Journal of the Nigerian Association of Mathematical Physics
***Volume28, No. 1*, (*November, 2014*), pp 83 – 88**
© J. of NAMP

Analytical as Well as Numerical Treatment of Viscous Ag-Water and Cu-Water Nanofluids Flow Over a Stretching Surface

[1]Umaru Mohammed, [1]Hakimi Danladi and [2]Aliyu Ishaku Ma'ali

[1]Department of Mathematics and Statistics,
Federal University of Technology, Minna, Nigeria.
[2]Department of Mathematics/computer Science,
Ibrahihim Badamasi Babangida University Lapai,Niger State, Nigeria.

Abstract

The viscous Ag-Water and Cu-Water nanofluids flow over a stretching surface is studied by means of approximate analytical as well as numerical method using Homotopy Perturbation Method (HPM) for the approximate solution and Runge-Kuttamethod (RKM)for the numerical solution of the flow problem. Comparisons aremade with published work which reveals an excellent agreement.

Keywords: Stretching sheet; Homotopy perturbation method (HPM); Runge-Kutta method (RKM)

1.0 Introduction

Nanofluids have attracted enormous interest from researchers due to their potential for high rate of heat exchange incurring either little or no penalty in the pressure drop. The convective heat transfer characteristics of nanofluids depend on the thermophysical properties of the base fluids and the nanoparticles, the flow pattern and the flow structure, the volume fraction of the suspended particles, the dimension and the shape of these particles. The utility of a particularnanofluid for a heat transfer application can be established by suitably. Recently, Santra et al.[1] numerically examined nanofluid laminar flow and heat transfer in a two dimensional (infinite depth) horizontal rectangular duct.They observed that heat transfer in such a duct increases with increasing nanofluid volume fraction. An excellent collection of articles on this topic can be found byBuongiorno [2], Kuznetsov and Nield [3], Nield and Kuznetsov [4], Bachok et al. [5] Khan and Pop [6], Rana and Bhargava [7], Makinde and Aziz [8] and Bachok et al. [9].

Analytical solutions of the above mentioned problems are not possible due to its non-linear nature. If analytical solutions exist for such flow problems, it deserves great attention, since it allows us to gain a deeper knowledge of underlying physical situation. Moreover, it provides the possibility to get a benchmark for numerical solvers with reference to basic flow configurations. Recently, Homotopy perturbation method (HPM) has been established by He [10-16]. The method does not depend on a small parameter in the equation, it has however, been applied to solve linear and nonlinear equations of heat transfer [17-19]. Ganji [20] have used HPM to solve boundary layer flow and convective heat transfer over a flat plate while Raftari and Yildirim [21] employedHPMs to analyzeMHD flow of UCM fluid above porous stretching sheets. Yildrim and Gülkanat[22] also obtained new solitary solutions with compact support for Boussinesq-like $B(2n, 2n)$ equations with fully nonlinear dispersion using the homotopy perturbation method (HPM). Saddiqui et al. [23] carried out the hydrodynamic squeezing flow of a viscous fluid between parallel plates using (HPM). Rashidi etal. [24] examined the hydrodynamic squeezing flow of viscous fluid by homotopy analysis method (HAM).In another work, Rashidi etal.[25] used the HAM for obtaining approximate analytical solution of the steady flow over a rotating disk in porous medium with heat transfer. The validity of the obtained solution is verified by numerical method.

The flow behaviour of viscous Ag-water and Cu-water nanofluids over a stretching surfacehas been investigated numerically usingKeller Box Method [26].The purpose of this work is to provide approximate analytical solution to the problem of viscous Ag-water and Cu-water nanofluid flow overa stretching sheet. In order to check the validity of HPM, the problem is solved by Runge-Kutta Method.

