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Analytical solution for viscous flow over a nonlinearly stretching sheet: A homotopy perturbation approach

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Abstract: This note is directed at highlighting the strength of the Homotopy Perturbation Method (HPM) over some known numerical and approximate methods. The HPM is employed to obtain the solution of a flow problem involving viscous incompressible fluid over a non-linearly stretching sheet. Aside the faster convergence of the method, a comparison made revealed that the HPM returns results with better accuracy than other approximate and numerical methods.

Keywords: approximate, convergence, flow problem, incompressible fluid, Numerical

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Introduction

Most scientific problems in engineering are inherently non-linear except a few numbers of them. One of the well known equations arising in fluid mechanics and boundary layer approach is Blassius equation (Blassius, 1950). Analytical solutions to non-linear problems are difficult to obtain therefore; different semi-analytical as well as numerical methods have been devised for the solution of such problems. The numerical techniques, such as the finite difference or finite element methods have been used by several authors to investigate physical and engineering problems (Vajravelu, 2001; Cortell, 2007; Cortell, 2008). However, the numerical results obtained by such techniques have been observed to suppress the relationships between the dependent and independent variables, it therefore limits the scope of analysis that could be done on the obtained results. Perturbation technique is another method that is used to deal with non-linearity that arises in model equations for engineering problems. The method assumes a perturbed solution with very small amplitude of one of the independent variables. Perturbation techniques have been used by several authors in the solutions of nonlinear equations that arise from environmental and engineering problems. The solution obtained by this technique is constrained to small amplitude of the perturbation parameter. In attempt to overcome the small amplitude constraint which characterizes the perturbation technique, other methods such as the homotopy analysis method (HAM), Adomian decomposition method (ADM) and the homotopy perturbation method (HPM) have been developed to obtain approximate solutions to nonlinear equations. Several authors have used the HPM to solve different nonlinear problems (Ganji, 2006; Esmaeilpour and Ganji, 2007; Raftari and Yildrin, 2010)).

In the present note, the method of HPM is used to investigate viscous flow over a nonlinearly stretching sheet. The same problem has been investigated in (Vajravelu, 2001) by the use of a fourth -order Runge-Kutta scheme. However, some limitations have been identified in the results and discussions in (Vajravelu, 2001) for which we use HPM to circumvent. For instance, the exact solution was obtained only for a linear stretching (n=1) of the sheet, while for the nonlinear cases, a relationship could not be established between the various parameters. This deficiency has however been taken care of in the present note in which the HPM is used to derive a relationship that could handle general cases involving both linear as well as nonlinear stretching of the boundary sheet.

Equation of motion

Consider the flow of a viscous flow over a nonlinearly stretching sheet. The basic boundary layer equation for steady flow can be written as (Vajravelu, 2001)

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2} \qquad (2)$$

Where u and v are the velocity components in x and y directions and ν is kinematic viscousity respectively. The boundary conditions to the problem are

$$u=Cx^n,\ v=0 \text{ at } y=0$$

$$u\to 0 \text{ as } y\to\infty \qquad (3)$$

These condition suggest a similarity transformation

$$\eta=y\sqrt{\frac{C(n+1)}{2\nu}}x^{\frac{n-1}{2}},$$

$$u=Cx^n f'(\eta),$$

$$v=-\sqrt{\frac{C\nu(n+1)}{2}}x^{\frac{n-1}{2}}\left[f+\frac{n-1}{n+1}\eta f'\right] \qquad (4)$$

Where a prime denotes differentiation with respect to η. Substitute Eq.(2) into Eq.(4), the governing equation and boundary conditions reduce to

$$f'''+ff''-\left(\frac{2n}{n+1}\right)(f')^2=0 \qquad (5)$$

$$f'=1,\ f=0\ at\ \eta=0$$

$$f'\to 0\ as\ \eta\to 0 \qquad (6)$$

Homotopy perturbation technique

We will use HPM in order to obtain the solution of Eq.(5). Assuming $f=\phi$ Eq.(5) can be written in following form:

$$\phi'''+F(\phi)=0 \qquad (7)$$

in which

$$F(\phi)=\phi\phi''-\left(\frac{2n}{n+1}\right)(\phi')^2 \qquad (8)$$

According to the homotopy perturbation method (He, 1998), we construct a homotopy in the form with the initial conditions

$$\phi'''-\alpha^2\phi'+p\left(F(\phi)+\alpha^2\phi'\right)=0 \qquad (9)$$

$$\phi'(0)=1,\ \phi(0)=0,\ \phi'(\infty)=0 \qquad (10)$$

When p=0 Eq. (9) becomes a lineralized equation $\phi'''-\alpha^2\phi'=0$, where α is an unknown parameter to be further determined, when p=1, the equation becomes original problem. The embedded parameter p monotonically increases from zero to one as the trivial problem, $\phi'''-\alpha^2\phi'=0$.

