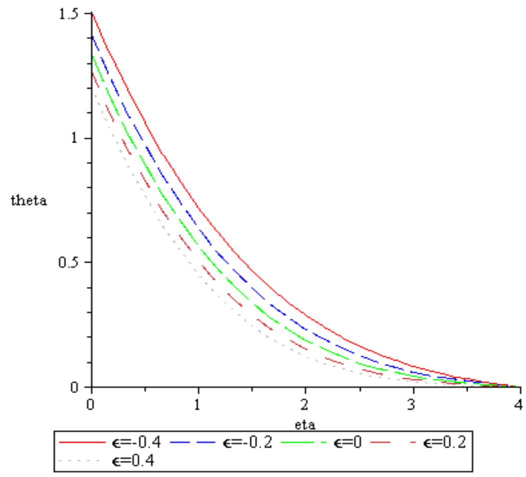

Fig.3. Variation o f the of the non-dimensional temperature as function of η for various values of ε

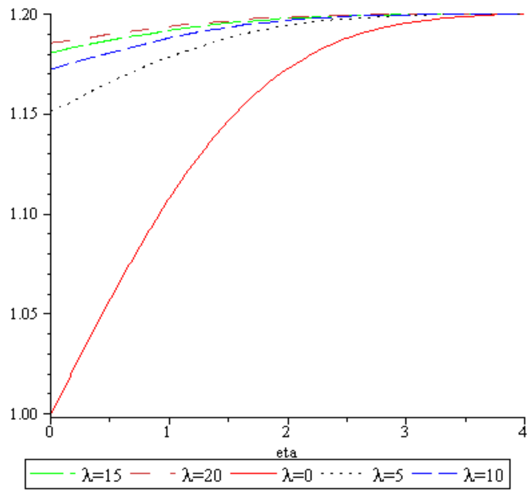

Fig.4. Variation of the velocity profile as function of η for various values of λ

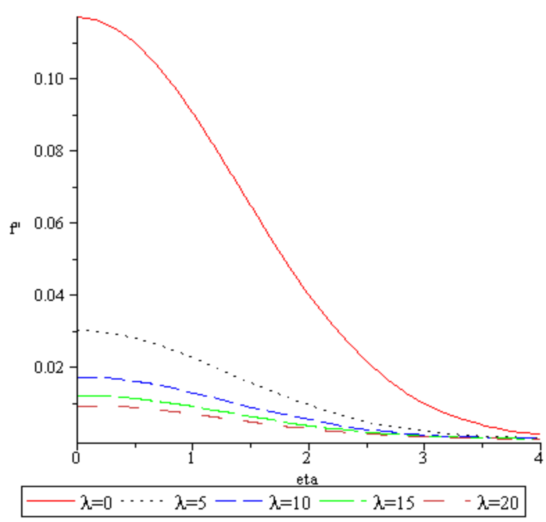

Fig.5. Variation o f the of the velocity gradient as function of η for various values of λ

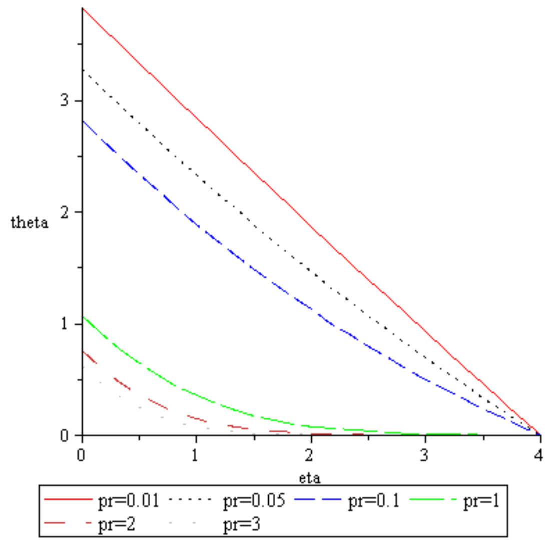

Fig.6. Variation o f the of the non-dimensional temperature as function of η for various values of pandtl

In order to get a clear insight of the physical problem, numerical computations have been carried out using the method described in the previous section for various values of different parameters such as velocity ratio parameter (ϵ), slip parameter (λ) and Prandtl number (Pr) encountered in this problem. For illustrations of the results, numerical values are plotted in Figs. 1–6. Table1, table2 and table3 represent the effect of pandtl number,ϵ and slip constant on skin friction and temperature at η=0.The influences of velocity ratio parameter (ϵ) on velocity, shear stress and temperature are presented in Figs. 1–3. With the increasing velocity ratio parameter, fluid velocity increases in Fig.1. Fig. 2 exhibits that the shear stress decreases with increasing velocity ratio parameter. It is quite obvious that the velocity within the boundary layer increases as the free stream velocity increases. The temperature is found to decrease with increasing ϵ (Fig. 3).Fig 5 and Fig 5 show variation of dimensionless velocity and velocity gradient as function of η. These figure indicate that in case of no slip condition (without micro channel), the velocity at the wall is equal to one. With the increasing values of λ, the fluid velocity increases monotonically.Due to the slip condition at the plate the velocity of fluid adjacent to the plate has some positive value and accordingly the thickness of momentum boundary layer decreases. . Increasing slip coefficient λ tends to decrease the skin friction coefficient in Fig. 5 because of increasing fluid velocity at the wall. In addition, the gradient of the velocity and boundary layer thickness tend to decrease by increasing λ. Fig.6 demonstrates the effect of the Prandtl number to the temperature distribution. The temperature (at a fixed η) as well as the thermal boundary layer thickness rapidly decreases with increasing values of Pr under slip condition. An increase in Prandtl number means an increase of fluid viscosity which causes a decrease in the flow velocity and the temperature decreases. This is consistent with the fact that the thermal boundary layer thickness decreases with increasing Prandtl number.

Conclusion

The boundary layer flow of rarefied gas over a flat plate with constant heat flux boundary conditionshave been obtainednumerically. Our study reveals that the parameters involved in thestudy viz. slip parameters; velocity ratio and Prandtl number significantly affect the flow field and temperature distribution. It is hoped that by our model study, the physics of flow over a flatplate can be utilized as the basis of many engineering and scientificapplications. The results pertaining to this study will serve as amotivation for future experimental work which seems to be lacking atthe present time.

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