Corresponding author:***Umaru Mohammed***E-mail: digitalumar@yahoo.com, Tel.: +2348117367980

2.0 Equation of Motion

Consider a steady two-dimensional laminar fluid flow over astretching sheet. The basic boundary layer equation can be written as [26]

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2} \tag{2}$$

where*u* and *v* are the velocity components along x and *y* direction respectively.
The boundary conditions for the problem are

$$u=bx,\ v=0 \text{ at } y=0$$
$$u\to 0 \text{ as } y\to\infty \tag{3}$$

The effective density of nanofluids is given as

$$(\rho)_{nf}=(1-\phi)\rho_f+\phi\rho_s \tag{4}$$

where ϕ is the solid volume fraction of nanoparticles.
The effective dynamic viscosity of the nanofluid given by Brinkman [27] as

$$\mu_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}} \tag{5}$$

Here the subscripts *nf*, *f* and *s* represent respectively the properties of the nnofluids, base fluid and the nano-solid particles.
By defining the similarity transformation

$$\eta=\sqrt{\frac{b}{v_f}}y,\ u=bxf'(\eta),\ v=-\sqrt{bv_f}\,f(\eta) \tag{6}$$

where prime denotes differentiation with respect to η. SubstitutingEq.(6) into Eq.(2), the governing equation and boundary conditions reduce to

$$f'''=(1-\varphi)^{2.5}\left[1-\varphi+\varphi\left(\frac{\rho_s}{\rho_f}\right)\right]\left\{f'^2-ff''\right\} \tag{7}$$

$$f=0,\ f'=1 \text{ at } \eta=0$$
$$f'\to 0 \text{ as } \eta\to\infty \tag{8}$$

3.0 Method of Solution

We will use HPM in order to obtain the solution of Eq.(7). Assuming *f*=*φ*Eq.(7) can be written in following form

$$\varphi'''+F(\varphi)=0 \tag{9}$$

in which

$$F(\varphi)=-(1-\phi)^{2.5}A\left(\varphi'^2(\eta)-\varphi(\eta)\varphi''(\eta)\right) \tag{10}$$

where

$$A=(1-\phi)+\phi\left(\frac{\rho_s}{\rho_f}\right) \tag{11}$$

Let

$$A^*=(1-\phi)^{2.5}A \tag{12}$$

According to the homotopy perturbation method [28], we construct a homotopy in the form

$$\varphi'''-\alpha^2\varphi'+p\left(F(\varphi)+\alpha^2\varphi'\right)=0 \tag{13}$$

with the initial conditions

$$\varphi'(0)=1,\ \varphi(0)=0,\ \varphi'(\infty)=0 \quad (14)$$

When p=0 Eq. (13) becomes a linearised equation $\varphi''' - \alpha^2 \varphi' = 0,$ where α is an unknown parameter to be further determined. When p=1, the equation becomes the original problem. The embedded parameter p monotonically increases from zero to unit as thetrivial problem,

$\varphi''' - \alpha^2 \varphi' = 0$, is continuously deforms to the original problem, Eq. (9). By introducing the HPM, we assume that the solution to Eq.(13) can be written as a power series in p.

$$\varphi = \varphi_0 + p\varphi_1 + p^2\varphi_2 + ... \quad (15)$$

Substituting Eq.(15) into Eq.(13) and equating the terms with the identical power of p, we have

$$p^0 : \varphi_0''' - \alpha^2 \varphi_0' = 0,\ \varphi_0'(0)=1,\ \varphi_0(0)=0,\ \varphi_0'(\infty)=0 \quad (16)$$

$$p^1 : \varphi_1''' - \alpha^2 \varphi_1' = -\left(F(\varphi) + \alpha^2 \varphi_0'\right),\ \varphi_1'(0)=0,\ \varphi_1(0)=0,\ \varphi_1'(\infty)=0 \quad (17)$$

Where $F(\varphi) = -A^*\left(\varphi_0'^2 - \varphi_0 \varphi_0''\right)$ (18)

The solution of Eqs.(17) and (18) can be readily obtained, which reads

$$\varphi_0 = \frac{1}{\alpha}\left(1-e^{-\alpha\eta}\right) \quad (19)$$

$$\varphi_1 = -\frac{\left(\alpha^2 - A^*\right)}{2\alpha^2}\eta e^{-\alpha\eta} \quad (20)$$

Where $\alpha = \pm\sqrt{A^*}$ (21)

Therefore, we obtain

$$\varphi(\eta) = \varphi_0(\eta) + \varphi_1(\eta) \quad (22)$$

Setting ϕ=0, the present problemreturns to the flow problem studied by Crane. [29].
The exact solution for the velocity fieldis

$$f(\eta) = 1 - e^{-\eta} \quad (23)$$

In the presence of nanofluid particles volume fraction ($\phi \neq 0$) the approximate solution of Eq.7 satisfying the required boundary conditions is given by

$$f'(\eta) = e^{-\alpha\eta} \quad (24)$$

Where

$$\alpha = \sqrt{(1-\phi)^{2.5}\left[(1-\phi) + \phi\frac{\rho_s}{\rho_f}\right]} \quad (25)$$

