

## DYNAMIC ANALYSIS OF TALL BUILDING UNDER PULSATION WIND EXCITATION

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### ABSTRACT

This paper presents analysis of tall building under wind load excitation taking account of pulsation. The path of the analysis using MatLAB, Lira and Russian national code SNiP are presented. Use of wind velocity-time graph is employed where the change in wind pressure (pulsation) with time is assumed to be proportional to a similar change in wind velocity. Results are compared and show a conformation to dynamic behaviour of the buildings under such excitations as pulsation, thus making the analysis adequate for assessing the dynamic behaviour of a structure.

**Keyword:** dynamic analysis, numerical method, wind load, pulsation, drift, acceleration

### 1. INTRODUCTION

Wind effects on structures continue to pose danger that continues to attract the attention of scientist round the globe. This is because of its trend of effect that changes with time, a dynamic problem, and inadequate information on the response of structures to wind action.

In a most recent report by the US National Institute of Standards and Technology makes 30 recommendations to improve the safety of tall buildings on the collapse of the World Trade Centre. Among the 8 core structural recommendations is: Increased structural integrity-the standards for estimating the load effects of potential hazards (such as progressive collapse, wind) and that the design of structural systems to mitigate the effect of those hazards should be improved to enhance structural integrity [1]. This suggests the importance and necessity of continues research on dynamic behaviour of structures.

In this study three buildings are considered: 10-storey [2]; 15-storey [2]; and 20-storey [3]. 10 and 15-storey consist of uniform cross-sectional area in struts and girders from reinforced concrete. The masses  $M_1=M_2=M_3=.....M_{14}=51\text{ton}$ , except  $M_{10}$  and  $M_{15}$  which are 60 ton each for 10 and 15-storey respectively. The 20-storey consists of structural steel frame from the class 26KY6 with masses  $M_1=M_2=....M_{19}=20\text{ton}$  and  $M_{20}=30\text{ton}$ . All

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the analytic frames have constant stiffness along the heights and all columns are fixed to the foundation and are all moments resisting frames. The cross sectional analytic frame models are shown in figures 1.

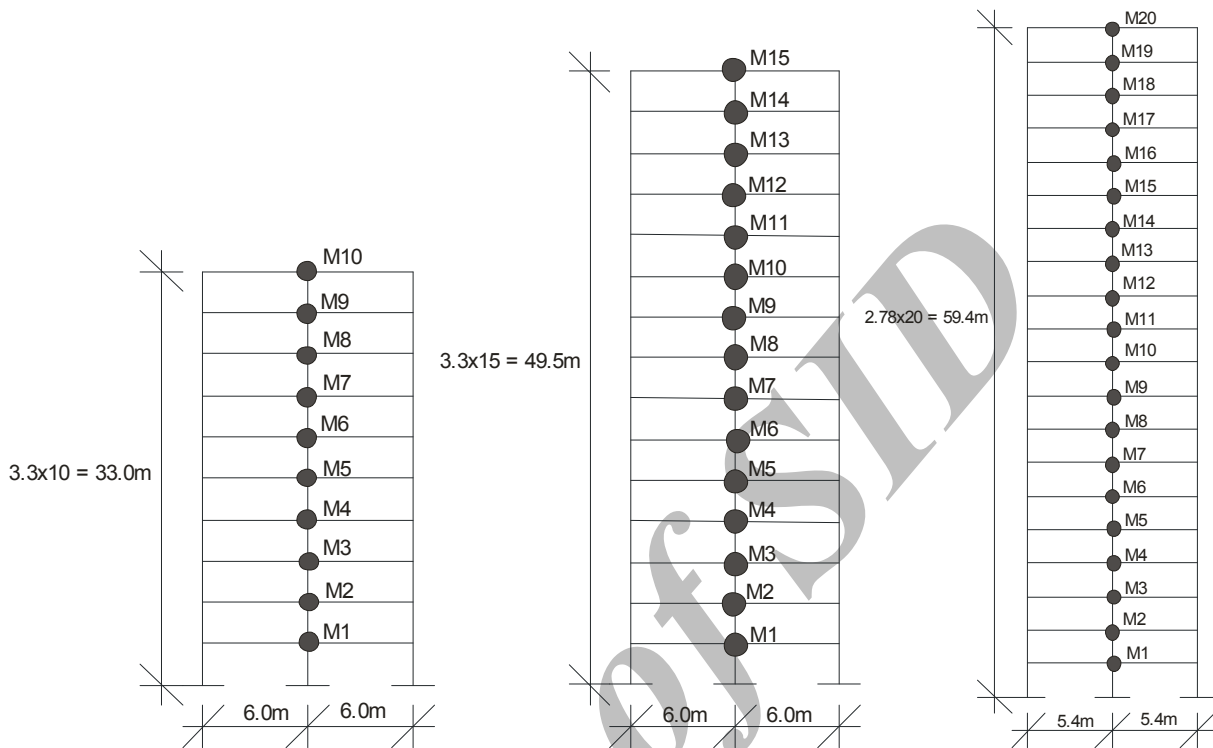


Figure 1. 10-storey, 15-storey and 20-storey cross-sectional analytic frames

## 2. THEORY OF METHODS

The computations of wind loads on tall buildings from Russian national code [4], is taken as the sum of statically obtained average wind load and pulsation wind load. While the former is computed from equation (1), the later is acceptable to be computed from equation (2) if the stiffness, mass and width of windward surface along height of the building are constant:

$$w_s = w_0 k c \quad (1)$$

$$w_p = 1.4(z/h) \xi w_{ph} \quad (2)$$

$$w_0 = 0.61 v_0^2 \quad (3)$$

$$w_{ph} = w_s \zeta v \quad (4)$$

Where  $w_0$  – is the nominal wind pressure (for Area II = 30 kgf/m<sup>2</sup>);  $k$  – height factor;  $c$  – aerodynamic coefficient +0.8 for windward side and -0.6 leeward side;  $\xi$  – dynamic coefficient;  $w_{ph}$  – nominal wind pressure due to pulsation at height  $h$  to be calculated from equation (4);  $v_0$  – wind velocity;  $v$  – coefficient of space correlation due to wind pulsation;  $\zeta$  –

coefficient of wind pulsation.

Lira is a software for structural analysis. Wind load values obtained from SNIp are also processed and analysed here for the sake of comparison.

Numerical method of analysis is also employed in this study. Differential equations of a moving mass can be integrated using computer analysis [5, 6]. In dynamic analysis these equations at any given time  $t$  are written in matrixes which take the form of equation (5):

$$M\ddot{Y}_t + C\dot{Y}_t + KY_t = R_t, \quad (5)$$

where  $M$ ,  $C$ ,  $K$ –matrix of masses, damping and stiffness;  $R_t$  – vector representing external excitations;  $Y_t$ ,  $\dot{Y}_t$ ,  $\ddot{Y}_t$ –vectors of displacements, velocities and accelerations of a system with definite number of degree of freedom. Numerical integration of these equations is done through an iterative process.

Table 1. Summary of wind load for 10-storey frame

| Average wind load,           |   | Pulsation wind load,         |   | Total wind load, F in kN                |   |
|------------------------------|---|------------------------------|---|---|---|
| $w_{si}$ , kN/m <sup>2</sup> | $F_{si}=A(w_{si})$ , kN<br>$A=14.85m^2$ | $w_{pi}$ , kN/m <sup>2</sup> | $F_{pi}=A(w_{pi})$ , kN<br>$A=14.85m^2$ | Windward,<br>$F_{wi} = F_{si} + F_{pi}$ | Leeward, $F_{li} = 0.75(F_{si} + F_{pi})$ |
| $w_{s1} = 0.18$              | $F_{s1} = 3.119$                        | $w_{p1} = 0.030$             | $0.5F_{p1} = 0.443$                     | $F_{w1} = 3.562$                        | $F_{l1} = 2.672$                          |
| $w_{s2} = 0.24$              | $F_{s2} = 3.564$                        | $w_{p2} = 0.060$             | $0.5F_{p2} = 0.885$                     | $F_{w2} = 4.449$                        | $F_{l2} = 3.337$                          |
| $w_{s3} = 0.24$              | $F_{s3} = 4.010$                        | $w_{p3} = 0.089$             | $0.5F_{p3} = 1.328$                     | $F_{w3} = 5.338$                        | $F_{l3} = 4.004$                          |
| $w_{s4} = 0.30$              | $F_{s4} = 4.455$                        | $w_{p4} = 0.119$             | $0.5F_{p4} = 1.770$                     | $F_{w4} = 6.225$                        | $F_{l4} = 4.669$                          |
| $w_{s5} = 0.30$              | $F_{s5} = 4.455$                        | $w_{p5} = 0.149$             | $0.5F_{p5} = 2.213$                     | $F_{w5} = 6.668$                        | $F_{l5} = 5.001$                          |
| $w_{s6} = 0.30$              | $F_{s6} = 4.901$                        | $w_{p6} = 0.179$             | $0.5F_{p6} = 2.655$                     | $F_{w6} = 7.556$                        | $F_{l6} = 5.667$                          |
| $w_{s7} = 0.36$              | $F_{s7} = 5.346$                        | $w_{p7} = 0.209$             | $0.5F_{p7} = 3.098$                     | $F_{w7} = 8.444$                        | $F_{l7} = 6.333$                          |
| $w_{s8} = 0.36$              | $F_{s8} = 5.346$                        | $w_{p8} = 0.238$             | $0.5F_{p8} = 3.540$                     | $F_{w8} = 8.886$                        | $F_{l8} = 6.665$                          |
| $w_{s9} = 0.36$              | $F_{s9} = 5.346$                        | $w_{p9} = 0.268$             | $0.5F_{p9} = 3.983$                     | $F_{w9} = 9.329$                        | $F_{l9} = 6.997$                          |
| $w_{s10} = 0.36$             | $F_{s10} = 2.673$                       | $w_{p10} = 0.298$            | $0.5F_{p10} = 2.213$                    | $F_{w10} = 4.886$                       | $F_{l10} = 3.665$                         |

