Stochastic Network Calculus Analysis of Energy Harvesting Rate in Wireless Networks with Delay and Energy Storage Constraints

Zhidu Li, Yuehong Gao, Bala Alhaji Salihu, Pengxiang Li, Lin Sang, Dacheng Yang
Wireless Theories and Technologies Lab
Beijing University of Posts and Telecommunications
Beijing, P.R. China, 100876
Email: prclzd@126.com

***Abstract*—Energy evaluation of wireless transmission for data service under different QoS (quality of service) constraints is a hot topic in green communications. In this paper, we employ the theory of the stochastic network calculus to investigate the relationship between the traffic arrival rate and energy harvesting rate under the delay and energy storage constraints. We first construct a wireless system model which works only by consuming the harvested renewable energy. Also, stochastic traffic arrivals, packet size as well as a two-state Markov chain interference on energy harvesting are jointly considered to ensure the accuracy of the analysis. We derive the minimum energy harvesting rate needed to satisfy the delay and energy storage constraints under a given average arrival rate. The efficiency of the harvested energy is also studied and shown to be convex in the numerical simulation. Numerical results illustrate that while the minimum energy harvesting rate is positively correlated with the packet size, it is negatively related to the delay requirement, under a given arrival rate. Additionally, the probabilistic bound of energy insufficiency is shown to be positively correlated with the transition cycle of the interference.**

Keywords*—Energy harvesting rate; delay constraint; energy storage constraint; stochastic network calculus.

I. Introduction

With development of energy harvesting technologies, renewable energy like solar energy and wind energy have been used to sustain the operations of some wireless communication systems (e.g., solar mobile phones and solar base stations) [1, 2]. Unlike traditional fossil fuel, energy in these systems is usually limited and harvested stochastically [3]. Solar energy, for instance, can only be collected during the daytime, and the amount of energy being harvested depends on other conditions. Moreover, data services especially real-time services like multimedia video and live broadcast have become more and more popular for users. These services usually generate high amount of traffic and require to be served under strict and sensitive delay constraints, which means high transmission rate and high transmission power are needed. Therefore, in order to guarantee the QoS of different services in wireless communication systems through consuming renewable energy, it is certainly worth studying the relationship between traffic arrivals and energy harvesting under delay and energy storage constraints.

Previous works of renewable energy usually focus on how much probability for a real smart grid system to guarantee cumulative energy demands [3–5]. Researchers thereof derived the relationship between energy harvesting and energy demands. However, the energy demands in these works are directly modeled based on the collected sampled data, which consequently neglected the traffic and transmission characteristics. Besides, researches jointly considering arrival traffic, transmission rate and energy consumption can be divided into two classes based on whether delay constraints are taken into account or not. For example, disregarding delay constraints, Gursoy and Qiao ascertained the relationship between the spectral efficiency and energy efficiency by using the theory of effective capacity [6, 7]. Differently, Zafer and Li, respectively, found minimum transmission powers to meet delay constraints under different traffic arrivals [8, 9]. However, all of these works (i.e., [6–9]) did not address energy harvesting, which means energy is assumed to be always sufficient for transmission. In addition, Gao constructed a wireless communication model with finite energy storage to analyze the battery life under delay constraints [10]. Though traffic and transmission characteristics as well as energy consumption were all considered in [10], the analysis also did not include energy harvesting. To the best of our knowledge, synthetical studies of the relationship between traffic arrivals and energy harvesting under delay and finite energy storage constraints have not been available.

Motivated by this, we focus on the relationship mentioned above. Specifically, we are mainly concerned with the minimum energy harvesting rate needed to support the transmission of a stochastic arrival traffic under delay and energy storage constraints. Also, our analysis can conversely obtain the maximum mean throughput rate sustained by the network when energy harvesting rate is known. Energy efficiency defined as energy per bit normalized to background noise spectral level is also one of our interests. Additionally, the probabilistic bound of energy insufficiency is investigated to provide a guideline on choosing appropriate energy storage capacity to ensure the system to work efficiently. In order to heighten the accuracy of our analysis, the impacts of packet size and interference are also taken into account.

This work is supported by the National Science Foundation of China under Grant No.61300185.

The analytical tool which enables the analysis in this paper is the stochastic network calculus [11]. The stochastic network calculus uses probabilistic bounds to characterize the arrival and service processes, and then to derive the performance measurements. The key idea in the stochastic network calculus is to transform a complex non-linear queueing system into an analytically tractable system where some violations of performance criteria can be tolerated. Thus, it is a powerful tool for QoS analysis especially in scenarios full of randomness (e.g. stochastic wireless networks). Applications of the stochastic network calculus could be found in [9, 10, 12–14] etc.