The shear stress at the surface of the sheet is defined as

$$\tau_w = \frac{\mu_{nf}}{\rho_f u_w^2}\left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{1}{(1-\phi)^{2.5}}(\mathrm{Re}_x)^{-\frac{1}{2}} f''(0) \quad (26)$$

Here $\mathrm{Re}_x = u_w x / \nu_f$ (Local Reynold number).
From Eqs.(19) and (20) we obtain

$$f''(0) = \varphi''(0) = \varphi_0''(0) + \varphi_1''(0) = -\frac{A^*}{\alpha} \quad (27)$$

4.0 Discussion

Figs. 1-2 have been made in order to see the effect of nanoparticles volume fraction parameter on the wall normal velocity profile and stream-wise velocity profile for both Cu-water and Ag-water nanofluids. They show that by increasing the values of a nanoparticle volume fraction, the wall normal velocity profile and the stream wise velocity profile decrease. The numerical values of the skin friction for several values of the nanoparticles volume fraction parameter are given in Tables 1-2, using HPM, R-K method and Vajravelu et al. [26].The results show that the methods are in good agreement with each other. It is clear from the tables that wall stress increases when the values of solid volume fraction increase.

5.0 Conclusion

In this study, we have applied homotopy perturbation method and Runge-Kutta method in solving the flow problem of viscous Ag-water and Cu-water nanofluids over a stretching surface. Comparisons between the methods show a remarkable agreement and reveal that the HPM need less computation work to achieve reasonable accuracy. The most important feature of the HPM that makes it favoured above the numerical schemes is that it provides the relation between dependent variable and the controlling parameters of the physical situation.

Table1 Comparison of the values of $f''(0)$ obtained by HPM, R-K and Vajravelu et al for the case of Cu-water nanofluid

	HPM	**RKM**	**Vajravelu et al.[26]**
ϕ	$-f''(0)$	$-f''(0)$	$-f''(0)$
0.0	1.000000000	1.0000011307476	1.001411
0.1	1.174746021	1.1747461841520	1.175203
0.2	1.218043809	1.2180439101310	1.218301

Table 2 Comparison of the values of $f''(0)$ obtained by HPM, R-K and Vajravelu et al for the case of Ag-water nanofluid

	HPM	**RKM**	**Vajravelu et al.[26]**
ϕ	$-f''(0)$	$-f''(0)$	$-f''(0)$
0.0	1.000000000	1.0000011307476	1.001411
0.1	1.225068142	1.2250682355712	1.222537
0.2	1.289788015	1.2897880605508	1.289507