Table 2. Summary of wind load for 15-storey frame

| Average wind load,           |  | Pulsation wind load,         |  | Total wind load, F in kN                  |  |
|------------------------------|--|------------------------------|--|---|--|
| $w_{si}$ , kN/m <sup>2</sup> | $F_{si}=A(w_{si}), \text{kN}$<br>$A=14.85\text{m}^2$ | $w_{pi}$ , kN/m <sup>2</sup> | $F_{pi}=A(w_{pi}), \text{kN}$<br>$A=14.85\text{m}^2$ | Windward, $F_{wi}$<br>$= F_{si} + F_{pi}$ | Leeward, $F_{li} =$<br>$0.75(F_{si} + F_{pi})$ |
| $w_{s1} = 0.18$              | $F_{s1} = 3.119$                                     | $w_{p1} = 0.021$             | $0.5F_{p1} = 0.316$                                  | $F_{w1} = 3.435$                          | $F_{l1} = 2.576$                               |
| $w_{s2} = 0.24$              | $F_{s2} = 3.564$                                     | $w_{p2} = 0.043$             | $0.5F_{p2} = 0.633$                                  | $F_{w2} = 4.197$                          | $F_{l2} = 3.148$                               |
| $w_{s3} = 0.24$              | $F_{s3} = 4.010$                                     | $w_{p3} = 0.064$             | $0.5F_{p3} = 0.949$                                  | $F_{w3} = 4.959$                          | $F_{l3} = 3.719$                               |
| $w_{s4} = 0.30$              | $F_{s4} = 4.455$                                     | $w_{p4} = 0.085$             | $0.5F_{p4} = 1.265$                                  | $F_{w4} = 5.720$                          | $F_{l4} = 4.290$                               |
| $w_{s5} = 0.30$              | $F_{s5} = 4.455$                                     | $w_{p5} = 0.106$             | $0.5F_{p5} = 1.581$                                  | $F_{w5} = 6.036$                          | $F_{l5} = 4.527$                               |
| $w_{s6} = 0.30$              | $F_{s6} = 4.901$                                     | $w_{p6} = 0.128$             | $0.5F_{p6} = 1.898$                                  | $F_{w6} = 6.799$                          | $F_{l6} = 5.099$                               |
| $w_{s7} = 0.36$              | $F_{s7} = 5.346$                                     | $w_{p7} = 0.149$             | $0.5F_{p7} = 2.214$                                  | $F_{w7} = 7.560$                          | $F_{l7} = 5.670$                               |
| $w_{s8} = 0.36$              | $F_{s8} = 5.346$                                     | $w_{p8} = 0.170$             | $0.5F_{p8} = 2.530$                                  | $F_{w8} = 7.876$                          | $F_{l8} = 5.907$                               |
| $w_{s9} = 0.36$              | $F_{s9} = 5.346$                                     | $w_{p9} = 0.192$             | $0.5F_{p9} = 2.846$                                  | $F_{w9} = 8.192$                          | $F_{l9} = 6.144$                               |
| $w_{s10} = 0.36$             | $F_{s10} = 5.346$                                    | $w_{p10} = 0.213$            | $0.5F_{p10} = 3.163$                                 | $F_{w10} = 8.509$                         | $F_{l10} = 6.382$                              |
| $w_{s11} = 0.36$             | $F_{s11} = 5.346$                                    | $w_{p11} = 0.234$            | $0.5F_{p11} = 3.479$                                 | $F_{w11} = 8.825$                         | $F_{l11} = 6.619$                              |
| $w_{s12} = 0.36$             | $F_{s12} = 5.346$                                    | $w_{p12} =$<br>$0.256$       | $0.5F_{p12} = 3.795$                                 | $F_{w12} = 9.141$                         | $F_{l12} = 6.856$                              |
| $w_{s13} = 0.41$             | $F_{s13} = 5.717$                                    | $w_{p13} = 0.277$            | $0.5F_{p13} = 4.112$                                 | $F_{w13} = 9.829$                         | $F_{l13} = 7.372$                              |
| $w_{s14} = 0.41$             | $F_{s14} = 6.089$                                    | $w_{p14} = 0.298$            | $0.5F_{p14} = 4.428$                                 | $F_{w14} = 10.517$                        | $F_{l14} = 7.888$                              |
| $w_{s15} = 0.41$             | $F_{s15} = 3.045$                                    | $w_{p15} = 0.319$            | $0.5F_{p15} = 2.372$                                 | $F_{w15} = 5.417$                         | $F_{l15} = 4.063$                              |

### 3. WIND LOAD CALCULATIONS FROM SNIP [3] AND LIRA

The analysis of the frames in figure 1 for wind load using SNiP [4] is calculated from the appropriate formulae (1, 2, 3 and 4). The buildings are assumed to be sited in type A – open country with no obstruction ( $\text{Area II} = 30 \text{ kgf/m}^2$ ). The wind load calculations is summarised in tables 1, 2 and 3. These values were then processed in Lira to calculate the maximum

moment in the middle columns and as well the drift at the top of the structure. Lira was also used to calculate displacements of the nodes from unit force which were then used in subsequent dynamic analysis with MatLAB. It is to be noted that the self weight of the frame was not considered while calculating for the bending moments of columns.