Main contributions of this paper are as follows:

- We derive a closed form expression to characterize the relationship between traffic arrival rate and energy harvesting rate under delay and energy storage constraints.
- Our analysis is general because the traffic arrivals and energy harvesting are considered to be stochastic processes. Moreover, while analyzing the violation probability of delay requirement, we synthetically consider the case of energy insufficiency which has not ever been taken into account in the literatures (e.g. [8–10]).

The remainder of this paper is organized as follows. Section II introduces the system model, where some definitions and assumptions are given. Section III demonstrates how we derive the relationship between traffic arrival rate and energy harvesting rate. Thereafter, numerical results are presented and discussed in section IV. Finally, section V concludes the paper.

II. System Model

We consider a wireless communication system as depicted in Fig.1, containing transmission channels, power controller and battery. The system model can be regarded as a working scenario of a solar mobile phone. Suppose the system works by only consuming renewable energy which is harvested from the energy source. The energy harvesting process and the energy demand are assumed to be independent. As the existence of interference, the energy harvesting process is considered to be stochastic. A battery with finite capacity is equipped to store the harvested energy and support the transmission. If the battery is fully charged, the surplus energy will be discarded. In addition, we assume that the perfect channel state information is available in the system, namely, the power controller always allocates appropriate transmission power to transmit the arrival traffic according to the give delay constraint and the long term traffic characteristics (e.g. traffic type, packet size and average arrival rate) as long as the harvested energy is sufficient.

A. Stochastic Network Calculus Basic Notions

In this paper, the cumulative traffic arrivals within time interval $[s, t]$ is denoted by $A(s, t)$, and $A(0, t) = A(t)$ for simplification. The cumulative departures of the system is similarly denoted by $A^*(t)$. Additionally, we let $A(0) = A^*(0) = 0$. The delay is denoted by $W(t) = \inf\{t_0 : A(t) \leq A^*(t+t_0)\}$, wherein $W(t)$ represents the delay of the last packet arriving at

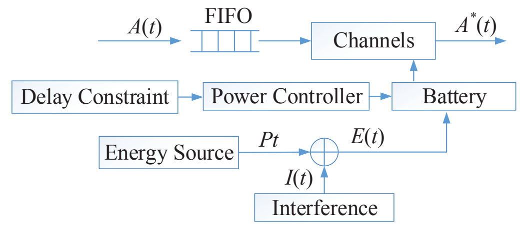

Fig. 1. System model

time t. In particular, The delay constraint, denoted by (ϵ_d, t_0), is defined as

$$Pr\{W(t) > t_0\} \leq \epsilon_d, \tag{1}$$

where ϵ_d is a probabilistic violation bound. The definition of the delay constraint means that the traffic delay exceeding t_0 should be controlled within a probability ϵ_d.

There are two basic concepts in the stochastic network calculus: the stochastic arrival curve and the stochastic service curve, which are used to describe the arrival process of the input traffic and the service process respectively.

A flow $A(t)$ is said to have a stochastic arrival curve $\alpha(t)$ with the bounding function $f(t)$, denoted by $A \sim< f, \alpha >$, if for all $t \geq s \geq 0$ and all $x \geq 0$, there holds [11]

$$Pr\{\sup_{0\leq s\leq t}\{A(s,t) - \alpha(s,t)\} > x\} \leq f(x). \tag{2}$$

The transmission channels are said to provide a stochastic service curve $\beta(t)$ with the bounding function $g(x)$, denoted by $S \sim< g, \beta >$, if for all $t \geq 0$ and all $x \geq 0$, there holds [11]

$$Pr\{A \otimes \beta(t) - A^*(t) > x\} \leq g(x), \tag{3}$$

where $\otimes$ is the minimum plus (min; +) convolution operation, and $A \otimes \beta(t) \triangleq \inf_{0\leq s\leq t}\{A(s) + \beta(s, t)\}$.

