Differential Transformation Method (DTM) for Solving Mathematical Modelling of Monkey Pox Virus Incorporating Quarantine

[*1]Somma S. A., [2]Akinwande N. I., [3]Abdurrahman N. O. and [4]Zhiri A. B.

[1,2,3]Department of Mathematics, Federal University of Technology, Minna

*[1]sam.abu@futminna.edu.ng

[2]ninuola.wande@futminna.edu.ng

[3]abdurrahmannurat@gmail.com

[4]a.zhiri@futminna.edu.ng

Abstract

In this paper the Mathematical Modelling of Monkey Pox Virus Incorporating Quarantine was solved semi-analytically using Differential Transformation Method (DTM). The solutions of difference cases were presented graphically. The graphical solutions gave better understanding of the dynamics of Monkey pox virus, it was shown that effective Public Enlightenment Campaign and Progression Rate of Quarantine are important parameters that will prevent and control the spread of Monkey Pox in the population.

Keywords: differential transformation method; Monkey Pox Virus; semi-analytically; solution

Introduction

Differential Transformation Method (DTM) is a semi-analytical technique for solving linear and non-linear ordinary differential equations (ODEs). Differential Transformation Method (DTM) is one of the method uses to solve linear and nonlinear differential equations. It was first proposed Zhou (1986), for solving linear and nonlinear initial value problems in electrical circuit analysis. The DTM construct a semi-analytical numerical technique that uses Taylor series for the solution of differential equations in the form of a polynomial. DTM is a very effective and powerful tool for solving different kinds of differential equations. This technique has been used to solve different kind of problem by different people such as; Arikoglu. and Ozkol, (2007), Momani *et al.* (2008) used it to solved fractional differential equations, Ayaz (2004) solved differential algebraic equations, Moustafa,(2008) solved nonlinear oscillatory system and Biazar and Eslami (2010) solved quadratic Riccati differential equation. Akinboro *et al.* (2014) obtained the numerical solution of Susceptible Infected Recovered (SIR) model, Batiha (2015) obtained the solution of prey and predator problem, Soltanalizadeh (2012), solved fourth-order parabolic partial differential equations, Odibat (2008) solved Volterra integral equations and Arikoglu and Ozkol (2006) solved difference equations. The main advantage of this method is that it can be applied directly to linear and nonlinear Ordinary Differential Equations (ODEs) without linearization, discretization or perturbation.

Monkey pox is caused by a rodent virus, which occurs mostly in West and Central Africa. The identification of monkey pox virus is based on biological characteristics and endonuclease patterns of viral DNA. In contrast to smallpox, monkeypox virus can infect rabbit skin and can be transmitted serially by intracerebral inoculation of mice. The four orthopoxviruses that may infect man produce macroscopically characteristic lesions on the inoculated chorioallantoic membrane of an embryonated chicken egg, (Jezek and Fenner, 1988).

The virus can spread from human to human by both respiratory (airborne) contact and contact with infected person's bodily fluids. Risk factors for transmission include sharing a bed, room, or using the same utensils with an infected patient. Increased transmission risk associated with factors involving introduction of virus to the oral mucosa, (Kantele, *et al.* 2016). Incubation period is 10–14 days. Prodromal symptoms include swelling of lymph nodes, muscle pain, headache, fever, prior to the emergence of the rash. The rash is usually only present on the trunk but has the capacity to spread to the palms and soles of the feet, occurring in a centrifugal distribution. The initial macular lesions exhibit a papular, then vesicular and pustular appearance, (Kantele, *et al.* 2016).

Literature Review

Che Hussin *et al.* (2010) proposed the generalization of differential transformation method to solve higher order of linear boundary value problem. They provided several numerical examples and compared the results with the exact solutions in order to show the accuracy of the proposed method.

Soltanalizadeh (2012) used the Differential Transformation method (DTM) to solved a fourth-order parabolic partial differential equations. In the end, some numerical tests are presented to demonstrate the effectiveness and efficiency of the proposed method.

Akinboro *et al.* (2014) investigated the application of differential transformation method and variational iteration method in finding the approximate solution of Epidemiology Susceptible Infected Recovered (SIR) model. Their result revealed that both methods are in complete agreement, accurate and efficient for solving systems of ODE.

Bhunu and Mushayabasa (2011) developed a mathematical modeling of pox- like infection, in their model they considered SIR model for both human and rodent/wild animals. They used Lyapunov approach to analyzed the global stability of the animal (nonhuman) endemic equilibrium.