Table 3. Summary of wind load for 20-storey frame

| Average wind load,           |  | Pulsation wind load,         |  | Total wind load, F in kN                |  |
|------------------------------|--|------------------------------|--|---|--|
| $w_{si}$ , kN/m <sup>2</sup> | $F_{si}=A(w_{si}),$ kN<br>$A=10.692\text{m}^2$ | $w_{pi}$ , kN/m <sup>2</sup> | $F_{pi}=A(w_{pi}),$ kN<br>$A=10.692\text{m}^2$ | Windward,<br>$F_{wi} = F_{si} + F_{pi}$ | Leeward, $F_{li} =$<br>$0.75(F_{si} + F_{pi})$ |
| $w_{s1} = 0.18$              | $F_{s1} = 2.245$                               | $w_{p1} = 0.016$             | $0.5F_{p1} = 0.169$                            | $F_{w1} = 2.414$                        | $F_{l1} = 1.811$                               |
| $w_{s2} = 0.24$              | $F_{s2} = 2.566$                               | $w_{p2} = 0.032$             | $0.5F_{p2} = 0.337$                            | $F_{w2} = 2.903$                        | $F_{l2} = 2.177$                               |
| $w_{s3} = 0.24$              | $F_{s3} = 2.887$                               | $w_{p3} = 0.047$             | $0.5F_{p3} = 0.506$                            | $F_{w3} = 3.393$                        | $F_{l3} = 2.545$                               |
| $w_{s4} = 0.30$              | $F_{s4} = 3.208$                               | $w_{p4} = 0.063$             | $0.5F_{p4} = 0.675$                            | $F_{w4} = 3.883$                        | $F_{l4} = 2.912$                               |
| $w_{s5} = 0.30$              | $F_{s5} = 3.208$                               | $w_{p5} = 0.079$             | $0.5F_{p5} = 0.843$                            | $F_{w5} = 4.051$                        | $F_{l5} = 3.038$                               |
| $w_{s6} = 0.30$              | $F_{s6} = 3.528$                               | $w_{p6} = 0.095$             | $0.5F_{p6} = 1.012$                            | $F_{w6} = 4.540$                        | $F_{l6} = 3.405$                               |
| $w_{s7} = 0.36$              | $F_{s7} = 3.849$                               | $w_{p7} = 0.110$             | $0.5F_{p7} = 1.181$                            | $F_{w7} = 5.030$                        | $F_{l7} = 3.773$                               |
| $w_{s8} = 0.36$              | $F_{s8} = 3.849$                               | $w_{p8} = 0.126$             | $0.5F_{p8} = 1.349$                            | $F_{w8} = 5.198$                        | $F_{l8} = 3.899$                               |
| $w_{s9} = 0.36$              | $F_{s9} = 3.849$                               | $w_{p9} = 0.142$             | $0.5F_{p9} = 1.518$                            | $F_{w9} = 5.367$                        | $F_{l9} = 4.025$                               |
| $w_{s10} = 0.36$             | $F_{s10} = 3.849$                              | $w_{p10} = 0.158$            | $0.5F_{p10} = 1.687$                           | $F_{w10} = 5.536$                       | $F_{l10} = 4.152$                              |
| $w_{s11} = 0.36$             | $F_{s11} = 3.849$                              | $w_{p11} = 0.174$            | $0.5F_{p11} = 1.856$                           | $F_{w11} = 5.705$                       | $F_{l11} = 4.279$                              |
| $w_{s12} = 0.36$             | $F_{s12} = 3.849$                              | $w_{p12} = 0.189$            | $0.5F_{p12} = 2.024$                           | $F_{w12} = 5.873$                       | $F_{l12} = 4.405$                              |
| $w_{s13} = 0.41$             | $F_{s13} = 4.116$                              | $w_{p13} = 0.205$            | $0.5F_{p13} = 2.193$                           | $F_{w13} = 6.309$                       | $F_{l13} = 4.732$                              |
| $w_{s14} = 0.41$             | $F_{s14} = 4.384$                              | $w_{p14} = 0.221$            | $0.5F_{p14} = 2.362$                           | $F_{w14} = 6.746$                       | $F_{l14} = 5.060$                              |
| $w_{s15} = 0.41$             | $F_{s15} = 4.384$                              | $w_{p15} = 0.237$            | $0.5F_{p15} = 2.530$                           | $F_{w15} = 6.914$                       | $F_{l15} = 5.186$                              |
| $w_{s16} = 0.41$             | $F_{s16} = 4.384$                              | $w_{p16} = 0.252$            | $0.5F_{p16} = 2.699$                           | $F_{w16} = 7.083$                       | $F_{l16} = 5.312$                              |
| $w_{s17} = 0.41$             | $F_{s17} = 4.384$                              | $w_{p17} = 0.268$            | $0.5F_{p17} = 2.868$                           | $F_{w17} = 7.252$                       | $F_{l17} = 5.439$                              |
| $w_{s18} = 0.41$             | $F_{s18} = 4.384$                              | $w_{p18} = 0.284$            | $0.5F_{p18} = 3.036$                           | $F_{w18} = 7.420$                       | $F_{l18} = 5.565$                              |
| $w_{s19} = 0.41$             | $F_{s19} = 4.384$                              | $w_{p19} = 0.300$            | $0.5F_{p19} = 3.205$                           | $F_{w19} = 7.589$                       | $F_{l19} = 5.692$                              |
| $w_{s20} = 0.41$             | $F_{s20} = 2.192$                              | $w_{p20} = 0.316$            | $0.5F_{p20} = 1.687$                           | $F_{w20} = 3.879$                       | $F_{l20} = 2.909$                              |

#### 4. DYNAMIC ANALYSIS ON MATLAB

Structural analysis becomes a dynamic analysis when load becomes a function of time. The change in wind pressure (pulsation) with time is assumed to be proportional to a similar change in wind speed. With this assumption it is sufficient to use a real wind velocity-time graph taken from [7]. From this graph, the wind pulsation load is deduced proportionately to nodal wind load from tables (1, 2, and 3) for each floor of the entire buildings. These values are by numerical method arranged in matrixes according to equation (5), which is then processed in MatLAB in order to produce dynamic characteristics of the buildings. Input data and computation analysis are shown in appendix I for 10-storey frame. The same process as shown in appendix I for 10-storey was followed for 15-storey and 20-storey frames respectively.

