

APPLICATION OF LINEAR PROGRAMMING TO OPTIMIZE SOAP PRODUCTION USING OIL FROM IRVINGIA GABONENSIS (OGBONO) SEED

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ABSTRACT

The efficiency of a soap produced with oil extracted from obgono (irvingia gabonensis) seed using Operational Research (linear programming) was investigated. Oil was first extracted from two (2) grounded sizes (1.4mm and 1.0mm) of ogbono seed using a soxhlet set-up and hexane as solvent. 47.98% and 43.64% of oil were extracted from 1.0mm and 1.4mm sizes respectively. This means that more oil was extracted from smaller grounded size. Four (4) pastes of soap were produced with 25ml each of 2 molar and 1 molar concentration of Sodium Hydroxide (NaOH) in four different mass of the extracted oil. 1 molar concentration of NaOH averagely produced more mass (15.5g) of soap than the 2 molar concentrations which averagely produced 8.25g of soap. The parameters used to produce the soap were optimised. Caustic soda (NaOH) was maximized being the objective function of the optimization model and it gave a value of 3.98 unit for every 1.99 unit of paste 1. The unused constrains value (rate of reaction of reagent i.e time and oil) for the production were 242.4 capacity and 0.2 capacity which at this point no further iteration can be done because the objective row in the last tableau table has no negative value.

KEY WORDS: **Linear Programming, Soap Production, Optimization, Ogbono Seed Oil.**

INTRODUCTION

The term "soap" refers to a particular type of detergent in which the water-solubilized group is carboxylate and the positive ion is usually sodium or potassium. Soaps are the product of the reaction between a fat and sodium hydroxide:



Soap is produced industrially in four basic steps. These steps are listed below according to the sequence of production; Saponification, Glycerine removal, Soap purification and Finishing(www.detergent.com, 2010).

Soap constituents (raw materials) are of three classes namely Oil and Fat, base (caustic soda) and perfume. The proportion and manner by which these materials are mixed depend on the process applied in the production (www.detergent.com, 2010). These are known as the tallow (animal fat), coconut and palm kernel (oil sources) which is commonly used in soap making. There are many other sources of oil which can be used for soap making (e.g. ogbono known as *Irvingia gabonensis*, melon, and mango seeds e.t.c). Oil extraction can be done mechanically with an oil press, expeller, or even with a wooden mortar and pestle—a traditional method that originated in India. Oils can also be extracted with solvents such as hexane, petroleum ether, etc. It can be extracted from nuts and seeds by heat. However, extraction by heat is not used commercially for vegetable oils. Pressure extraction separates the oil from the solid particles by simply squeezing the oil out of the crushed mass of seeds (Janet, 2001).

Sodium hydroxide (**NaOH**), also known as **lye** and **caustic soda**, is a caustic metallic base. It is used in many industries, mostly as a strong chemical base in the manufacture of pulp and paper, textiles, drinking water and soaps and as a drain cleaner. Pure sodium hydroxide is a white solid; available in pellets, flakes, granules and as a 50 % saturated solution. It is hygroscopic and readily absorbs water from the air. So it should be stored in an airtight container. It is very soluble in water with liberation of heat. (Sodium hydroxide – wikipedia, 2010)

In chemistry, **soap** is a salt of a fatty acid. Soap is mainly used for washing and cleaning, but soaps are also important components of lubricants. Soaps for cleansing are obtained by treating vegetable or animal oils and fats with a strongly alkaline solution. The alkaline solution, often **lye**, promotes what is known as **saponification**. In saponification, **fats** are broken down (**hydrolyzed**) yielding **crude soap**, i.e. impure salts of **fatty acids** and **glycerol**. When used for cleaning, soap serves as a surfactant in conjunction with water. The cleaning action of this mixture is attributed to the action of micelles; tiny spheres coated on the outside

with polar carboxylate groups, encasing a hydrophobic (lipophilic) pocket that can surround the grease particles, allowing them to dissolve in water. The hydrophobic portion is made up of the long hydrocarbon chain from the fatty acid. In other words, whereas normally oil and water do not mix, the addition of soap allows oils to dissolve in water, allowing them to be rinsed away (soap-wikipedia, 2010).