Usman and Adamu (2017) developed a mathematical model for the dynamics of the transmission of monkeypox virus infection with control strategies of combined vaccine and treatment interventions. They used standard approaches to established two equilibria for the model namely: disease-free and endemic. They analyzed the disease-free equilibrium and endemic equilibrium. Basic reproduction numbers for the humans and the non-human primates of the model were computed using next generation matrix

This paper reviews the paper of Bhunu and Mushayabasa (2011), by incorporating quarantine class and an enlightenment campaign parameter into the human population to control the spread of the disease in the population. The model equations are solve analytically and present the solutions graphically.

Table 2.1: The Fundamental Mathematical Operations by Differential Transformation Method (DTM)

Original Function	Transformed Function
$y(t)=f(t)\pm g(t)$	$Y(k)=F(k)\pm G(k)$
$y(t)=af(t)$	$Y(k)=aF(k)$
$y(t)=\frac{df(t)}{dt}$	$Y(k)=(k+1)F(k+1)$
$y(t)=\frac{d^2f(t)}{dt^2}$	$Y(k)=(k+1)(k+2)F(k+2)$
$y(t)=\frac{d^mf(t)}{dt^m}$	$Y(k)=(k+1)(k+2)\ldots(k+m)F(k+m)$
$y(t)=1$	$Y(k)=\delta(k)$
$y(t)=t$	$Y(k)=\delta(k-1)$
$y(t)=t^m$	$Y(k)=\delta(k-m)=\begin{cases}1, & k=m\\ 0, & k\neq m\end{cases}$
$y(t)=f(t)g(t)$	$Y(k)=\sum_{m=0}^{k}G(m)f(k-m)$
$y(t)=e^{(\lambda t)}$	$Y(k)=\frac{\lambda^k}{k!}$
$y(t)=(1+t)^m$	$Y(k)=\frac{m(m-1)\ldots(m-k+1)}{k!}$

Source: Hassan (2008).

Methodology

Model Equations

$$\frac{dS_h}{dt} = \Lambda_h + (1-\varepsilon)S_h - \left(\frac{\alpha_1 I_r}{N_h} + \frac{\alpha_2 I_h}{N_h}\right)S_h - \mu_h S_h \tag{3.1}$$

$$\frac{dI_h}{dt} = \left(\frac{\alpha_1 I_r}{N_h} + \frac{\alpha_2 I_h}{N_h}\right)S_h - (\mu_h + \delta_h + \tau)I_h \tag{3.2}$$

$$\frac{dQ_h}{dt} = \tau I_h - [\mu_h + \gamma_h + (1-\theta)\delta_h]Q_h \tag{3.3}$$

$$\frac{dR_h}{dt} = \gamma_h Q_h + \varepsilon S_h - \mu_h R_h \tag{3.4}$$

$$\frac{dS_r}{dt} = \Lambda_r - \frac{\alpha_3 I_r S_r}{N_r} - \mu_r S_r \tag{3.5}$$

$$\frac{dI_r}{dt} = \frac{\alpha_3 I_r S_r}{N_r} - (\mu_r + \delta_r)I_r \tag{3.6}$$

$$\left.\begin{array}{l} 0 \le \varepsilon \le 1 \\ 0 \le \theta \le 1 \end{array}\right\} \tag{3.7}$$

$$N_h = S_h + I_h + Q_h + R_h \tag{3.8}$$

$$N_r = S_r + I_r \tag{3.9}$$

Table 3.1: Definition of Variables and Parameters

Variables/Parameters	Definition

S_h	susceptible Humans
I_h	Infected Humans
Q_h	Quarantine Infected Humans
R_h	Recovered Humans
S_r	Susceptible Rodents
I_r	Infected Rodents
Λ_h	Recruitment Rate of Humans
Λ_r	Recruitment Rate of Rodents
α_1	Contact Rate of Rodents to Humans
α_2	Contact Rate of Humans to Humans
α_3	Contact Rate of Rodents to Rodents
μ_h	Natural Death Rate of Humans
δ_h	Disease Induced Death Rate of Humans
γ_h	Recovery Rate of Humans
τ	Progression Rate from Infected to Quarantine
ε	Effectiveness Public Enlightenment Campaign
θ	Effectiveness of Quarantine and Treatment
μ_r	Natural Death Rate of Rodents
δ_r	Disease Induced Death Rate of Rodents
N_h	Total Population of Humans
N_r	Total Population of Rodents

Differential Transformation Method (DTM)

An arbitrary function $f(t)$ can be expanded in Taylor series about a point $t = 0$ as

$$f(t) = \sum_{k=0}^{\infty} \frac{t^k}{k!} \left[\frac{d^k f}{dt^k} \right]_{t=0} \tag{3.10}$$

The differential transformation of $f(t)$ is defined as

$$F(t) = \frac{1}{k!} \left[\frac{d^k f}{dt^k} \right]_{t=0} \tag{3.11}$$

Then the inverse differential transform is

$$f(t) = \sum_{k=0}^{\infty} t^k F(t) \tag{3.12}$$

In Hassan (2008) if $y(t)$ and $g(t)$ are two uncorrelated functions with t where $Y(k)$ and $G(k)$ are the transformed functions corresponding to $y(t)$ and $g(t)$ then, the fundamental mathematical operations performed by differential transform can be proof easily and are listed as follows

Analytical Solution of the Model Equations using Differential Transformation Method (DTM)

In this section we are going to apply Differential Transformation Method to the Model equation and solve.