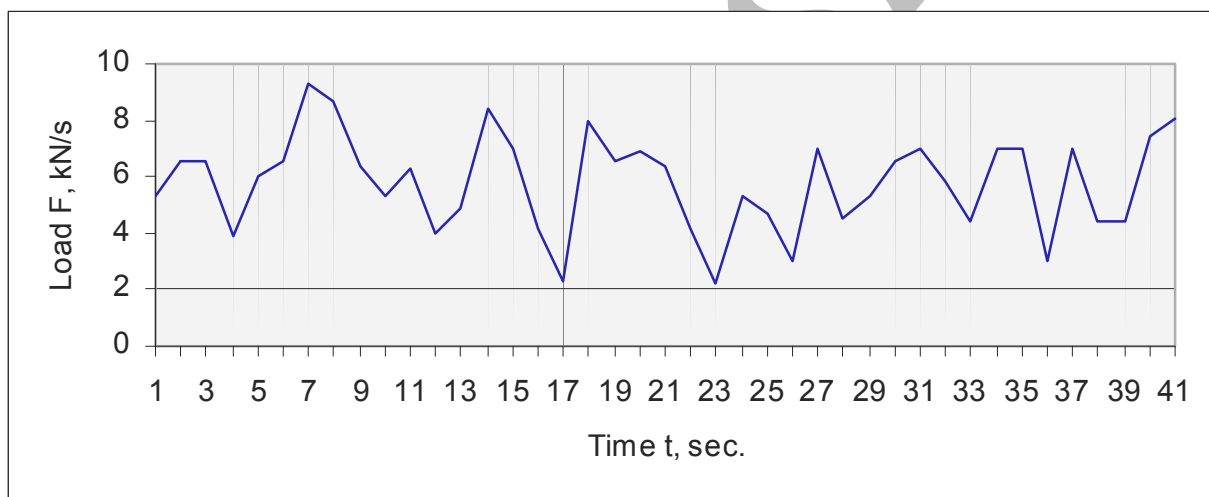


Figure 2. Wind load variation with time at the 9<sup>th</sup> floor of the 10-storey frame

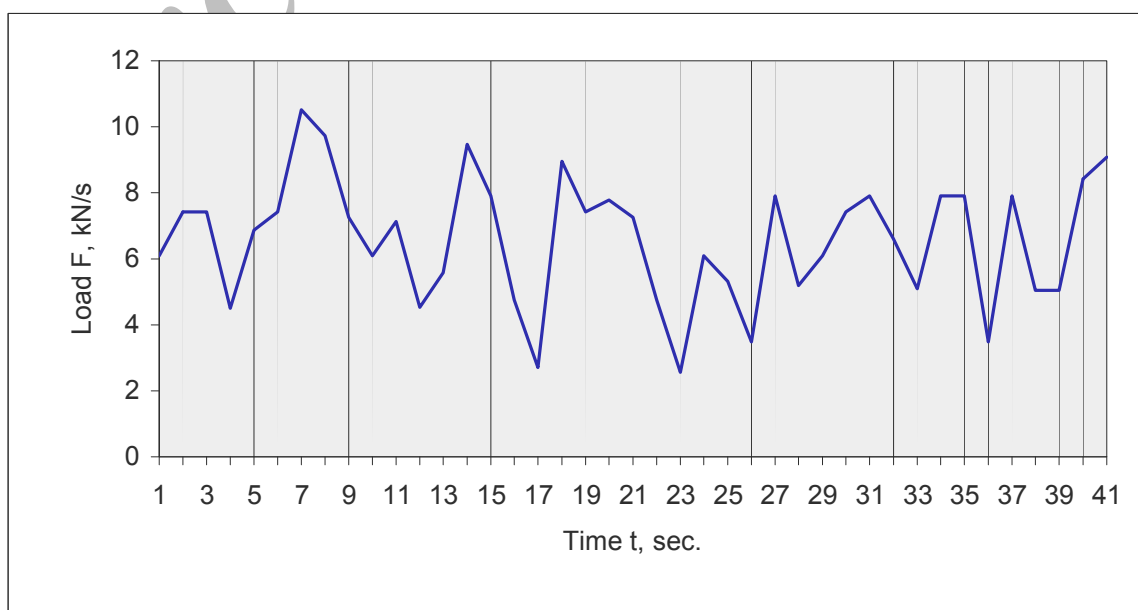


Figure 3. Wind load variation with time at the 14<sup>th</sup> floor of the 15-storey frame

An example of this operation is on the 10-storey frame. The maximum amplitude is in the 9th floor, 3.983kN which is added to the statistical wind load 5.35kN to produce a total of 9.33kN. The illustration is shown in figure 2. For the period of 40seconds the peak is located in the 6th second. All other floors are equally computed for in relation to the 9th floor. The same procedure is followed for the 15-storey and 20-storey frames with their maximum amplitude appearing at 14th and 19th floor respectively as illustrated in figure 3 and 4.