B. Traffic, Transmission and Energy Harvesting Models

In this paper, we use the $(\sigma(\theta), \rho(\theta))$ traffic model which is proposed by Chang in [15] to perform our analysis. A process A is said to have a stochastic arrival curve $A \sim< e^{-\theta x}, \rho(\theta)t + \sigma(\theta) >$ with respect to a chosen $\theta > 0$, if A has stationary and independent increments and for all $t \geq 0$, there holds [11]

$$\frac{1}{\theta}\ln \mathrm{E}[e^{\theta A(0,t)}] \leq \rho(\theta)t + \sigma(\theta).$$

It has been showed that many types of traffic can be represented by using the $(\sigma(\theta), \rho(\theta))$ model [11][15], including exponential ON-OFF process, Markov modulated process and Poisson process.

For the transmission channels, we denote the transmission rate and the transmission power by C and P respectively. Also, the bandwidth of the channels and the power spectral density of background noise are denoted by W and N_0 respectively. We assume that the channels do not suffer any fading and the transmission is lossless, i.e., the transmission rate only depends on P, W and N_0, there holds

$$C = W\log_2(1 + \frac{P}{N_0 W}). \tag{4}$$

In this paper, the transmission power P depends on the traffic characteristics and the delay constraint (ϵ_d, t_0). In addition, we adopt a sufficient condition for a stability system as $C \geq \rho(\theta)$, which is also widely used in the stochastic network calculus framework (e.g. [9], [10] and [13] etc).

For the energy harvesting model, we assume the energy harvesting rate is constant and denoted by P_0. Consider a two-state Markov chain interference process $I(t)$ with ON state and OFF state. The energy being harvested would be exhausted when interference process is in ON state and would be well preserved in OFF state. The state transition times from ON to OFF and from OFF to ON are both exponentially distributed with mean values $1/\lambda$ and $1/\mu$ respectively, which means the average state transition cycle T equals to $(1/\lambda+1/\mu)$. Also, we assume the interference process has independent and stationary increments, i.e. for any nonnegative variables s, t, k and x, $I(s,t)$ is independent of $I(s+k, t+k)$, and

$$Pr\{I(s,t) > x\} \equiv Pr\{I(s+k, t+k) > x\}.$$

Consequently, the energy harvesting process can be denoted by $E(t)$, and

$$E(t) = P_0 t - I(t).$$

Also, we denote the energy demand process by $D(t)$. For holding the system stable, we assume that for any $\theta > 0$ and $t > 0$, there holds

$$\mathrm{E}[e^{\theta E(0,t)}] \geq \mathrm{E}[e^{\theta D(0,t)}]. \tag{5}$$

However, since $E(t)$ and $D(t)$ are both random variables, $D(t) \geq E(t)$ may hold at some time. Thus, we use $B(t)$ to denote the cumulative amount of energy deficit up to t, and

$$\begin{aligned} B(t) &= \max\{0, B(t-1) + D(t-1,t) - E(t-1,t)\} \\ &= \sup_{0\leq s\leq t} \{D(s,t) - E(s,t)\} \end{aligned}. \tag{6}$$

Suppose the battery is fully charged at time 0. We remark that if energy deficit is greater than the battery capacity, the energy will be insufficient. With a given battery capacity b, the probabilistic bound of energy insufficiency is defined as

$$Pr\{B(t) > b\} \leq \epsilon_b, \tag{7}$$

where ϵ_b is a probabilistic violation bound. Here, (7) is also called the energy storage constraint and denoted by (ϵ_b, b).

Additionally, we use the definition in [6, 7] to measure the efficiency of the harvested energy. Energy efficiency, denoted by $\frac{E_b}{N_0}$ (in decibels), is defined as energy per bit normalized to spectral density of background noise, i.e.

$$\frac{E_b}{N_0} = 10\log_{10}(\frac{P_0}{rN_0}), \tag{8}$$

where r denotes the average arrival rate.

Based on the descriptions and assumptions above, the minimum energy harvesting rate or the maximum sustained throughput rate can be derived reciprocally under the delay constraint (ϵ_d, t_0) and the energy storage constraint (ϵ_b, b). Specifically, an optimization problem is formulated as follows;

$$\begin{array}{lll} \min & P_0(r, (\epsilon_d, t_0), (\epsilon_b, b)) \text{ or } & \max \ r(P_0, (\epsilon_d, t_0), (\epsilon_b, b)) \\ \text{s.t.} & C \geq \rho(\theta), & Pr\{W(t) > t_0\} \leq \epsilon_d \\ & \mathrm{E}[e^{\theta E(0,t)}] \geq \mathrm{E}[e^{\theta D(0,t)}], & Pr\{B(t) > b\} \leq \epsilon_b \end{array}. \tag{9}$$

For ease of understanding the subsequent analysis, we will use Poisson traffic as a specific type of $(\sigma(\theta), \rho(\theta))$ traffic to derive the relationship between traffic arrival rate and energy harvesting rate, and then obtain the solution of (9).