By introducing the HPM, we assume that the solution to Eq.(9) can be written as a power series in p

$$\phi=\phi_0+p\phi_1+p^2\phi_2+\ldots \qquad (11)$$

Substituting Eq.(11) into Eq.(9) and equating the terms with the identical power of p, we have

$$p^0:\phi_0'''-\alpha^2\phi_0'=0,\ \phi_0'(0)=1,\ \phi_0(0)=0,\ \phi_0'(\infty)=0 \qquad (12)$$

$$p^1:\phi_1'''-\alpha^2\phi_1'=\left(\frac{2n}{(n+1)}-1\right)e^{-2\alpha\eta}+\left(1-\alpha^2\right)e^{-\alpha\eta},\ \phi_1'(0)=1,\ \phi_1(0)=0,\ \phi_1'(\infty)=0 \qquad (13)$$

The solution of Eqs.(12) and (13) can be readily obtained, which reads

$$\phi_0=\frac{1}{\alpha}\left(1-e^{-\alpha\eta}\right) \qquad (14)$$

$$\phi_1=\left(\frac{2n}{(n+1)}-1\right)\frac{1}{6\alpha^3}\left(1-e^{-2\alpha\eta}\right)+\frac{\left(1-\alpha^2\right)}{2\alpha^2}e^{-\alpha\eta}\eta \qquad (15)$$

Where $\alpha^2=\frac{(5n+1)}{3(n+1)}$ (16)

Therefore, we obtain

$$\phi(\eta)=\phi_0(\eta)+\phi_1(\eta) \qquad (17)$$

Setting n=1 represent the flow problem studied by (Troy *et al.*, 1987). For n=1, the exact solution for the velocity field f is

$$f(\eta)=1-e^{-\eta} \qquad (18)$$

Moreover, the exact solution is unique. That is, for the linearly stretching boundary problem the solution is unique

The shear stress at the surface of the sheet is defined as

$$\tau_w=\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}=C\mu\sqrt{\frac{C(n+1)}{2\nu}}x^{\frac{(3n-1)}{2}}f''(0) \qquad (19)$$

From Eqs.(14) and (15) we obtain

$$f''(0) = \phi''(0) = \phi_0''(0) + \phi_1''(0) = \left(1 - \frac{2n}{n+1}\right)\frac{2}{3\alpha} - \frac{1}{\alpha} \quad (20)$$

Discussion

Homotopy perturbation method has been used to study fluid flow over a nonlinearly stretching sheet. The hydrodynamic response of the fluid is presented in Figs. 1 and 2 for variations in the stretching parameter *n*. From Fig. 1, the stream function *f* is observed to decrease changes in *n*. This shows that f is highest when the sheet is linearly stretched while increasing the nonlinearity of the stretching sheet results to a monotonic decrease in f. The velocity of the fluid is also seen to decrease with growing nonlinearity of the stretching sheet.

The skin-friction on the surface of the stretching sheet is compared with those from different works of various authors and presented in the Table. It is found that the skin friction increases monotonically with stretching parameter *n*. A matlab program has been included in Appendix for the readers who are familiar with the software.

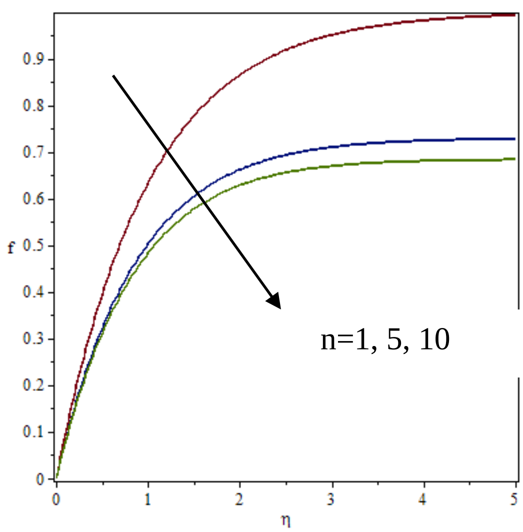

Table 1: The velocity Gradient at wall $f''(0)$

	(vajravelu, 2001)	**(Cortell, 2007)**	**(Cortell, 2008)**	Present Result
N	$-f''(0)$	$-f''(0)$	$-f''(0)$	$-f''(0)$
0.1	-	-	0.705897	0.6742
0.2	-	0.766758	-	0.7454
0.3	-	-	0.815696	0.8006

Fig. 1. *f* versus η

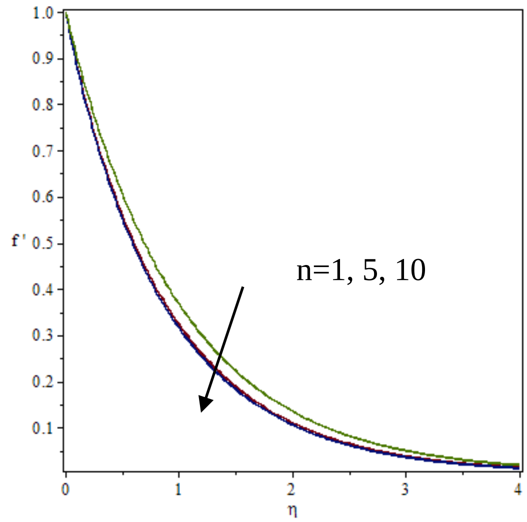

Fig. 2. f' **versus η**

Conclusion

In this study, we have applied homotopy perturbation method in solving the viscous flow over a nonlinearly stretching sheet. Comparison between HPM and fourth-order Runge-Kutta methods show a remarkable agreement and reveal that the HPM need less computation work. Also, it provides the relation between dependent variable and the controlling parameter of the physical situation.

0.5	-	0.889477	-	0.8819
0.6	-	-	0.918176	0.9129
0.75	-	0.953786	-	0.9512
0.9	-	-	0.983242	0.9823
1.0	-	1.000000	1.000000	1.0000
1.5	-	1.061587	1.061587	1.0646
3.0	-	1.148588	1.148588	1.1547
5.0	1.1945	-	-	1.2019
7.0		1.216847	-	1.2247
10	1.2348	1.234875	1.234875	1.2432
20	-	1.257418	-	1.2662
100	-	1.276768	-	1.2859

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Appendix A: Skin Friction

```
n=0.1;

x1=(2.*n)./(n+1.);

x2=1.-x1;

a=(5.*n+1.)./(3.*(an+1.));

α=sqrt(a);

x3=2./(3.*α);
```

$f''(0)$ **=x3.*(x2)-(1./α)**