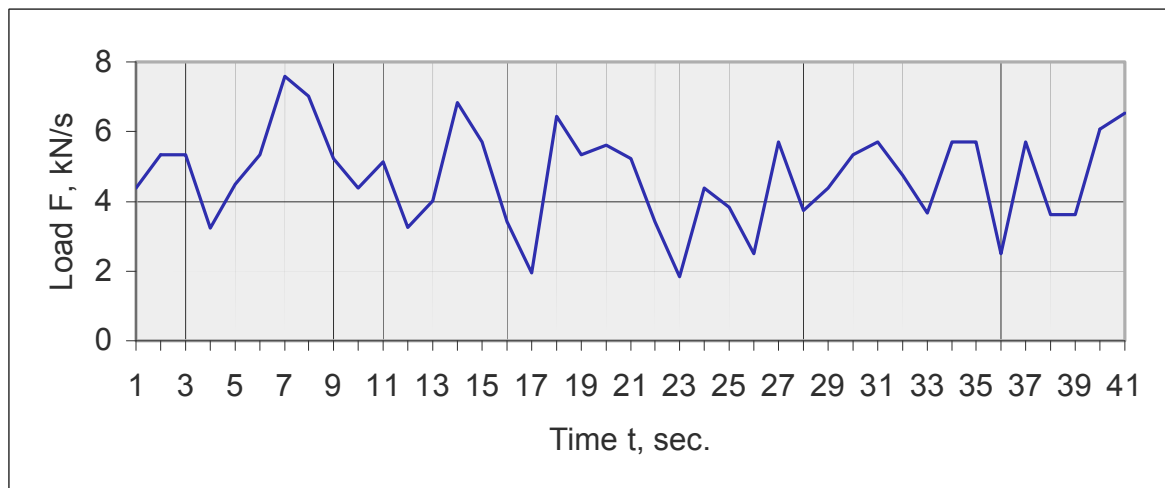


Figure 4. Wind load variation with time at the 19<sup>th</sup> floor of the 20-storey frame

## 5. RESULTS AND DISCUSSION

The result of the analysis is presented in table 4. Results from the dynamic analysis on MatLAB and equivalent calculations from SNIIP and Lira are closely similar with slight deflections. The following deductions can be made:

1. Maximum drift  $\delta_{\max}$  generally remain within the limit  $H/500$  [4] except in the case of 10-storey where  $\delta_{\max}$  obtained from MatLAB was beyond limit by +29%, thus indicating an effect of wind pulsation;
2. Maximum acceleration in all cases remains within limit of  $0.1\text{m/s}^2$  [8], however this was obtainable only from dynamic analysis on MatLAB;
3. Maximum bending moment  $M_{\max}$  obtained from MatLAB was always more than that obtained from LIRA and SNIIP by 26%, 33% and 22% for 10, 15 and 20-storey frame respectively. This is also indicating an effect of wind pulsation;
4. The summary of wind effect due to pulsation can be presented graphically in figure 5, where  $w_s$  – is statically defined average wind pressure;  $w_p$  – is the pulsation wind pressure. In this illustration it is shown that at any given time the value of pulsation wind pressure can be more than its equivalent calculated value given by SNIIP. This can only be detected by dynamic analysis on MatLAB.

Table 4. Summary of result

|  | SNiP[3]  | Lira   | MatLAB |        |
|--|--|--------|--------|--------|
| 15-storey analytical frame                       | Mode frequency,                                  |        |        |        |
|  | $f_1$ , Hz                                       | 0.565  | 0.565  | 0.565  |
|  | $f_2$ , Hz                                       | 1.727  | 1.727  | 1.727  |
|  | $f_3$ , Hz                                       | 3.004  | 3.004  | 3.004  |
|  | Drift, $\delta_{max}$ , mm                       | 24     | 24     | 85     |
|  | Drift limit, $\delta_l$ , mm [4]                 |        | 66     |        |
|  | $M_{max}$ , kN.m                                 | 89.97  | 89.97  | 113.43 |
|  | Acceleration, $a_{max}$ , m/s <sup>2</sup>       | -      | -      | 0.105  |
|  | Acceleration limit, $a_l$ , m/s <sup>2</sup> [8] |        | 0.100  |        |
|  | Mode frequency,                                  |        |        |        |
|  | $f_1$ , Hz                                       | 0.369  | 0.369  | 0.369  |
|  | $f_2$ , Hz                                       | 1.127  | 1.127  | 1.127  |
|  | $f_3$ , Hz                                       | 1.962  | 1.962  | 1.962  |
|  | Drift, $\delta_{max}$ , mm                       | 63.5   | 63.5   | 85     |
| Drift limit, $\delta_l$ , mm [4]                 |  | 99     |        |        |
| $M_{max}$ , kN.m                                 | 153.54   | 153.54 | 203.80 |        |
| Acceleration, $a_{max}$ , m/s <sup>2</sup>       | -  | -      | 0.099  |        |
| Acceleration limit, $a_l$ , m/s <sup>2</sup> [8] |  | 0.100  |        |        |
| 20-storey analytical frame                       | Mode frequency,                                  |        |        |        |
|  | $f_1$ , Hz                                       | 0.396  | 0.396  | 0.396  |
|  | $f_2$ , Hz                                       | 1.222  | 1.222  | 1.222  |
|  | $f_3$ , Hz                                       | 2.174  | 2.174  | 2.174  |
|  | Drift, $\delta_{max}$ , mm                       | 94     | 94     | 106    |
|  | Drift limit, $\delta_l$ , mm [4]                 |        | 119    |        |
|  | $M_{max}$ , kN.m                                 | 143.85 | 143.85 | 175.00 |
|  | Acceleration, $a_{max}$ , m/s <sup>2</sup>       | -      | -      | 0.100  |
|  | Acceleration limit, $a_l$ , m/s <sup>2</sup> [8] |        | 0.100  |        |