III. Performance Analysis

In this section, Poisson traffic with constant packet size is employed as a stochastic arrival traffic to perform our analysis. The analysis is divided into two parts. One is to find out the relationship between traffic arrival rate and transmission rate under the delay and energy storage constraints. The other part is to ascertain the relationship between transmission power and energy harvesting rate under energy storage constraints.

A. Traffic Arrival Rate and Transmission Rate

At first, we give the stochastic arrival curve and the stochastic service curve for the considered system. The stochastic arrival curve with the bounding function is deduced as follows [15, 16]

$$\begin{aligned} &\alpha(t) = \frac{1}{\theta}\ln \mathrm{E}[e^{\theta A(0,t)}] = \frac{rt}{L\theta}(e^{\theta L} - 1) \\ &f(x) = e^{-\theta x} \end{aligned}, \tag{10}$$

where r denotes the average arrival rate, L denotes the packet size and θ is a non-negative optimization parameters.

In the light of the assumption in Section II, the transmission rate is fixed when traffic characteristics and delay constraint are given. Hence, the service curve of the channels holds as follows [15, 16]

$$\begin{aligned} &\beta(t) = Ct \\ &g(x) = 0 \end{aligned}. \tag{11}$$

For holding the system stable, we have

$$C \geq \frac{r}{L\theta}(e^{\theta L} - 1). \tag{12}$$

While giving a certain delay requirement t_0 and sufficient energy, the probabilistic delay bound is derived as follows:

$$\begin{aligned} &Pr\{W(t) > t_0\} \\ \leq& Pr\{A(t) - A^*(t+t_0) > 0\} \\ \leq& Pr\{\sup_{0\leq s\leq t}\{A(s,t) - \alpha(s,t) + \alpha(s,t) - \beta(s,t+t_0)\} \\ &+ A\otimes\beta(t+t_0) - A^*(t+t_0) > 0\} \\ \leq& Pr\{\sup_{0\leq s\leq t}\{A(s,t) - \alpha(s,t)\} + A\otimes\beta(t+t_0) - A^*(t+t_0) \\ &> \inf_{0\leq s\leq t}\{\beta(s,t+t_0) - \alpha(s,t)\}\} \\ \leq& f\otimes g(Ct_0) \\ =& e^{-\theta C t_0} \end{aligned}$$

Here, the fourth line is based on the Lemma 1.5 in [11] and the last line uses the definition of min-plus convolution. Furthermore, while considering the case of energy insufficiency, the relationship between the probabilistic delay bound and the probabilistic bound of energy insufficiency turns to be

$$Pr\{W(t) > t_0\} = Pr\{W(t) > t_0 | B(t) \leq b\} + Pr\{B(t) > b\},$$

i.e.,

$$\epsilon_d = e^{-\theta C t_0}(1 - \epsilon_b) + \epsilon_b.$$

Consequently, there holds

$$e^{-\theta C t_0} = \frac{\epsilon_d - \epsilon_b}{1-\epsilon_b}.$$

Let $\epsilon = \frac{\epsilon_d - \epsilon_b}{1-\epsilon_b}$, the transmission rate holds as

$$C = \frac{\ln(1/\epsilon)}{\theta t_0}. \tag{13}$$

Combining (12) and (13), the relationship between traffic arrival rate and transmission rate is obtained as

$$C \geq \frac{L\ln(1/\epsilon)}{t_0 \ln(\frac{\ln(1/\epsilon)}{rt_0/L}+1)}, \tag{14}$$

where the equality holds if and only if

$$\theta = \frac{\ln(\frac{\ln(1/\epsilon)}{rt_0/L}+1)}{L}. \tag{15}$$

Note that (14) implies the minimum transmission rate needed to satisfy the delay and energy storage constraints for the arrival traffic. Conversely, if the transmission rate is given, the maximum sustained throughput rate can be figured out according to (14).