Let the model equation be a function $h(t)$, $h(t)$ can be expanded in Taylor series about a point $t = 0$ as

$$h(t) = \sum_{k=0}^{\infty} \frac{t^k}{k!} \left[\frac{d^k h}{dt^k} \right]_{t=0} \tag{3.13}$$

Where,

$$h(t) = \{s_h(t), i_h(t),\ q_h(t),\ r_h(t), s_r(t), i_r(t)\} \tag{3.14}$$

The differential transformation of $h(t)$ is defined as

$$H(t) = \frac{1}{k!}\left[\frac{d^k h}{dt^k}\right]_{t=0} \tag{3.15}$$

Where,

$$H(t) = \{S_h(t), I_h(t), Q_h(t), R_h(t), S_r(t), I_r(t)\} \tag{3.16}$$

Then the inverse differential transform is

$$h(t) = \sum_{k=0}^{\infty} t^k H(t) \tag{3.17}$$

Using the fundamental operations of differential transformation method in table 2.1, we obtain the following recurrence relation of equation (3.1) to (3.6) as

$$S_h(k+1) = \frac{1}{k+1}\left[\Lambda_h - \frac{\alpha_1}{N_h}\sum_{m=0}^{k} S_h(m) I_r(k-m) - \frac{\alpha_2}{N_h}\sum_{m=0}^{k} S_h(m) I_h(k-m) - A_1 S_h(k)\right] \tag{3.18}$$

$$I_h(k+1) = \frac{1}{k+1}\left[\frac{\alpha_1}{N_h}\sum_{m=0}^{k} S_h(m) I_r(k-m) + \frac{\alpha_2}{N_h}\sum_{m=0}^{k} S_h(m) I_h(k-m) - A_2 I_h(k)\right] \tag{3.19}$$

$$Q_h(k+1) = \frac{1}{k+1}\left[\tau I_h(K) - A_3 Q_h(k)\right] \tag{3.20}$$

$$R_h(k+1) = \frac{1}{k+1}\left[\gamma_h Q_h(k) + \varepsilon S_h(k) - \mu_h R_h(k)\right] \tag{3.21}$$

$$S_r(k+1) = \frac{1}{k+1}\left[\Lambda_r - \frac{\alpha_3}{N_r}\sum_{m=0}^{k} S_r(m) I_r(k-m) - \mu_r S_r(k)\right] \tag{3.22}$$

$$I_r(k+1) = \frac{1}{k+1}\left[\frac{\alpha_3}{N_r}\sum_{m=0}^{k} S_r(m) I_r(k-m) - A_4 I_r(k)\right] \tag{3.23}$$

Where,

$$A_1 = [\mu_h - (1-\varepsilon)],\ A_2 = (\mu_h + \delta_h + \tau),\ A_3 = [\mu_h + \gamma_h + (1-\theta)\delta_h] \text{ and } A_4 = (\mu_r + \delta_r) \tag{3.24}$$

We consider $k = 0, 1, 2, 3$

We are considered six cases, the cases are variation of different values of effective enlightenment campaign ε and Quarantine Progression rate τ

Case 1: $\varepsilon = 0.25$

$$\left.\begin{aligned}
S_h(t) &= 160000 + 125065.8824t + 49646.1959t^2 + 14545.7717t^3 + 4433.5931t^4 \\
I_h(t) &= 500 - 242.6324t + 65.4105t^2 - 10.6703t^3 + 1.4717t^4 \\
Q_h(t) &= 700 + 136.0750t - 71.7312t^2 + 14.7932t^3 - 1.9357t^4 \\
R_h(t) &= 400 + 40100.200t + 15402.8397t^2 + 4071.9851t^3 + 897.4495t^4 \\
S_r(t) &= 5000 - 100.4000t + 256.0816t^2 + 156.4111t^3 + 120.3063t^4 \\
I_r(t) &= 200 - 53.6000t + 7.1784t^2 - 0.6337t^3 + 0.0441t^4
\end{aligned}\right\} \tag{3.25}$$