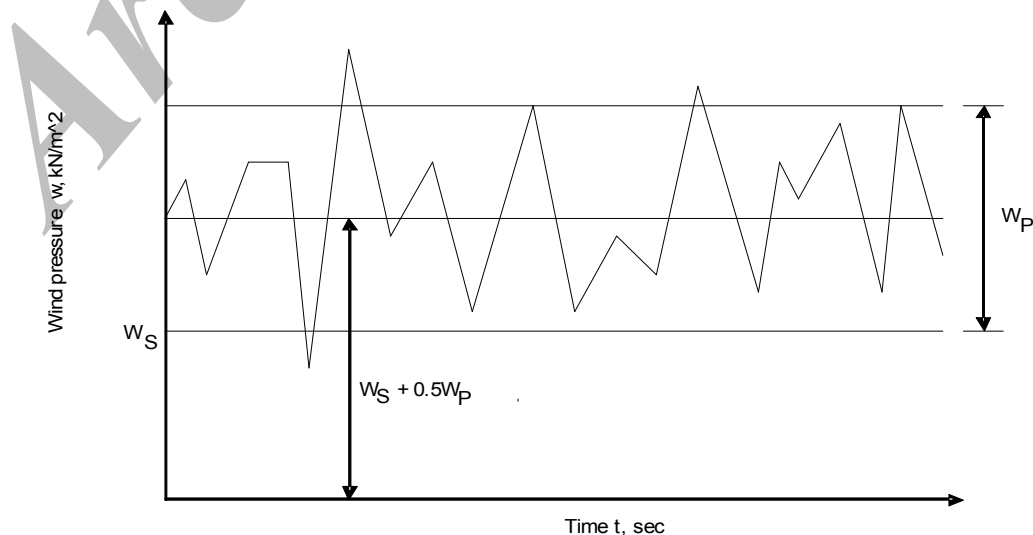


Figure 5. Summary of the wind effect due to pulsation.



## 5. CONCLUSIONS

Dynamic analysis of tall building under pulsation wind load excitation is presented. Results show reliably, dynamic behaviour of the building since they are generally similar. The traditional way of limiting the lateral drift at the top of the building to a fraction of the height under the statically equivalent design wind load prescribed in SNiP is not sufficient to spare occupants discomfort. This is because comfort of occupants on the top floor is closely related to building acceleration which is a dynamic problem due to wind pulsation as analysed in this paper, therefore dynamic analysis on MatLAB will reasonably serve in designing and studying the behaviour of a multi-storey building.