Irvingia gabonensis (African bush mango), locally known as *ogbono* in Nigeria, is an economic tree cultivated mainly for its seeds. The leaves and stem bark are employed in the traditional African medicine against fever and stomachache. The seeds are popularly used as soup thickener and are responsible for the characteristic appetising flavour of the Nigeria *ogbono* soup (Ekpo, *et al.*, 2007). There are two varieties of *Irvingia gabonensis* namely: the *gabonensis* and the *wombolu* varieties. The *wombolu* variety fruits during the dry season (November to March) while the *gabonensis* variety fruits during the rainy season (April to June). The *gabonensis* variety has an edible yellowish pulp, when ripened, with a turpentine flavour while the *wombolu* variety has a bitter, inedible and acrid pulp. The seeds of the *wombolu* variety are smaller in size but are preferable in soup due to their greater drawability in soup. They are used as soup thickeners because they gelatinise into a thick viscous source base when mixed with oil and water (Onyeike, *et al.*, 1995). The seeds of both varieties are commonly mixed together by the *ogbono* marketers probably to avoid differential pricing of their goods by the customers. The few reports on *Irvingia gabonensis* seeds (Eka, 1980 and Onyeike *et al.*, 1995) are apparently based on the mixed seeds as the samples were purchased from the market.

Linear programming is an optimizing approach which is used to maximize an available resources set for an objective, that must be satisfied by constraints, each of which must be a linear inequality or linear equality (www.columbia.edu.com, 2010). Linear programming is a mathematical technique used for determining the optimum allocation of scarce resources for obtaining a particular objective. Although allocating resources to activities is the most common type of application, linear programming has numerous other important applications as well. Production allocation model, blending model and product mix model are some of the most common areas of applications. In product mix selection, the decision maker wishes to determine the level for a number of production activities during the specified period of time.

These techniques have universal applicability and they are of great help in making rational decisions on different aspects of business. In this research, one of the popular resource management techniques viz., Linear Programming is applied to find out the optimum model pertaining to product mix. Linear programming can only be applied to those problem which satisfy; Objective function, Constraints, Linearity, Non- negativity, and Finiteness conditions. Thus, linear programming is applicable to problems involving quantitative variables which are linear in nature. Given these quantitative variables the maximization or minimization of objective function can be achieved (www.eurojournals.com, 2010). This work intend to employ the technique of linear programming to suggest suitable model that will maximize the efficiency of soap production using oil extracted from *Irvingia gabonensis* (ogbono).

EXPERIMENTAL METHODS

The experiment was carried out in two stages using the same material but different formulation or quantity. The first stage (Experiment A) involves extraction of oil while the second stage (Experiment B) involves preparation and characterization of soap (surfactant), using different concentration of sodium hydroxide solution.

Oil was extracted from two (2) sample sizes (1.4mm and 1.0mm) of ogbono seed using Soxhlet extraction method and hexane as solvent. Two (2) caustic soda solutions of 2 molar and 1 molar concentrations were prepared by dissolving 40g and 10g of caustic soda in 500 ml and 250 ml of water respectively.

Four (4) different masses (24, 20, 16 and 12g) of the extracted oil were measured into calibrated reactors and soap prepared by mixing each with 25ml of the 2 molar concentration of caustic soda prepared earlier on. Using the same masses of oil but mixed with 25 ml of 1 molar concentration of caustic soda paste 2 was prepared. The pH values, the foamibility and the cleanability of the soap produced were investigated and recorded.

Derivation of Objective Function

The aim on the model was to maximize the amount of sodium hydroxide dissolved/used in the paste (soap) produced so that:

- The quantity of oil used to obtain the product (paste) was utilised.

- The time taken to obtain the product (paste) was reduced by getting homogenous solution (product) in quicker time.
- Less effort (energy) would be used in stirring, thereby conserving human or machine energy

Objective Function

The first step in linear programming is to decide what result is required, that is the objective function. This is to maximize the yield or contribution in this project. However, the contribution of each constituent (raw material) will be maximized. Once the objective function has been defined, maximization of the products contribution is then stated mathematically with the element involved. The objective function for the surfactant, soap produced in this project was based on the quantity produced in which contribution was done by sodium hydroxide (NaOH), reacting on other constituents and was maximized.

Optimization of Surfactant (Soap) Production

In this project, the contribution of sodium hydroxide on each other raw materials was maximized. The optimum value (maximum or minimum) of a function, $f(X_1, X_2, \dots, X_n)$ of n real variables. Since the functions refer to the contribution obtained by producing quantities of X_1 of product P_1 , thus our desire will be to maximize the function.

The circumstances that govern the achievement of an objective function were identified as;

- I. Quantity of oil material
- II. Rate of reaction of raw material (time)
- III. Cost of raw materials
- IV. Contribution of sodium hydroxide in each paste respectively.

Maximization was achieved by the simplex tableau method. The decision variables were X_1 , X_2 and the slack variables S_1 , S_2 , and S_3 which were referred to as solution variables. The resources limitation in this project follows a typical pattern of less than or equal to (\leq).