Case 2: $\varepsilon = 0.50$

$$\left.\begin{aligned}
S_h(t) &= 160000 + 85065.8823t + 24254.7254t^2 + 6278.5947t^3 + 2515.9404t^4 \\
I_h(t) &= 500 - 242.6324t + 63.6458t^2 - 10.7201t^3 + 1.4244t^4 \\
Q_h(t) &= 700 + 136.0750t - 71.7312t^2 + 14.4990t^3 - 1.9399t^4 \\
R_h(t) &= 400 + 80100.2000t + 20796.0750t^2 + 3955.6833t^3 + 773.5010t^4 \\
S_r(t) &= 5000 - 100.4000t + 256.0816t^2 + 156.4111t^3 + 120.3063t^4 \\
I_r(t) &= 200 - 53.6000t + 7.1784t^2 - 0.6337t^3 + 0.0441t^4
\end{aligned}\right\} \tag{3.26}$$

Case 3: $\varepsilon = 0.75$

$$\left.\begin{aligned}
S_h(t) &= 160000 + 45065.8823t + 8863.2548t^2 + 3036.3687t^3 + 1930.6308t^4 \\
I_h(t) &= 500 - 242.6324t + 61.8812t^2 - 10.4758t^3 + 1.3780t^4 \\
Q_h(t) &= 700 + 136.0750t - 71.7312t^2 + 14.2049t^3 - 1.8874t^4 \\
R_h(t) &= 400 + 120100.2001t + 16189.3103t^2 + 2147.4699t^3 + 563.4094t^4 \\
S_r(t) &= 5000 - 100.4000t + 256.0816t^2 + 156.4111t^3 + 120.3063t^4 \\
I_r(t) &= 200 - 53.6000t + 7.1784t^2 - 0.6337t^3 + 0.0441t^4
\end{aligned}\right\} \tag{3.27}$$

Case 4: $\tau = 0.25$

$$\left.\begin{aligned}
S_h(t) &= 160000 + 125065.8824t + 49645.6077t^2 + 14545.4655t^3 + 4433.5076t^4 \\
I_h(t) &= 500 - 117.6324t + 19.2009t^2 - 0.9992t^3 + 0.1967t^4 \\
Q_h(t) &= 700 + 11.0750t - 15.6053t^2 + 2.4467t^3 - 0.1620t^4 \\
R_h(t) &= 400 + 40100.200t + 15393.4647t^2 + 4074.7799t^3 + 896.9590t^4 \\
S_r(t) &= 5000 - 100.4000t + 256.0816t^2 + 156.4111t^3 + 120.3063t^4 \\
I_r(t) &= 200 - 53.6000t + 7.1784t^2 - 0.6337t^3 + 0.0441t^4
\end{aligned}\right\} \tag{3.28}$$

Case 5: $\tau = 0.50$

$$\left.\begin{aligned}
S_h(t) &= 160000 + 125065.8824t + 49646.1959t^2 + 14545.7717t^3 + 4433.5931t^4 \\
I_h(t) &= 500 - 242.6324t + 65.4105t^2 - 10.6703t^3 + 1.4717t^4 \\
Q_h(t) &= 700 + 136.0750t - 71.7312t^2 + 14.7932t^3 - 1.9357t^4 \\
R_h(t) &= 400 + 40100.200t + 15402.8397t^2 + 4071.9851t^3 + 897.4495t^4 \\
S_r(t) &= 5000 - 100.4000t + 256.0816t^2 + 156.4111t^3 + 120.3063t^4 \\
I_r(t) &= 200 - 53.6000t + 7.1784t^2 - 0.6337t^3 + 0.0441t^4
\end{aligned}\right\} \tag{3.29}$$

Case 6: $\tau = 0.75$

$$\left.\begin{aligned}
S_h(t) &= 160000 + 125065.8824t + 49646.1959t^2 + 14545.7717t^3 + 4433.5931t^4 \\
I_h(t) &= 500 - 367.6324t + 142.8701t^2 - 35..8982t^3 + 6.9457t^4 \\
Q_h(t) &= 700 + 261.0750t - 159.1071t^2 + 44.3491t^3 - 8.5354t^4 \\
R_h(t) &= 400 + 40100.200t + 15412.2157t^2 + 4067.6378t^3 + 898.5840t^4 \\
S_r(t) &= 5000 - 100.4000t + 256.0816t^2 + 156.4111t^3 + 120.3063t^4 \\
I_r(t) &= 200 - 53.6000t + 7.1784t^2 - 0.6337t^3 + 0.0441t^4
\end{aligned}\right\} \quad (3.30)$$

Results and Discussions

Difference variations of effective enlightenment campaign ε and progression rate from infected to quarantine τ are considered. Cases one to cases three are the variations of effective enlightenment campaign ε while cases four to cases six the variations of progression rate from infected to quarantine τ.