B. Transmission Power and Energy Harvesting Rate

In this subsection, we are going to be concerned with the relationship between the transmission power P and the energy harvesting rate P_0. We begin by deriving the probabilistic bound of energy deficit. Giving the traffic characteristics, the energy demand can be fixed as $D(t) = Pt$. According to (6), the probabilistic bound of energy deficit is derived as

$$\begin{aligned}
&Pr\{B(t) > x\} \\
=&Pr\{\sup_{0\leq s\leq t}\{P(t-s) - E(s,t)\} > x\} \\
=&Pr\{\sup_{0\leq s\leq t}\{e^{\xi(P(t-s)-E(s,t))}\} > e^{\xi x}\} \\
\leq&Pr\{e^{\xi(P-E(1))} > e^{\xi x}\} \\
\leq&e^{-\xi x}\mathrm{E}(e^{\xi(P-E(1))}) \\
\leq&e^{-\xi x}
\end{aligned}, \tag{16}$$

where ξ is an non-negative free parameter. Here, the third line and the fourth line of (16) hold because $e^{\xi(P(t)-E(t))}$ is a supermatingale, which is proved in Appendix A. In the last line we directly use the stability condition in (5).

Note that (16) also implies that the energy insufficiency probability is bounded by $e^{-\xi x}$ when the battery capacity equals to x. For a given energy storage constraint (ϵ_b, b) , there holds

$$\xi = \frac{\ln(1/\epsilon_b)}{b}. \tag{17}$$

Moreover, the relationship between transmission power and energy harvesting rate is obtained according to the stability condition in (5)

$$\begin{aligned}
P &\leq \frac{1}{\xi t}\ln\mathrm{E}[e^{\xi E(t)}] \\
&= \frac{1}{\xi t}\ln\mathrm{E}[e^{\xi(P_0 t - I(t))}] \\
&\leq \frac{1}{2\xi}(P_0\xi - \lambda - \mu + \sqrt{(P_0\xi+\lambda-\mu)^2 + 4\lambda\mu}) \\
&\triangleq \gamma(P_0, \xi)
\end{aligned}, \tag{18}$$

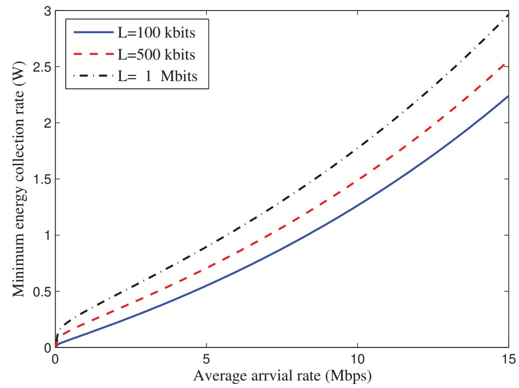

Fig. 2. Minimum energy harvesting rate vs. average arrival rate for different packet sizes

where ξ is obtained in (17).

C. Solution of the Optimization Problem

Combining (4), (17) and (18), the relationship between transmission rate and energy harvesting rate under a given energy storage constraint is directly obtained as follows:

$$C \leq W\log_2(1 + \frac{\gamma(P_0, \ln(1/\epsilon_b)/b)}{N_0 W}), \tag{19}$$

where $\gamma(P_0, \ln(1/\epsilon_b)/b)$ can be ascertained according to (18). The solution of inequality (19) then yields the minimum energy harvesting rate P_{0min} to sustain the traffic input with average arrival rate r under given delay and energy storage constraints.

$$P_{0min} = \frac{(2^{\frac{C_{min}}{W}}-1)N_0W((2^{\frac{C_{min}}{W}}-1)N_0W\xi+\lambda+\mu)}{(2^{\frac{C_{min}}{W}}-1)N_0W\xi+\lambda}, \tag{20}$$

where C_{min}, depending on r, is the right hand side of (14) and ξ is achievable in (17).

Similarly, the maximum sustained throughput rate r_{max} could be conversely deduced if the information of energy harvesting rate, delay constraint and energy storage constraint are known, there holds

$$r_{max} = \frac{K}{e^{K/C_{max}}-1}, \tag{21}$$

where $K = \frac{L\ln(1/\epsilon)}{t_0}$ and $\epsilon = \frac{\epsilon_d-\epsilon_b}{1-\epsilon_b}$. C_{max} is the righthand side of (19) and depends on P_0. Hence, the optimization problem in (9) is successfully solved.

IV. Numerical Results

In this section, we present some selected numerical results of our analysis for discussion. We let the power spectral density of background noise $N_0 = 10^{-7}$ W/Hz and the bandwidth of the channels W=11 MHz as also used in [17]. Additionally, the relationship between the mean state transition rates of the interference from ON to OFF λ and from OFF to ON μ are fixed to $\frac{\lambda}{\lambda+\mu} = 0.8$.