In addition, a general limitation applicable to any maximization problem is that it is not possible to make negative quantities of a column

$$X_1 \geq 0, X_2 \geq 0.$$

RESULTS AND DISCUSSIONS

The Tables 4.1 and 4.2 show the quantity of oil extracted from different masses and two sizes (1.4 mm and 1.0 mm) of grounded ogbono seed using hexane as solvent.

Table 4.1 Quantity of oil extracted from grounded sample (ogbono seed) of size 1.4 mm.

S/N	Weight of rapped sample before extraction (g)	Weight of rapped sample after extraction (g)	mass of oil extracted (g)	Percentage of oil extracted. (%)	Average Percentage of oil extracted. %
1	15	8	7	46.67	43.64
2	16	10	6	37.50	
3	16	9	7	43.75	
4	15	8	7	46.67	

Table 4.2 Quantity of oil extracted from grounded sample (ogbono seed) of size 1.0 mm.

S/N	Weight of rapped sample before extraction (g)	Weight of rapped sample after extraction (g)	Mass of oil extracted (g)	Percentage of oil extracted. (%)	Average Percentage of oil extracted. %
1	8	4	4	50.00	47.98
3	9	4	5	55.56	
5	10	5	5	50.00	
7	11	7	4	36.36	

Table 4.3 and 4.4 show the quantity of soap produced from different measures of oil extracted from grounded ogbono seed with the same volume of caustic soda solution (25ml) but with two different concentrations (2 molar and 1 molar) respectively.

Table 4.3 Quantity of soap produced from the different paste 1 formed using 2 molar concentration of sodium hydroxide solution (NaOH).

No. of beaker	Mass of oil in beaker (g)	Volume of NaOH solution added (ml)	Boiling point rate of oil (sec)	Time of paste formation (sec)	pH value	Mass of soap produced (g)	Mass of soap produced (g)
1	24	25	60	180	5	6	8.25
2	20	25	60	240	6	10	
3	16	25	60	360	7	9	
4	12	25	60	420	9	8	

Table 4.4 Quantity of soap produced from the different paste 2 formed using 1 molar concentration of sodium hydroxide solution (NaOH).

No. of beaker	Mass of oil in beaker (g)	Volume of NaOH solution added (ml)	Boiling point rate of oil (sec)	Time of paste formation (sec)	pH value	Mass of soap produced (g)	Mass of soap produced (g)
1	24	25	60	240	5	16	15.50
2	20	25	60	360	7	18	
3	16	25	60	480	8	15	
4	12	25	60	600	9	13	

Optimization by Simple Tableau Method

Objective Function Formulation

From Table 4.3 and 4.4, let the pastes produced in both analysis be X_1 and X_2 .

2g of NaOH was dissolved in 25ml of water to give a concentration of 2 molar. This was used to produce paste 1.

2g of NaOH was dissolved in 25ml of water to give a concentration of 1 molar. This was used to produce paste 2.

The objective function is thus expressed as (P).

$$P = 2X_1 + X_2$$

Limitation or Constrain

From the table 4.3 and 4.4, following other constrains, the oil constrain can be deduced that,

$$20X_1 + 20X_2 \leq 40$$

Time constrain $240X_1 + 360X_2 \leq 600$

Cost constrains $222X_1 + 219X_2 \leq 441$ [Cost of Raw Material neglecting labour cost]

Problem statement;

Maximize $P = 2X_1 + X_2$ ----- (1)

Subject to

$20X_1 + 20X_2 \leq 40$ ----- (2)

$240X_1 + 360X_2 \leq 600$ ----- (3)

$222X_1 + 219X_2 \leq 441$ ----- (4)

Initial Simplex Tableaux

Basis	X ₁	X ₂	S ₁	S ₂	S ₃	P	Value
S ₁	20	20	1	0	0	0	40
S ₂	240	360	0	1	0	0	600
S ₃	222	219	0	0	1	0	441
P	-2	-1	0	0	0	1	0

Second Simplex Tableau

Basis	X ₁	X ₂	S ₁	S ₂	S ₃	P	Value
X ₁	0	0.2	1	0	-0.0045	0	0.2
S ₂	0	122.4	0	1	-1.08	0	242.4
S ₃	1	0.99	0	0	0.0045	0	1.99
P	0	0.98	0	0	0.009	1	3.89

Discussion of Result

Extraction of Oil from *Irvingia Gabonensis* (Obgono) Seed

Extraction was done using soxhlet apparatus with different sizes of grounded obgono seed. The solvent used was hexane. Using the different sizes of sample, it was observed that higher mass of oil with an average percent of 47.98% was obtained from 1.00mm size of the sample while from 1.4mm size of the sample an average percentage extraction of 43.64% was obtained. This shows that the smaller the sample size, the more the oil extracted. This is because the smaller size sample provided wider and more surface area for the oil in the sample to be squeezed out.