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## Appendix I

```
% Numerical method of analysis of dynamic equations on MatLAB
```

```
% Input data
```

```
dt=1;a=0.5;b=0.25;a0=1/(b*dt^2);a1=a/(b*dt);a2=1/(b*dt);a3=(a-2*b)*dt/(2*b);
a4=(a-b)/b;a5=(0.5-b)/b;
bs=[-0.667;-0.758;-0.819;-0.861;-0.870;-0.875;-0.876;-0.877;-0.877;-0.877];
sf=89.97;
f=1e-3*[0.017 0.022 0.022 0.023 0.023 0.023 0.023 0.023 0.023 0.023
0.022 0.049 0.055 0.056 0.056 0.056 0.057 0.057 0.057 0.057
0.022 0.055 0.084 0.090 0.091 0.092 0.092 0.092 0.093 0.093
0.023 0.056 0.090 0.119 0.126 0.127 0.128 0.128 0.129 0.130
0.023 0.056 0.091 0.126 0.155 0.162 0.164 0.165 0.166 0.167
0.023 0.056 0.092 0.127 0.162 0.192 0.200 0.202 0.203 0.205
0.023 0.057 0.092 0.128 0.164 0.200 0.230 0.238 0.241 0.243
0.023 0.057 0.092 0.128 0.165 0.202 0.238 0.269 0.278 0.281
0.023 0.057 0.093 0.129 0.166 0.203 0.241 0.278 0.310 0.319
0.023 0.057 0.093 0.130 0.167 0.205 0.243 0.281 0.319 0.353];
M1=[51 51 51 51 51 51 51 51 61];m=diag(M1);
be1=[0.090 0.276 0.481 0.691 0.934 1.143 1.404 2.120 1.658 1.806];be=diag(be1);
% Input data wind load at joints of 10 storey frame
p0=1.75*[3.12 3.25 3.25 2.96 3.20 3.25 3.56 3.48 3.24 3.12 3.22 2.96 3.07 3.46
3.30 2.99 2.78 3.41 3.25 3.29 3.24 2.99 2.77 3.12 3.04 2.86 3.30 3.03 3.12 3.25
3.30 3.17 3.02 3.30 3.30 2.86 3.30 3.02 3.02 3.35 3.42 ;3.56 3.82 3.82 3.24
3.72 3.82 4.44 4.29 3.79 3.56 3.77 3.25 3.46 4.23 3.92 3.30 2.89 4.13 3.82 3.90
3.79 3.30 2.86 3.56 3.40 3.04 3.92 3.38 3.56 3.82 3.92 3.66 3.36 3.92 3.92 3.04
3.92 3.35 3.35 4.03 4.16;4.01 4.41 4.41 3.53 4.24 4.41 5.34 5.10 4.36 4.01 4.32
3.54 3.86 5.02 4.56 3.61 3.00 4.87 4.41 4.52 4.36 3.61 2.96 4.01 3.78 3.23 4.56
3.74 4.01 4.41 4.56 4.16 3.71 4.56 4.56 3.23 4.56 3.70 3.70 4.71 4.91;4.46 4.99
4.99 3.83 4.77 4.99 6.23 5.92 4.93 4.46 4.88 3.84 4.26 5.81 5.19 3.93 3.11 5.61
4.99 5.14 4.93 3.93 3.06 4.46 4.15 3.42 5.19 4.10 4.46 4.99 5.19 4.66 4.06 5.19
5.19 3.42 5.19 4.04 4.04 5.39 5.65 ;4.46 5.12 5.12 3.66 4.85 5.12 6.67 6.28
5.04 4.46 4.98 3.68 4.20 6.15 5.37 3.80 2.77 5.89 5.12 5.31 5.04 3.80 2.70 4.46
4.07 3.16 5.37 4.01 4.46 5.12 5.37 4.72 3.97 5.37 5.37 3.16 5.37 3.94 3.94 5.63
5.96 ;4.90 5.69 5.69 3.95 5.37 5.69 7.55 7.09 5.60 4.90 5.53 3.97 4.59 6.93
5.99 4.11 2.87 6.62 5.69 5.92 5.60 4.11 2.79 4.90 4.43 3.34 5.99 4.36 4.90 5.69
5.99 5.21 4.31 5.99 5.99 3.34 5.99 4.27 4.27 6.30 6.69 ;5.35 6.28 6.28 4.24
5.89 6.28 8.45 7.90 6.17 5.35 6.08 4.26 4.99 7.72 6.63 4.42 2.98 7.36 6.28 6.19
6.17 4.42 2.89 5.35 4.81 3.53 6.63 4.72 5.35 6.28 6.63 5.71 4.66 6.63 6.63 3.53
6.63 4.62 4.62 6.98 7.44 ;5.35 6.41 6.41 4.08 5.97 6.41 8.89 8.27 6.28 5.35
6.19 4.11 4.94 8.05 6.81 4.29 2.65 7.64 6.41 6.71 6.28 4.29 2.54 5.35 4.73 3.27
6.81 4.63 5.35 6.41 6.81 5.76 4.56 6.81 6.81 3.27 6.81 4.51 4.51 7.22 7.74
;5.35 6.54 6.54 3.92 6.05 6.54 9.33 8.63 6.40 5.35 6.29 3.95 4.89 8.39 6.99
4.16 2.31 7.93 6.54 6.88 6.40 4.16 2.19 5.35 4.65 3.01 6.99 4.54 5.35 6.54 6.99
5.81 4.46 6.99 6.99 3.01 6.99 4.41 4.41 7.45 8.04 ;2.67 3.33 3.33 1.87 3.06
3.33 4.88 4.49 3.25 2.67 3.19 1.89 2.41 4.36 3.58 2.01 0.98 4.10 3.33 3.52 3.25
2.01 0.91 2.67 2.28 1.37 3.58 2.22 2.67 3.33 3.58 2.93 2.18 3.58 3.58 1.37 3.58
2.15 2.15 3.84 4.17];
k=inv(f); c=m*be;r0=1.75*[3.12;3.56;4.01;4.46;4.46;4.90;5.35;5.35;5.35 2.67];
yn=inv(m)*r0;k1=a0*m+a1*c;k2=a2*m+a4*c;k3=a5*m+a3*c;k01=k+k1;g1=inv(k01);w1=g1*r0;
v1=a1*w1;y1=a0*w1;
% Iterative process
for j=1:40
p=p0(:,j);r1=p+k1*w1+k2*v1+k3*y1;w3=g1*r1;v3=a1*(w3-w1)-a4*v1-a3*y1;
y3=a0*(w3-w1)-a2*v1-a5*y1;
w1=w3;v1=v3;y1=y3;z=(w1(10))';y=(y1(10))';w=z+0.0629;t=1*j;s=-m*y1;my=bs'*s+sf;
t=1.0*j;
sm=my';yz=[t sm];%yz=[t w];%yz=[t y]; disp(yz);
end.
```