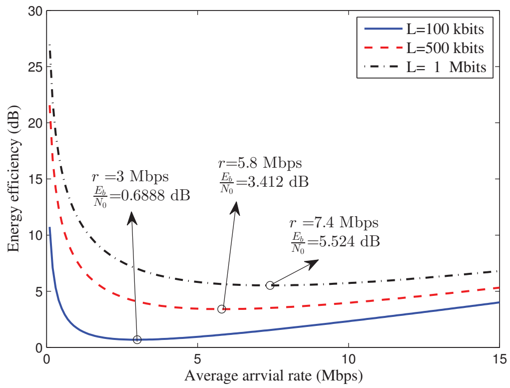

Fig. 3. Energy efficiency vs. average arrival rate for different packet sizes

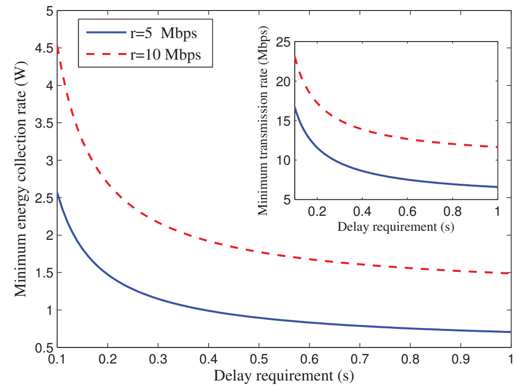

Fig. 4. Energy harvesting rate and transmission rate satisfying different delay constraints

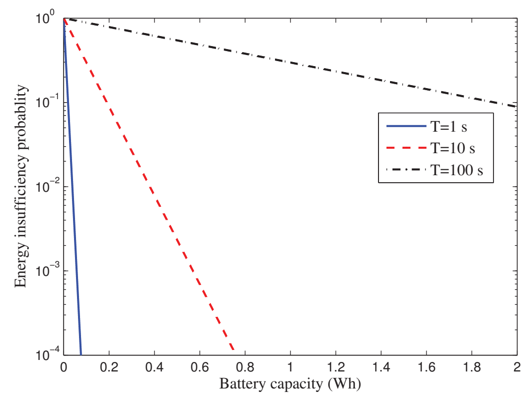

Fig. 5. Energy insufficiency probability vs. energy storage capacity

A. Impact of Packet Size on the Minimum Energy Harvesting Rate and the Energy Efficiency

Fig. 2 and Fig. 3 exhibit the variation trends of the minimum energy harvesting rate and the energy efficiency respectively in different packet size cases. The delay constraint (ϵ_d, t_0) and the energy storage constraint (ϵ_b, b) are set to (0.002, 1s) and (0.001, 1Wh (where 1Ws=1J)) respectively. The state transition cycle T (defined in Section II) of the interference process is assumed to be 1s. Moreover, to account for the impacts of packet size, we used more than one size by setting $L = 100\text{kbits}, 500\text{kbits}$ and 1Mbits as different cases.

Fig. 2 implies that the optimization problem (9) has unique solution. The average arrival rate is positively correlated with the minimum energy harvesting rate. Besides, the system needs more energy to transmit the traffic with larger packet size though other conditions are the same. This is because larger packet size leads to more stochastic in actual arrival rate, which needs larger transmission rate and then higer energy harvesting rate to ensure the delay and energy storage constraints. On the other hand, the maximum sustained throughput rate can also be observed in Fig. 2 if we regard P_0 as the independent variable and r as the dependent variable.

Fig. 3 shows that the efficiency of harvesting energy $\frac{E_b}{N_0}$. According to the definition in (8), less $\frac{E_b}{N_0}$ means higher energy efficiency. Hence, energy efficiency is negatively correlated to the packet size. In addition, we observe that energy efficiency is a convex function related to the arrival rate r. Consequently, the optimal arrival rate can be obtained to maximize energy efficiency as pointed in Fig. 3.

B. Impact of Delay Requirement on Minimum Transmission rate and Minimum Energy Harvesting Rate

In this subsection, the delay requirement is variable and the probabilistic delay bound is assumed to be $\epsilon_d = 0.002$. we set the energy storage constraint to (0.001, 1Wh). The packet size is fixed to 500kbits and the average arrival rate r is set to 5Mbps and 10Mbps as two cases. The state transition cycle T is assumed to be 1s.