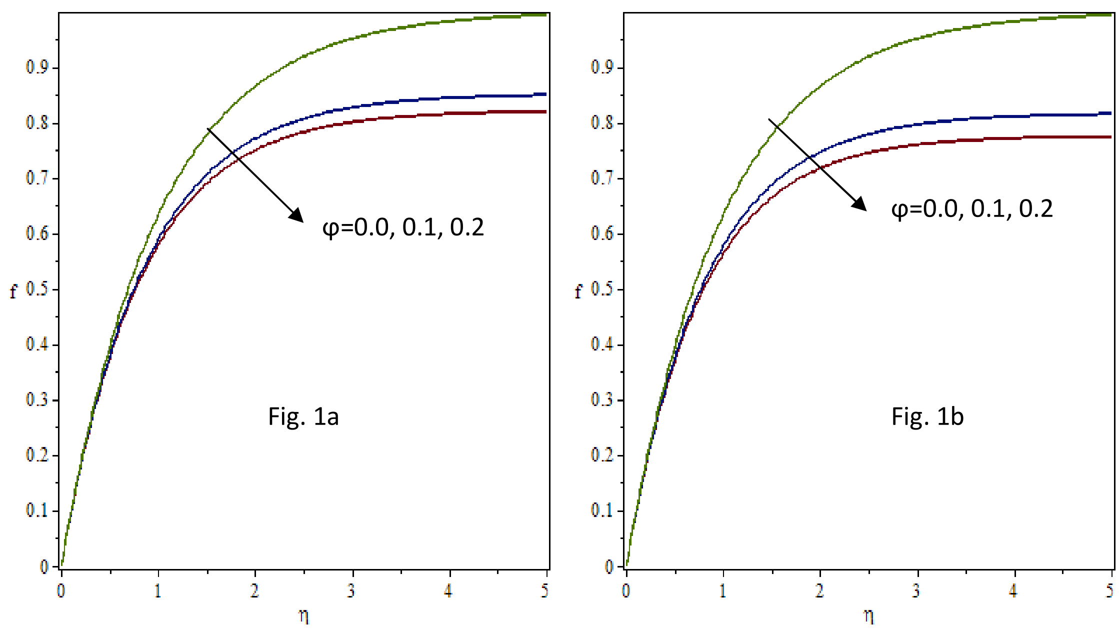

Fig. 1. Wall normal velocity profile f vs. η for different values of ϕ for the case. (a) Cu-water and (b) Ag-water.

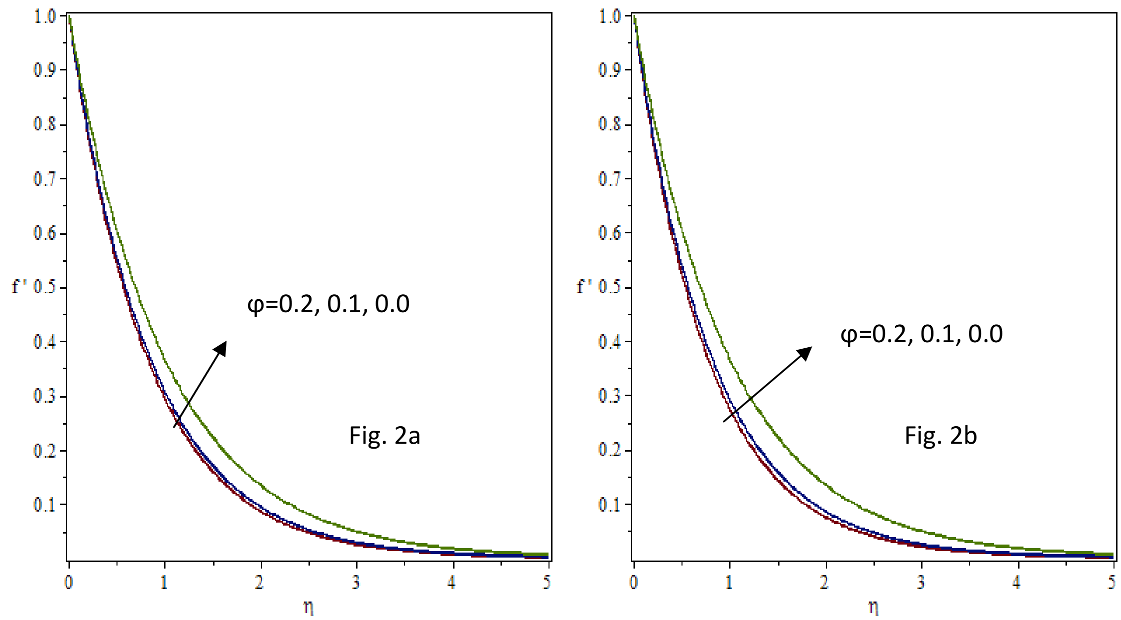

Fig. 2. Streamwise velocity profile*f '* vs. η for different values of ϕ for the case. (a) Cu-water and (b) Ag-wat

References

[1] Santra AK., Sen S, Chakraborty N. Study of heat transfer due to laminar flow of copper-water nanofluid through two isothermally heated parallel plates.*International Journal of Thermal Science*s 2009; 48 :391-400.

[2] Buongiorno J. Convective transport in nanofluids. *Journal of Heat Transfer* 2006;128: 240–250.

[3] Kuznetsov AV, Nield DA. Natural convective boundary-layer flow of a nanofluid pasta vertical plate.*International Journal of Thermal Science*s2010;49:243–247.

[4] Nield DA, Kuznetsov AV. The Cheng–Minkowycz problem for natural convective boundarylayer flow in a porous medium saturated by a nanofluid.*Int. J. Heat Mass Transfer*2009 ;52:. 5792–5795.

[5] Bachok N, Ishak A, Pop I. Boundary-layer flow of nanofluids over a moving surface in a flowing fluid.*Int. J. Therm. Sci.*2010 ; 49:1663–1668.

[6] Khan WA, Pop I. Boundary-layer flow of a nanofluid past a stretching sheet.*Int. J. Heat Mass Transfer* 2010 ;53: 2477–2483.