Figures 4.1 to 4.4 are graphical solutions of each compartment of human population and different proportions of effective enlightenment campaign ε. Figure 4.1 shows that as effective enlightenment campaign ε increases the susceptible population decreases. The more the people are being sensitize about the risk of monkeypox the less the transmission in the population. Figure 4.2 reveals that as the enlightenment campaign ε increases the infected human population decreases. It is assumed that those who have being enlightened effectively will not contact the disease and hence, moved to recovered class from susceptible class. Figure 4.3 is the graph of quarantine human population against the effective enlightenment campaign ε and it is observe that there is no effect of enlightenment campaign on quarantine population because people are already sensitized and so there no cause for outbreak in the population. Figure 4.4 shows that as enlightenment campaign ε increases the recovered human population increases.

Figures 4.5 to 4.8 are graphical solutions of each compartment of human population and different proportions of progression rate from infected to quarantine τ. Figure 4.5 shows no effect of the progression rate because it is the infected population that are quarantined. In Figure 4.6 it is observe that as the progression rate increases the infected population decreases. The more the people are quarantined the less the infected population. Figure 4.7 reveal that as progression rate increases the quarantine population increases. Figure 4.8 shows no effect of progression rate.

The solution of the model is for seven systems of ordinary differential equation while that Akinboro *et al.* (2014) is only three seven systems of ordinary differential equation. The graphical solutions makes this paper richer than theirs and also shows the usefulness of the method.

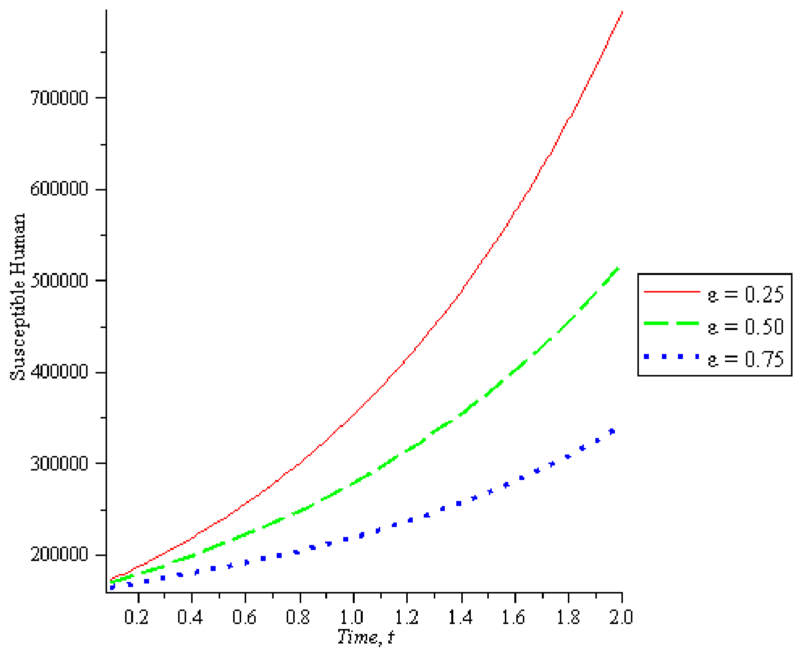

Figure 4.1: Susceptible Human against difference variation of Effective Enlightenment Campaign

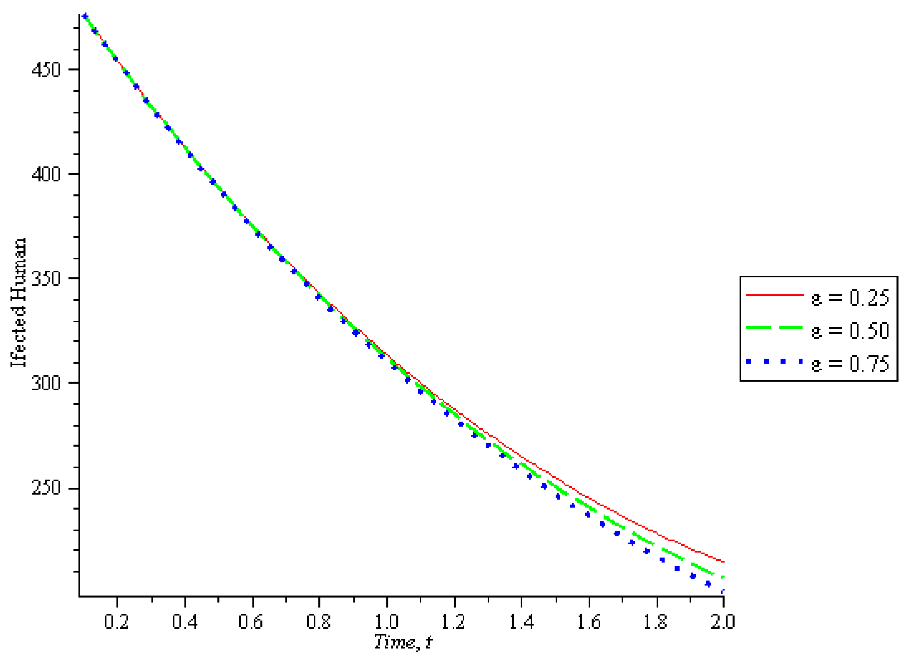

Figure 4.2: Infected Human against difference variation of Effective Enlightenment Campaign