Fig. 4 depicts the minimum transmission rate and minimum energy harvesting rate needed to assure different delay constraints. The transmission rate and energy harvesting rate are both negative correlated with the delay requirement. Concretely, looser delay constraint will lead to lower transmission rate and then lower energy harvesting rate. It is also clear that the decreasing trends of the four curves in Fig. 4 are more and more smoother as the delay requirement increases. Moreover, according to the right hand side of (14), the transmission rate will be close to the arrival rate provided that the delay requirement is infinite.

C. Impact of State Transition Cycle on Energy Insufficiency Probability

In this subsection, the relationship between the probabilistic bound of energy insufficiency and energy storage capacity is discussed. The energy harvesting rate and average arrival rate are fixed to 0.6W and 5Mbps respectively. We set the packet size 500kbits and the delay constraint (0.002, 1s). The state transition cycle of the interference process is set to T=1s, 10s, and 100s as three cases.

In Fig. 5, the probabilistic bound of energy insufficiency decays as the battery capacity increases. The reason is that battery with larger capacity can store more remaining energy to support the transmission at the moment when energy being harvested is insufficient. An interesting phenomenon is that the system needs larger battery capacity to satisfy the delay and energy storage constraints while suffering an interference with longer state transition cycle. This is because in one cycle time T, the average time during which the harvested energy is exhausted by the interference is $1/\mu$. Longer cycle means larger energy deficit more easily occurs when the interference process is in ON state. As a result, the system requires larger battery capacity to store the remaining energy and then support the transmission. On the other hand, Fig. 5 also provides a useful guideline on how large energy storage capacity should be chosen to ensure the system works regularly under different interference cases and different tolerable probabilities of energy insufficiency.

V. Conclusions

This paper has proposed a stochastic network calculus approach to investigate the relationship between traffic arrival rate and energy harvesting rate in a wireless communication system which operates by only consuming renewable energy. Considering the impacts of packet size and interference, minimum energy harvesting rate has been derived to sustain a given traffic arrival rate under the delay and energy storage constraints. In addition, energy efficiency and probabilistic bound of energy insufficiency were also discussed. It is highlighted that the approach in this paper is generally applicable to any $(\sigma(\theta), \rho(\theta))$ traffic even though the analysis in Section III is based on Poisson traffic. This is because the difference only addresses the stochastic arrival curve while employing other traffic to conduct the analysis.

Appendix A
Proof of Sumpermartingale

To prove $e^{\xi(P(t)-E(t))}$ is a sumpermartingale, we firstly introduce the definition and property of supermartingale [16][18].

Suppose a stochastic process U_n with finite mean, it is said to be a supermartingale iff for all $n, n = 1, 2, ...,$ there holds

$$\mathrm{E}[U_{n+1}|U_1, U_2, ..., U_n] \le U_n.$$

And if $\{U_k, 1 < k < n\}$ is a supermartingale, and all U_k is nonnegative, then for any $x > 0$, there holds:

$$Pr\{\sup_{1\le k\le n}\{U_k\} \ge x\} \le \frac{\mathrm{E}[U_1]}{x}.$$

Consider a sequence of non-negative random variables $\{V_s\}, s = 1, 2, ..., t$, formed by,

$$V_s = e^{\xi(Ps - E(t-s,t))} = e^{\xi(Ps - \sum_{k=t-s+1}^{t} X_k)},$$

where $X_k = E(k-1, k)$. Since $I(t)$ has independent and stationary increments and $E(t) = P_0 - I(t)$, $E(t)$ also has independent and stationary increments. We then have

$$\begin{aligned}
&\mathrm{E}[V_{s+1}|V_1, V_2, ...V_n] \\
=&\mathrm{E}[V_{s+1}|X_t, X_{t-1}, ...X_{t-s+1}] \\
=&\mathrm{E}[V_s e^{\xi P - \xi X_{t-s}}|X_t, X_{t-1}, ...X_{t-s+1}] \\
=&\mathrm{E}[V_s|X_t, X_{t-1}, ...X_{t-s+1}]\mathrm{E}[e^{\xi P - \xi X_{t-s}}], \\
=&V_s(\mathrm{E}[e^{\xi P}] - \mathrm{E}[e^{\xi E(1)}]) \\
\le&V_s
\end{aligned}$$

where the fourth line is due to $E(t)$ has the independent and stationary increments, and consequently

$$\mathrm{E}[V_s(X_t, X_{t-1}, ...X_{t-s+1})|X_t, X_{t-1}, ...X_{t-s+1}] = V_s,$$
$$\mathrm{E}[e^{\xi X_{t-s}}] = \mathrm{E}[e^{\xi E(t-s-1,t-s)}] = \mathrm{E}[e^{\xi E(1)}].$$

In the last line, we use the stability condition (5). Therefore, using the definition of supermartingale ends the proof.