Preparation of Surfactant, Soap

The dissolution of caustic soda in water generated heat (exothermic reaction). On adding this solution into the extracted oil with adequate stirring in a reactor, neutralization took place. The mixture was allowed to cool after paste formation was observed. Two layers were observed after cooling for 24 hours, with the upper layer as the soap and the down layer as the glycerol.

From tables 4.3 and 4.4, it can be observed that the time of soap formation increases as the mass of oil decreases. This means that the more the oil the faster the rate of reaction with NaOH and thus the faster the formation of soap. These two tables also show that the less the oil, the higher the pH value. This is because; NaOH being an alkaline is neutralized in the

presence of oil. This means that oil lowers the alkalinity of NaOH and so the more the oil, the lower the pH. Also the highest mass of soap was produced with 20g of oil and decreases as the mass of oil decreases. It can then be adduced that the optimal mass of oil required to produced maximum amount of soap with 25ml of both 2 molar and 1 molar concentration of NaOH is 20g. However, more mass of soap (18g) was produced and more time (360 sec) was required when 1 molar concentration of NaOH was used. This means that the more the concentration of NaOH, the faster the formation of soap but the less the mass of soap produced and vice versa.

Characterization of Surfactant, Soap

The pH of the soap was tested and fall within the standard range of 5 - 9 which is equivalent for most soap as shown in Table 4.3 and 4.4 above. The cleanability of the soap is in accordance when tested on fabrics and as multipurpose wash because it cleans well. The foamability test was carried out using length of 100 ml column, filled with warm water and a portion of mass of soap produced to 30 ml, after 20 second of agitation the text tube foam to a level increased by 25 ml for every soap produced from the different concentration of sodium hydroxide used.

Optimization of Surfactant (Soap) Production

From the simplex tableau, the last tableau table show that row S_3 turned to X_1 which has a value of 1.99, which implies that 1.99 amount of X_1 was produced. The P row is the contribution of the quantity 2 of X_1 at 1.99 is 3.98. The iterative method stop at this tableau table, since there are no negative values on the P row that is, the P row has zero and positive values.

The optimum mixture of production to attain maximum contribution for this project is as follows:

$X_1 = 1.99$ produced 1.99 quantity of the mixture.

$X_2 = 0$ i.e produced no quantity of the mixture

The value of the slack variable is:

$S_3 = 0$

From the values of S_1 and S_2 it was decided that no value was gained by altering or varying the contribution of rate of reaction of raw material (time) and oil used in production. This is because there is a spare in each of the constrain;

Contribution of rate of reaction of raw material (time) $S_2 = 242.4$ capacity

Oil for production

$S_1 = 0.2$ capacity

From the initial simplex tableau, the capacities of these constraints were given as 600 and 441 respectively. Although, the contribution of rate of reaction of raw material (time) and cost was utilized which shows a positive effect on the project contribution. It is a general rule that constraints only have a value when they are fully utilized. The values are known as the shadow prices or shadow cost, or dual prices or simplex multiples.

CONCLUSIONS

This is one of the main trends in the production of surfactant, soap, but one must not forget the fact that many other types of surfactant (detergent) are being produced in large or small quantities concurrently with the method described in this project. Each has a definite place and uses, but the vast majorities are modifications of the type described in this project. Following the scope of the work and the result obtained, the below conclusion can be drawn.

1. The contribution of sodium hydroxide in each paste produced was obtained using a mathematical model.
2. Optimization of the produced surfactant (soap) was achieved by maximizing the contribution of sodium hydroxide in each paste produced subject to some constraints. The simplex tableau method was used in achieving the optimization.
3. The use of decantation of the soap from glycerol gave a better soap.
4. The optimum contribution of caustic soda was obtained when 1.99 unit of paste (1) was produced
5. The total contribution of caustic soda at the optimum conditions of production is 3.98 units.
6. The work carried out on the project reveals that the quantity of raw materials used in the production of soap is easily affordable, which will encourage individuals to embark on the production if desired.

RECOMMENDATION

Based on some observations made in the course of this research work, the following recommendations are hereby made:

- The efficiency of soap produced from other types of oil use by some soap makers should be checked by the use of the same linear programming approach applied in this project.
- The technique of soap making used in this work should be studied despite the fact that there is another method to be used.

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