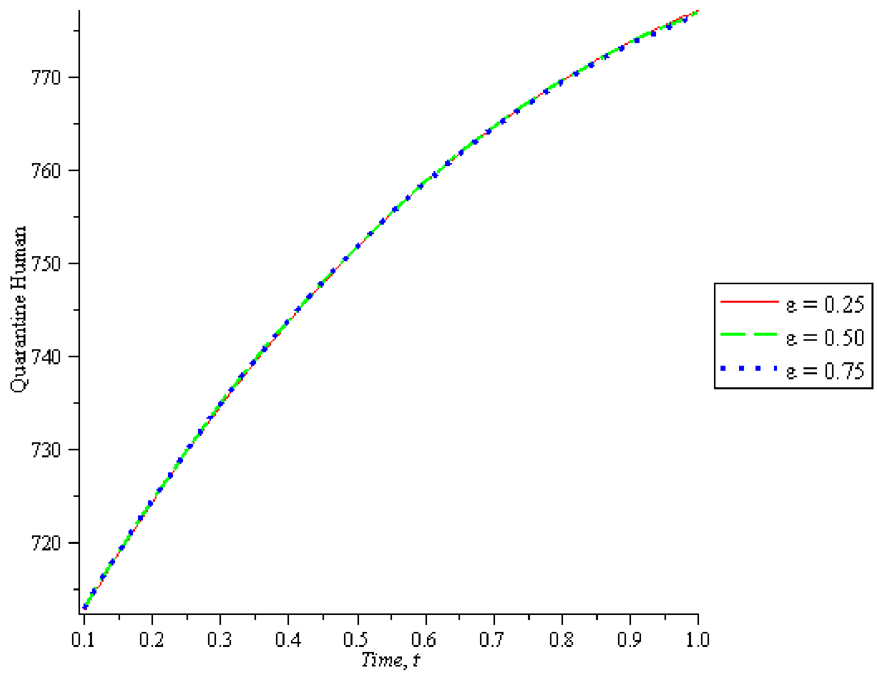

Figure 4.3: Quarantine Human against difference variation of Effective Enlightenment Campaign

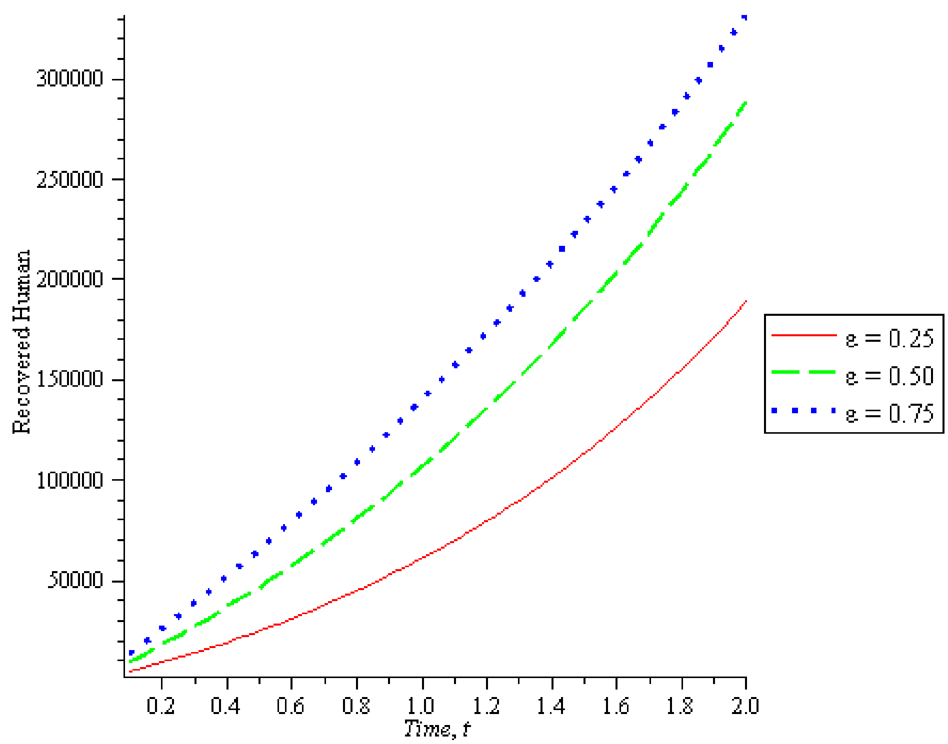

Figure 4.4: Recovered Human against difference variation of Effective Enlightenment Campaign