[7] Rana P, Bhargava R. Flow and heat transfer of a nanofluid over a nonlinearly stretching sheet: A numerical study.*Commun. Nonlinear Sci. Numer. Simul.*, doi: m10.1016/j.cnsns.2011.05.009.

[8] Makinde OD, Aziz A. Boundary layer flow of a nanofluid past a stretching sheet with a convective boundary condition.*Int. J. Therm. Sci*. 2011;50:1326–1332.

[9] Bachok N, Ishak A, Pop I. Unsteady boundary-layer flow and heat transfer of a nanofluidover a permeable stretching/shrinking sheet.*Int. J. Heat Mass* Transfer 2012;55: 2102–2109.

[10] He JH.Homotopyperturbationtechnique.*ComputMethApplMechEng* 1999;178: 257-262.

[11] He JH.Acoupling method ofhomotopy technique andperturbation technique fornonlinear problems.*IntJNon-linearMech* 2000;35:37-43.

[12] He JH. Some asymptotic methods for strongly nonlinear equations.*Int. J modern Physics* 2006 ;20(10):1141-1199.

[13] He JH.Recentdevelopmentof homotopyperturbationmethod.Topol.*MethNonlinearAnal* 2008;31:205-9.

[14] HeJH.Thehomotopyperturbationmethodfornonlinearoscillatorswithdiscontinuities.*ApplMathComput* 2004;151:287-92.

[15] He JH.Application of homotopy perturbation method to nonlinear wave equations.*Chaos SolitonFract* 2005 ;26:695-700.

[16] He JH.Limitcycleandbifurcationofnonlinearproblems.*ChaosSolitonFract* 2005 ;26:827-33.

[17] Rajabi A,Ganji DD. Application of homotopy perturbation method in nonlinear heat conductionandconvectionequations.*PhysLettA* 2007;360:570-3.

[18] Ganji DD,Sadighi A.Applicationofhomotopyperturbation andvariationaliterationmethods tononlinearheattransferandporousmediaequations, J.Comput.ApplMath,207,(2007)24-34.

[19] D.D. Ganji, The application of Hes homotopy perturbation method to nonlinear equations arisinginheattransfer,*PhysLettA* 2006;355:337-41.

[20] Esmaeilpour M, Ganji DD. Application ofHe's homotopy perturbation method toboundary layerflowandconvectionheattransferoveraflatplate.*PhysicsLettersA*2007;372:33–38.

[21] Raftari B, Yildrin A. The application of homotopy perturbation method forflows of UCM fluids above porous stretching sheet.*Comput. and maths. with Appl.* 2010;59:3328-3337.

[22] Yildrim A, Gülkanat Y.Homotopy perturbation method to obtain new solitary solutions with compact support for Boussinesq-like $B(2n, 2n)$ equations with fully nonlinear dispersion. *International Journal for Numerical Methods in Fluids*2009;65:699–706,

[23] Siddiqui AM,Irum S,Ansari AR. Unsteadysqueez- ingflowofaviscousMHDfluidbetweenparallelplates, asolutionusingthehomotopyperturbation method.*Math ModelAnal*2008;13:565–576.

[24] Rashidi MM,Shahmohamadi H,Dinarvand S.Analyticapproximatesolutionsfor unsteadytwo-dimensional andaxisymmetricsqueezingflowsbetweenparallelplates. *MathProblEng*2008;93:5095.

[25] Rashidi MM, Mohimanian Pour SA, Hayat T, Obaidat S. Analytical approximate solutions for steady flow over a rotating disk in porous medium with heat transfer by homotopy analysis method. *Comput. And Fluid* 2012;54 :1-9.

[26] Vajravelu K, Prasad KV, Lee J, Pop I, Van Gorder RA. Convective heat transfer in the flow of viscous Ag-water and Cu-water nanofluids over a stretching surface.*Int. J. of Thermal Sci.* 2011;50:843-851.

[27] Brinkman HC. The viscosity of concentrated suspensions and solution.*J. Chem. Phys.*1952; 20 :571-581.

[28] He JH. An approximate solution technique depending upon an artificial parameter,*Commun.Nonlinear sci. Numer. Simul.*1998;3 :92-97.

[29] Crane LJ. Flow pass a stretching plate.*Z.Angew. Math. Phys.*1970;21: 645-647