References

[1] A. Kwasinski and A. Kwasinski, "Increasing sustainability and resiliency of cellular network infrastructure by harvesting renewable energy," *IEEE Commun. Mag.*, vol. 53, no. 4, pp. 110–116, 2015.

[2] R. Copperwhite, C. McDonagh, and S. O'Driscoll, "A camera phone-based uv-dosimeter for monitoring the solar disinfection (sodis) of water," *IEEE Sensors J.*, vol. 12, no. 5, pp. 1425–1426, 2012.

[3] Y. Ghiassi-Farrokhfal, S. Keshav, C. Rosenberg, and F. Ciucu, "Solar power shaping: An analytical approach," *IEEE Trans. Sustainable Energy*, vol. 6, no. 1, pp. 162–170, Jan 2015.

[4] K. Wang, S. Low, and C. Lin, "How stochastic network calculus concepts help green the power grid," in *Proc. IEEE Int. Conf. Smart Grid Commun. (SmartGridComm)*, Oct 2011, pp. 55–60.

[5] K. Wang, F. Ciucu, C. Lin, and S. Low, "A stochastic power network calculus for integrating renewable energy sources into the power grid," *IEEE J. Sel. Areas Commun.*, vol. 30, no. 6, pp. 1037–1048, July 2012.

[6] M. C. Gursoy, D. Qiao, and S. Velipasalar, "Analysis of energy efficiency in fading channels under qos constraints," *IEEE Trans. Wireless Commun.*, vol. 8, no. 8, pp. 4252–4263, 2009.

[7] D. Qiao, M. C. Gursoy, and S. Velipasalar, "The impact of qos constraints on the energy efficiency of fixed-rate wireless transmissions," *IEEE Trans. Wireless Commun.*, vol. 8, no. 12, pp. 5957–5969, 2009.

[8] M. A. Zafer and E. Modiano, "A calculus approach to energy-efficient data transmission with quality-of-service constraints," *IEEE/ACM Trans. Netw.*, vol. 17, no. 3, pp. 898–911, 2009.

[9] Z. Li, Y. Gao, L. Sang, and D. Yang, "Analysis on the energy consumption in stochastic wireless networks," in *IEEE ICC Workshops*, June 2014, pp. 866–870.

[10] Y. Gao and Y. Jiang, "Analysis on the battery lifespan of a mobile terminal under probabilistic delay constraint," in *Proc. IEEE e-Energy*. IEEE, 2012, pp. 1–4.

[11] Y. Jiang and Y. Liu, *Stochastic network calculus*. Springer, 2008.

[12] F. Ciucu, O. Hohlfeld, and P. Hui, "Non-asymptotic throughput and delay distributions in multi-hop wireless networks," in *Proc. Annu. Allerton Conf. Commun., Control, and Computing*, Sept 2010, pp. 662–669.

[13] F. Ciucu, "On the scaling of non-asymptotic capacity in multi-access networks with bursty traffic," in *Proc. IEEE Int. Symp. Inf. Theory (ISIT)*, July 2011, pp. 2547–2551.

[14] K. Zheng, F. Liu, L. Lei, C. Lin, and Y. Jiang, "Stochastic performance analysis of a wireless finite-state markov channel," *IEEE Trans. Wireless Commun.*, vol. 12, no. 2, pp. 782–793, February 2013.

[15] C.-S. Chang, *Performance guarantees in communication networks*. Springer Science & Business Media, 2000.

[16] Y. Jiang, "A note on applying stochastic network calculus. http://q2s.ntnu.no/ jiang/publications.html," 2010.

[17] L. Liu, P. Parag, J. Tang, W.-Y. Chen, and J.-F. Chamberland, "Resource allocation and quality of service evaluation for wireless communication systems using fluid models," *IEEE Trans. Inf. Theory*, vol. 53, no. 5, pp. 1767–1777, May 2007.

[18] J. L. Doob, *Stochastic processes*. New York Wiley, 1953, vol. 101.