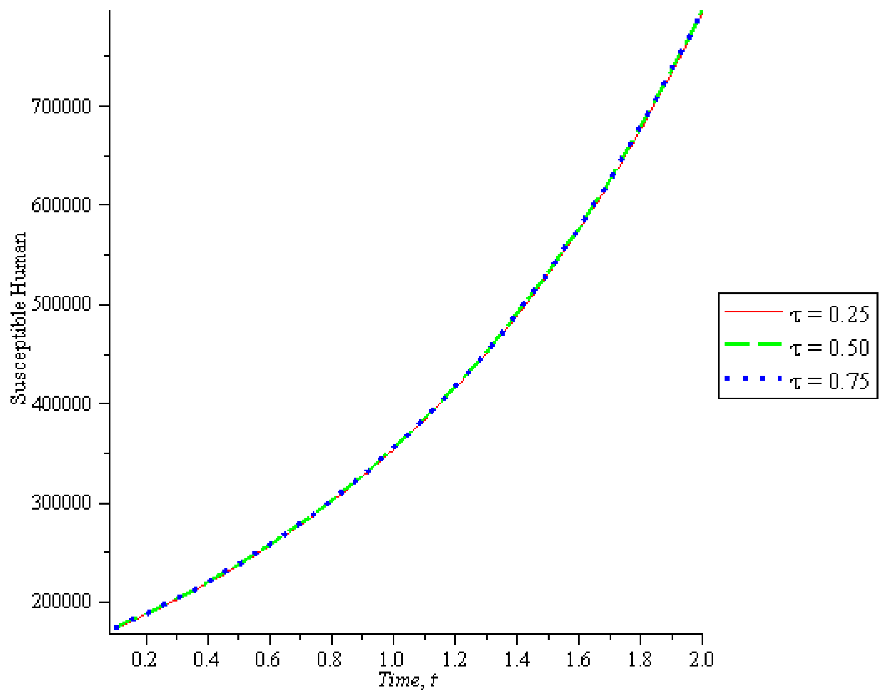

Figure 4.5: Susceptible Human against difference variation of Quarantine Progression Rate

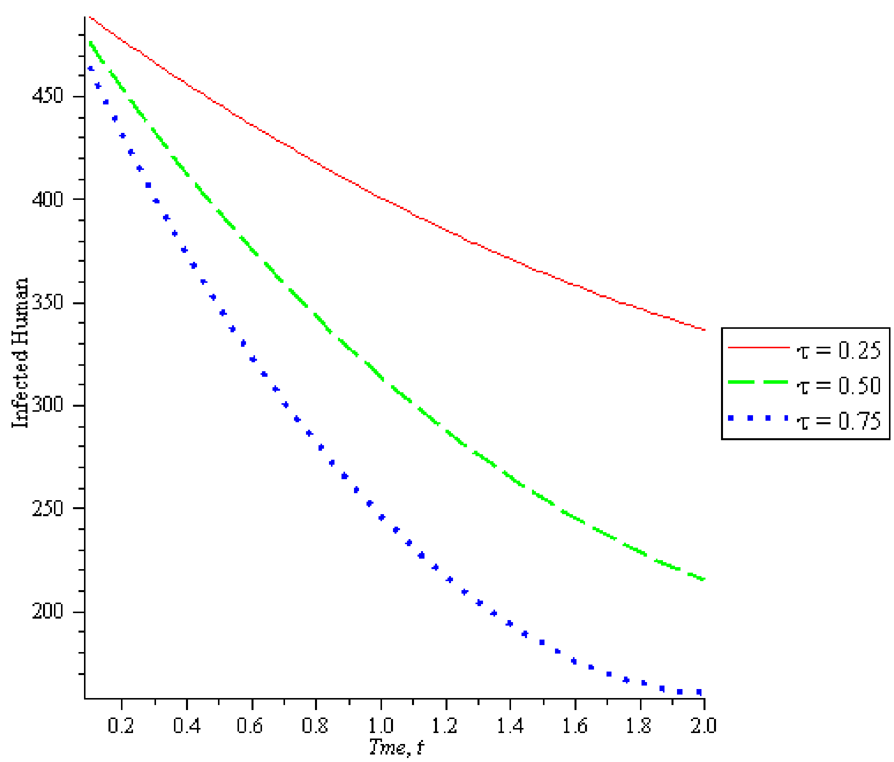

Figure 4.6: Infected Human against difference variation of Quarantine Progression Rate

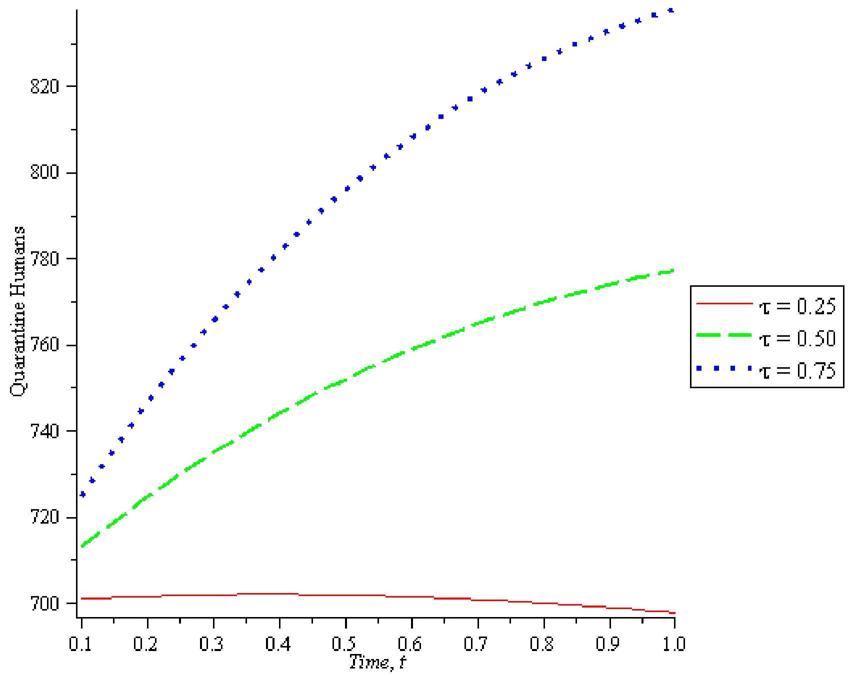

Figure 4.7: Quarantine Human against difference variation of Quarantine Progression Rate

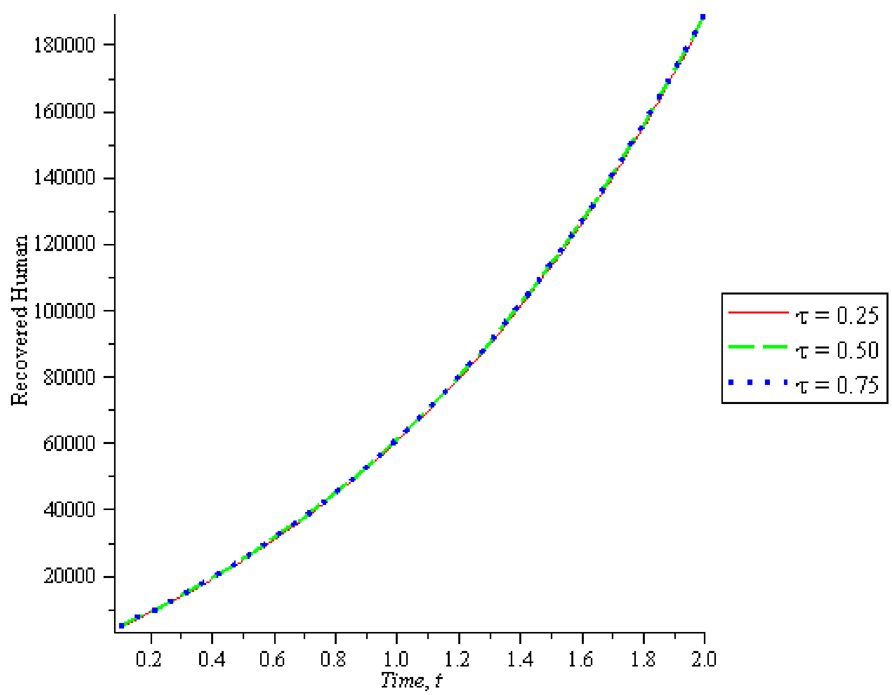

Figure 4.8: Recovered Human against difference variation of Quarantine Progression Rate

Conclusion

The Differential Transformation Method (DTM) is good method for solving non – linear differential equations. The solutions were presented graphically and it makes the understanding of the dynamics richer. The graphical solutions also show the parameters of the model that are important in eradicating monkey pox in the population. More effort should be made by relevant organizations to enlighten the public about the danger of the disease and how to go about the infected persons and also make effort to isolate the infected people from the others (Susceptible individuals).

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